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PROCEEDINGS

OF THE

ROYAL SOCIETY OF EDINBURGH

PROCEEDINGS

OF

THE ROYAL SOCIETY

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OF THE

ROYAL SOCIETY OF EDINBURGH.

VOL. XXII. 1897–98.

THE 115TH SESSION.

GENERAL STATUTORY MEETING.

Monday, 22nd November 1897.

The following Council were elected :—

President.

THE RIGHT HON. LORD KELVIN, G.C.V.O., LL.D., D.C.L., F.R.S.

Vice-Presidents.

The Hon. Lord M'LAREN, LL.D.
The Rev. Professor FLINT, D.D.
Professor JOHN G. M'KENDRICK, M.D., LL.D., F.R.S.
Professor GEORGE CHRYSTAL, LL.D.
Sir ARTHUR MITCHELL, K.C.B. LL.D.
Sir W. TURNER, M.B., F.R.S.

General Secretary—Professor P. G. TAIT.

Secretaries to Ordinary Meetings.
Professor CRUM BROWN, F.R.S.
JOHN MURRAY, Esq., D.Sc., LL.D.

Treasurer—PHILIP R. D. MACLAGAN, Esq., F.F.A.

Curator of Library and Museum—ALEXANDER BUCHAN, Esq., M.A., LL.D.

Ordinary Members of Council.

Sir STAIR AGNEW, K.C.B.
Dr JAMES BURGESS, C.I.E., M.R.A.S.
JOHN S. MACKAY, Esq., LL.D.
Professor COPELAND, Astronomer-Royal for Scotland.
Professor D'ARCY W. THOMPSON, C.B.
The Rev. Professor DUNS.
Lieut.-Col. BAILEY, R.E.
Professor JAMES GEIKIE, LL.D., F.R.S.
Sir J. BATTY TUKE, M.D.
A. BEATSON BELL, Esq., Advocate.
Professor SHIELD NICHOLSON, M.A., D.Sc.
Professor JOHN GIBSON, Ph.D.

Honorary Representative on George Heriot's Trust—JOHN MURRAY, Esq., D.Sc., LL.D.

By a Resolution of the Society (19th January 1880), the following Hon. Vice-Presidents, having filled the office of President, are also Members of the Council :—

HIS GRACE THE DUKE OF ARGYLL, K.G., K.T., LL.D., D.C.L.
SIR DOUGLAS MACLAGAN, M.D., LL.D., F.R.C.P.E.

THE RIGHT HON. LORD KELVIN, President, in the Chair.

Chairman's Opening Address.

(Read December 6, 1897.)

The PRESIDENT, on opening the Session, stated that—

At the statutory meeting, held on 23rd November 1896, the President was appointed (with power to name a substitute) to represent the Society at the British Association Commission on a National Physical Laboratory.

At a meeting of Council on 7th May last, the President and Dr Murray were appointed members of the Consultation Committee in connection with the proposed Catalogue of Scientific Literature.

At a meeting on 21st May 1897, Dr Murray, Dr Traquair, and Mr Cadell of Grange were appointed to represent the Society at the International Russian Congress at St Petersburg, and Principal Talmage was subsequently added to the above-mentioned representatives.

Professor D'Arcy Thompson, a member of our Council, was reappointed a Commissioner to report on the aspects of the Seal Fishery in Behring's Straits, and on its international relations.

In consequence of a letter from Mr Wragge, requesting the Council to express an opinion as to the establishment of High and Low Level Meteorological Stations on and near Mount Wellington, New Zealand, in view of the establishment by the local authorities there of such Observatories, a Committee of our Council was appointed to consider the subject, and reported on the great importance of obtaining a fuller knowledge of the meteorology of the countries bordering on the Southern Ocean, and on the deficiencies that at present exist as regards Low Level Meteorological Stations to the south of the Argentine Republic, to the north of Cape Colony, and in the Islands of the Southern Ocean south of latitude 30°, to which may be added the observations that might be obtained through antarctic expeditions; and that these deficiencies are still more conspicuous in regard to High Level Observatories and contiguous Low Level Observatories, good positions for which might be obtained in Australia, New Zealand, and Tasmania; and the Committee further thought that as the atmospheric pressure was singularly low and uniform over the Southern Hemisphere, where a close and intimate connection is shown between low pressure and cyclones with the attendant anticyclones, that this is a part of the world where problems of weather can be most successfully investigated.

Professor Playfair states in his Life of Hutton (published in vol. v. of our *Transactions*) that the two volumes on the Theory of the Earth, which that great geologist gave to the world in 1795, "do not complete the Theory, a third, necessary for that purpose, remained behind, and is still in manuscript" (January 1803). It was at first supposed that as Hutton's collection of minerals had been presented to the Society, this unpublished MS. volume might also have been presented, and might be in the archives of the Society, but the most exhaustive search led to no indication of its ever having been in our possession. The last trace of it that could be found is in Note C. to a paper on Glen Tilt, read to our Society on 16th May 1814, in which Lord Webb Seymour, one of the authors of it, states that Professor Playfair had promised to give him a sight of the manuscript of the third volume of Hutton's Theory of the Earth which "was left unpublished at the time of Hutton's death, and is nearly ready for the press."

After the failure of our search, we were agreeably surprised to learn from Sir Archibald Geikie (in a letter dated 16th June 1897) that he had discovered a portion of the MS. volume in the possession of the Geological Society of London; that portion having been acquired by Leonard Horner, and presented by him to the Geological Society's Library. Unfortunately it is only a small part of the whole volume, beginning with Chapter IV., and ending with Chapter IX.

The Astronomer-Royal for Scotland is about to proceed to India, with adequate instruments, to observe the Eclipse, which will take place on the 22nd of January next, and the Society will be much interested in the results obtained by such a competent observer.

During the last Session twenty-one Fellows have been elected, while nine Ordinary and three Honorary Fellows have been t from us by death.

The deceased are :—*Ordinary Fellows.*—The Rev. John Wi Admiral Sir Alexander Milne, Professor Henry Drumm Edmond Chisholm Batten of Aigas, James Greig Smith, M Professor Matthew Charteris, George Elder, Knock Castle, Th Brumby Johnston, Rev. Dean Montgomery.

Honorary Fellows.—James Joseph Sylvester, Emile Du Reymond, Joannes Iapetus Smith Steenstrup.

It is usual, on the opening of a new Session, to say a few w respecting each of the Members of the Society who have during the preceding Session, and I now proceed to discharge duty; hoping, however, that fuller and more adequate obitu may be communicated by the pious solicitude of friends relatives for the memory of the departed, which the short n I now give are not intended to supersede.

Of Mr Edmond Chisholm Batten an obituary notice has prepared by his son, Major Chisholm Batten, and will be re an early meeting.

Professor Matthew Charteris was born in the villa Wamphray, in Dumfriesshire. His father was parish schoolm there, and it was from his father that he received his ea education. From the parish school he proceeded to the versity of Edinburgh, where, in his Arts course, he distingui himself by the composition of Latin verses. Applying himse the study of Medicine, he graduated as M.D. in 1863. leaving college he practised successively in London, Airdrie, Glasgow. He was appointed Physician to the Glasgow I Infirmary, and in 1876 he obtained the Chair of Practic Medicine in Anderson's College, Glasgow. In 1880 he appointed to the Chair of Materia Medica and Therapeuti the University of that city. He was well known to the profe for his researches into salicylic acid and chloroform. He wa author of a work entitled *The Practice of Medicine*, and also handbook on *Health Resorts*. He wrote the paper on "The ment of Diseases by Climate," in Quain's *Dictionary of Med* He died on 7th June 1897. He was elected a Fellow of Society in 1896.

Professor Henry Drummond was born in 1851 in Stirling. He was educated there and in Crieff. Thence he proceeded to Edinburgh University, and was medallist in the Geological Class under Professor Archibald Geikie. He afterwards went to the New College, Edinburgh, where he qualified for the ministry of the Free Church, and he also studied at Tübingen, in Germany. He left College in 1876, and in the following year was appointed to the Lectureship on Natural Science in Glasgow Free Church College. In 1879 he accompanied Professor Geikie upon a scientific expedition to the Rocky Mountains and Colorado. Three years later he was sent out to East Central Africa, for the purpose of reporting upon the natural resources of Nyassaland. Just before going upon this journey he published his *Natural Law in the Spiritual World.* This book had a large sale. In 1884, the Lectureship in the Free Church College, Glasgow, having been endowed, he became its first occupant. Some time after he published his *Tropical Africa,* a record of his travels in the dark continent. During the spring of 1890 he went out to Australia, and from there he crossed to the South Sea Islands, and visited Java, the Malay Peninsula, and Japan, returning to Scotland through North America. Among the works he published were, *The Greatest Thing in the World, The Programme of Christianity, Pax Vobiscum.* In 1893 he went to Boston, U.S.A., to deliver the Lowell Institute Lectures, and in the following summer he lectured at several American universities. These lectures were published in the spring of 1894 under the title of *The Descent of Man.* In the autumn of 1894 he was seized with persistent rheumatic pains and catarrh of the stomach. He repaired, on medical advice, to the South of France; but after residing there for several months, he returned to England, taking up his residence at Tunbridge Wells, where he died on 11th March 1897. He was elected a Fellow of this Society in 1880, and contributed a paper to its *Proceedings* on "The Termite as the Tropical Analogue of the Earthworm."

Mr George Elder was born at Kirkcaldy in November 1816, and was educated at the High School there. As a very young man he was attracted by the enterprising life of a colonist, and in the early days of the Australian colonies he had serious thoughts of becoming a pioneer settler there. Circumstances, however, vented the fulfilment of his wish at this time; but after a ye two spent in Canada, Mr Elder, who in the meantime had ma Miss Jean Balfour, was able to accomplish his early intention, sailed to Australia with his wife, where he joined the bus already established in Adelaide, South Australia, by his brother, Mr Alexander Lang Elder, with whom in those day associated the second brother, Mr William Elder. After five of great commercial success he returned to his native land, on the shores of the Clyde lived a secluded and thoughtful He was elected a Fellow of this Society in 1869, and di Knock Castle, Largs, on 22nd July 1897.—(*Communicate Mr Frederick Elder*, 21 *Cleveland Gardens*, *Hyde Park*, *Lon*

Professor MATTHEW FOSTER HEDDLE was a native of Or He was appointed to the Chair of Chemistry in St Andrews versity in 1862, on the death of Professor Connell, whose ass he was, and held the professorship till 1884, when he retired. was widely known as a distinguished mineralogist, and was el a Fellow of this Society in 1876, and was awarded the Prize. He had been President of the Scottish Geological So and was a member of other learned bodies. He died on November 1897, about 70 years of age.

Mr THOMAS BRUMBY JOHNSTON was the son of Andrew ston, Penicuik, and Isobel, daughter of Archibald Keith, own Polton Paper Mill. He was born in Perth on the 28th Jan 1814, and after being educated at several private schools in burgh, joined his brothers—the late Sir William Johnston Alexander Keith Johnston, LL.D.—in the firm of W. & Johnston. In 1843 he married Jane, daughter of Thomas R man, and last, it is believed, of the family of that emi philologist and grammarian. In 1872 he published his *Histo Geography of the Clans of Scotland*, and was also the auth several other geographical works. He was a member of Scottish Society of Antiquaries, who, in 1872, presented him a handsome piece of plate as a token of their grateful estimati his services to that Society. He was also a member of the graphical Societies of Edinburgh and London, and was elec Fellow of this Society in 1867. He died on 2nd September.

Admiral Sir ALEXANDER MILNE, Bart., G.C.B., was born at Inveresk in 1806. His father was Sir David Milne, G.C.B. Sir Alexander received his education successively at Musselburgh, Portsmouth, Bordeaux, Halifax, and Bermuda, his places of schooling being in a great measure determined by the situation of the commands held by his father. He was subsequently entered as a pupil at the Edinburgh Royal High School, and thereafter, in 1817, he entered the navy as a volunteer and midshipman on the "Leander," which carried his father's flag. In 1825 he was lieutenant, in 1830 commander; and while in command of the "Snake," in 1837–38, he captured four slavers off the coast of Cuba, setting free 952 slaves. He was acting captain in 1839, and in 1842 became flag captain to his father. In 1847, while commanding Nelson's ship "Victory," Captain Milne was installed a Junior Naval Lord of the Admiralty, and was nominated Superintending Lord of the Store, Victualling, Transport, and Medical Departments,—in all of which he introduced great reforms. He was at the Admiralty for upwards of eleven years. In recognition of his work he received in 1855 a good service pension, was gazetted a G.C.B. after the Crimean War, and was promoted to rear-admiral's rank in 1858. In 1859 he quitted Whitehall to take the command of the North American and West Indian Stations. He was appointed Vice-Admiral in 1865, Senior Naval Lord of the Admiralty in 1866, Commander-in-Chief of the Mediterranean Station in 1868, and Admiral in 1870. He was again invited to become a Lord of the Admiralty in 1872, a position which he resigned in 1876. He was created a Baronet in 1875, and became Admiral of the Fleet in 1881.

In dealing with the grant of £30,000 a year which had been made by Parliament for the benefit of petty officers and seamen, his plan for the introduction of good conduct badges and pay was adopted. Previously there was nothing to mark the good from the bad or indifferent men. At his instance the first Signal Book was drawn up by Captain Wilmot, and he established the first Cipher Secret Signal Book for communication between the Commander-in-Chief and the Admiralty. He was the means of bringing under the notice of the authorities at Whitehall the defenceless state of Jamaica, Antigua, Bermuda, and Barbadoes, and of securing the strengthening of their defences.. He was one of the Commissioners of the 1851 Exhibition, of the British Colonial Department of the 1878 Paris Exhibition, was Vice-President of the Geographical Societies of London Edinburgh, and Fellow of the Royal Society. He was electe Fellow of this Society in 1833. On the death of Lady Miln gave £500 to establish what is called the Lady Milne Mem Fund, to perpetuate the charities which she had been in the h of giving. He died on the 29th December 1896.

James Francis Montgomery was born in Edinburgh in 1 and was the son of Mr Robert Montgomery, who for some y held the office of Lord Treasurer's Remembrancer. His grandfa was Sir James Montgomery, Bart., of Stobo Castle, Peebless Lord Chief Baron of the Exchequer. He was educated thr private tuition, and entered the law classes at Edinburgh Unive In 1840 he was called to the Bar. The bent of his mind lay in the direction of the ministry, and with the view of quali himself for that vocation he studied at the University of Dur He was appointed to the curacy of Puddleton, Dorsetshire, and spending two years there he returned to Edinburgh as curate to B Terrot, who then held the incumbency of St Paul's, York Plac 1864 Mr Montgomery was chosen as junior incumbent, and o death of Bishop Terrot in 1872 Mr Montgomery was promot the incumbency. Bishop Cotterell appointed him to the off dean, rendered vacant by the death of Dean Ramsay. But fo infirmity of partial deafness there can be little doubt he would been selected for even higher promotion. The fine peal of placed in the tower of St Mary's Cathedral was his gift. On occasion of his being promoted to the deanery, his *alma m* conferred on him the degree of D.D. He was occupied wit sorts of philanthropic work, and his tall and commanding pres will be greatly missed in his native city. He was elected a Fe of this Society in 1868. He died on 21st September 1897.

Professor James Greig Smith was born in 1854, near Aberd and was sent to the Grammar School and the University of that In 1876 he obtained the degrees of Bachelorship of Medicine Mastership of Surgery. In that year he was elected to the po Assistant or Junior House Surgeon at the Bristol Royal Infirm and in 1879 was placed on the staff as full surgeon. Some time this appointment he published papers on the Pathology and T ment of Chronic Osteoarthritis and the Growth of Spicular Osteophytes. From 1883 to 1890 he edited the *Bristol Medico-Chirurgical Journal.* In 1888 he began to lecture on Surgery. His best known work, entitled *Abdominal Surgery,* has gone through six editions, and has been translated into French, German, and Italian. He died on 29th May 1897. He was elected a Fellow of this Society in 1883.

Rev. JOHN WILSON.—A large circle of the Fellows of this Society, more especially those interested in mathematical science, will retain an affectionate remembrance of the late Rev. John Wilson, M.A. I am glad to say that a notice of him has been prepared by Dr Knott, and will be read at next meeting.

EMILE DUBOIS-REYMOND was born in Berlin, of French-Swiss parents, and studied first at the French Gymnasium and afterwards at the University of Berlin. Under the tuition of Johannes Müller he devoted himself to the investigation of the phenomena of animal magnetism, and his researches led him to discomfit the vitalist school of physiologists, who affirmed the existence of a general magnetic vital fluid. With Helmholtz and Mayer, he showed that biological phenomena are governed by physical and chemical laws. His famous 'ignoravimus,' as opposed to the 'ignoramus' of the Agnostics, bore witness to his readiness to dogmatise wherever he felt certain of his conclusions. Besides conducting successfully scientific investigations, he acquired great renown as a populariser of science. He was elected an Honorary Fellow of this Society in 1892, and died on 26th December 1896, at the age of 78.

Professor JOANNES IAPETUS SMITH STEENSTRUP of Copenhagen, after having acted as Lecturer on Mineralogy at Soröe, was appointed in 1845 Professor of Zoology and Director of the Zoological Museum at Copenhagen. He retired from his professorial office in 1885. He was the author of a number of scientific publications, and was elected a Fellow of this Society in 1881. He died on the 20th of June 1897, at the age of 84.

SYLVESTER (JAMES JOSEPH), the youngest son of Abraham Joseph Sylvester, was born in London on 3rd September 1814, and from the Royal Institution, Liverpool, went to St John's College, Cambridge. He was Second Wrangler in 1837. As a

Jew he could not take his degree, nor compete for the Sm Prize, still less obtain a Fellowship. He was called to the B 1850, but he mainly devoted himself to teaching. He was fessor of Natural Philosophy at University College, London, 1837 to 1844, then Professor of Mathematics at the Universi Virginia. Returning to England, he was made, in 1855, Prof at the Royal Military Academy, Woolwich. Fifteen years he retired, and in 1877 he was appointed the first Profess Mathematics in the newly founded Johns Hopkins Universi Baltimore, and was editor of the *American Journal of M matics.* In 1883 he was elected Savilian Professor of Geometry in Oxford. Elected a Fellow of the Royal Sc in 1839, he received a Royal Medal in 1860, and the C Medal in 1880. It is unnecessary to mention the various fc societies of which he was an Associate. He was elected a F of this Society in 1874. His contributions to mathematical s are thus described by Professor Cayley:—"They relate chie finite analysis, and cover by their subjects a large part of algebra, determinants, elimination, the theory of equations, tions, tactic, the theory of forms, matrices, reciprocants, the I tonian numbers, etc." He died on 15th March 1897.

A Short Note on the Disturbance of the Magnetical and Meteorological Instruments at the Colaba Observatory during the Earthquake of 12th June 1897.

By **N. A. Moos**, Director of the Observatory.

(Read December 6, 1897.)

The four instruments, the traces of which are examined, are the declination, horizontal force, and vertical force magnetographs, and the barograph. The instruments being not seismological, the following discussion of the character of the disturbance, based upon the traces of these instruments, must obviously be regarded as an attempt at explanation, and as an attempt only. Colaba, it appears, was situated well outside the area of perceptible shaking, and the distance of the Observatory from the centre of disturbance, whether suboceanic or otherwise, was sufficiently large to render the disturbance as it reached Colaba very feeble, and yet it was just sufficiently strong to leave some record of its peculiar characteristics. All traces have been enlarged about two and a half times, but the exact time and measurements have been derived from the original traces.

It must be noted here that the disturbance fortunately occurred just after 16 hours, the time of hourly eye observations. A valuable opportunity therefore was secured, and it became possible to note and study the peculiar behaviour of the instruments during their disturbances, some of which, by the rapidity of their movements, could not have been photographed. There may be some doubt as to whether the disturbance in the magnetographs was due to mechanical or magnetic action, but no such doubt can exist for the barograph. The disturbance of this instrument must have been caused by distortional waves, or due to tilting of the instrument. It will be noticed that the maximum effect of the disturbance, as photographed in the barograph trace, followed the largest wave shown by declination and vertical force magnetographs by about one minute.

With regard to the disturbed trace of the declination mag graph, on the other hand, it is difficult to conceive how cond tion or distortional waves could affect the suspended ma under an exhausted receiver, so as to set up large vibrations simulating magnetic action. The magnet was disturbed mec cally, no doubt, by the seismic waves, and these disturbances typical of *mechanical* action. The three motions (beside vibratory motion) observed in the declinometer at the ti disturbance showed (1) motion of the whole magnet parall itself (east and west); (2) motion of the magnet as a whole and south; and (3) the slight bobbing motion of the ends o magnet up and down. But neither of these motions would increase of the scale reading, nor would vibratory motion, if s by mechanical causes, *permanently* show increase or decrea declination. It is true that violent motion of any one of the referred to may so disturb the magnet as to bring one end o other of the magnet in slight contact with the damper, which result in vibrations; but the intensity of the mechanical di ance was feeble, and the motion *observed* at the time of the turbance showed no evidence of any great, much less vio movements. Finally, the peculiar character of the trace of instrument, as will be seen later on, leaves no reasonable d that the disturbance was due to magnetic action which have accompanied the seismic disturbance, possibly as effec a cause.

The case of the horizontal force magnetograph is, howe different. From the peculiar nature of the bifilar suspensio becomes obvious that the tension of the wires being a prin factor in the general formula for the condition of equilibriu must be affected by any sudden displacement of the point suspension. A distortional wave, for instance, suddenly li the points of suspension, would result in increasing the tension, any sudden depression would for a moment reduce the ten A lateral movement, also, is likely to temporarily alter the ditions of equilibrium, and this magnetograph therefore appea be disturbed by the combined action of both the mechanical magnetical disturbances—in this instance more, perhaps, by the latter than by the former, which was feeble, as stated above. The trace shows some evidence of being affected by two causes superimposed upon each other, specially at the end of the disturbance, where the blurred trace, probably due to change in the focal distance caused by a slight displacement of the mirror, appears suddenly to pass from a faint to a somewhat deep impression, bounded by *convex* curves.

Movement in the vertical force magnetograph now remains to be inquired into. This instrument is under an exhausted receiver, but its knife-edge is somewhat faulty, and this magnetograph is therefore peculiarly sensitive to shocks—an accidental fall of a lamp-chimney, heavy tread of visitors, and even a slight knock of the hand, results in *dislocation* of the curve. And such dislocations being frequent, special care is always taken to guard the instrument from such accidents.

And yet no dislocation of the curve is noticed during the earthquake, which more or less establishes the fact that the movement of the earth must have been very feeble. But the instrument does show vibrations, which naturally leads to the inference that they must have been caused by some magnetic action, the first vibration of which is timed to have taken place exactly when the declination trace is just lost by the second wave. And it would thus appear as if the seismic convulsion was in some way the cause of the magnetic action, the latter phenomenon running parallel to the former, increasing as it increased, and subsiding as it subsided, every seismic wave having its companion effect in a magnetic wave..

With regard to the direction, since the barograph would not show any indication of direction, nor the magnetographs, all of which appear to have been disturbed by the magnetic action, it is difficult to come to any definite conclusion. Except, perhaps, in the case of declination, the suggestive explanation given later on, if true, would point, at least, that the direction was inclined more to 'east to west' than to 'north to south.' And from the mechanical disturbance of the magnets of the two instruments, declination and horizontal force, which are suspended at right angles to each other, the more pronounced motion *observed during* *the disturbance* in both was from east to west. The focal dist of the latter instrument also appears to be slightly affected, dı motion of the mirror east and west; it strengthens, therefore, presumption that the direction of the waves must have had a st easterly component. And the comparative feeble motion of parallel to north and south direction, shows that the north component was present but was feeble. We shall now exa each trace separately.

The photograph shown here (fig. 1) is a copy (enlarged about

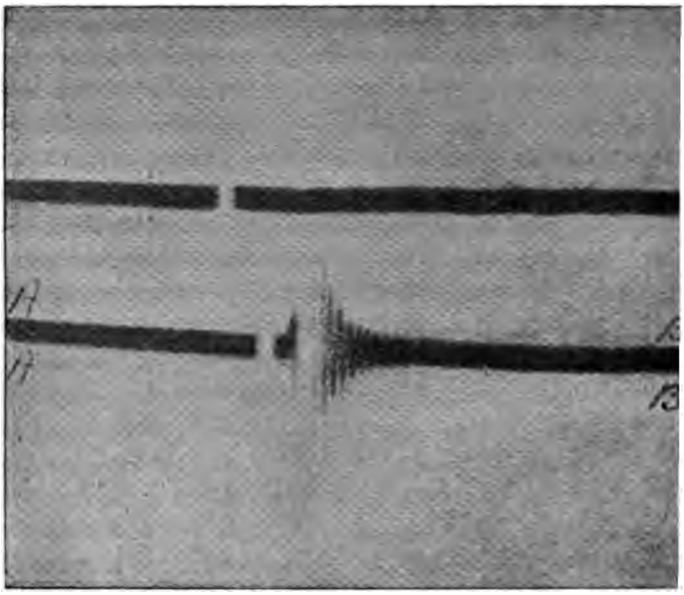

Fig. 1.

times) of the trace obtained from the declination magnetogra at the Colaba Observatory. The time of vibration of the magn is 5·33 seconds, and the original trace allows of an accura up to one minute in the determination of time. AB shows part of the trace of the 12th June. The regular break seen the trace is due to a metallic fan which automatically cuts off light every two hours for four minutes and a quarter. The middle of this break, therefore, represents 16 hours on the 12th June. The usual sensitiveness of the photographic paper used at this Observatory has been ascertained by experiments with a steady light, and found that an exposure of about four seconds to the usual kerosene burners (of about four candle-powers intensity) used, is enough for action; and with regard to impression of light during the vibration of the magnet, it is found that oscillations of amplitude of 3½ division of the scale, equal to a displacement of about 28 minutes of declination and upwards, are not recorded; the velocity of the speck of light, together with the motion of the paper, a little over half-an-inch in one hour, precludes photographic action, unless the amplitudes are brought within the above limit; and naturally the first impressions produced are at the extreme ends of the amplitude, where the velocity is a minimum. It is only when the vibrations fall to within about 1½ divisions of scale that the paper shows the impression of light in the middle of the curve—that is, where the velocity is a maximum.

It is assumed in what follows that every seismic wave was accompanied by a temporary disturbance of the magnetic circulation. The first wave, which appears to have just commenced the disturbance *in the instrument*, must have passed Colaba at about 5 minutes past four. The seismic disturbance then seems to have grown in intensity, and about 7 minutes past four the increased amplitudes (which, by the way, it must be noted, were first performed about a higher zero showing a decreased easterly declination, movement of the curve in the downward direction showing an increase) passed the limit above referred to, and the trace is entirely lost. Immediately after this, however, the impulsive force appears to have ceased, and the damper of the magnetograph reducing the amplitudes [the logarithmic decrement has been ascertained to be = ·035] brings them within the limit of photographic action, and the trace is just photographed at about nine minutes. This marks the time of temporary lull, for immediately afterwards the trace is lost, to appear faintly again, but more strongly than before, at about eleven minutes. This goes on; but the magnetic action now becomes less and less intense (presumably, therefore, the cause of the disturbance the seismic waves also); and the amplitudes of the vibrations falling within the limit of photographic action, the record runs beautifully clear after this, and every wave is registered—following its predecessor with more or less rhythmic precision, the intensity of the disturbance getting less and less, till at about 34 minutes past the hour the disturbed trace runs into normal curve once more.

However, one singular feature in the trace, which is clearly visible in the latter part of the disturbed curve, must also be explained, viz., the running of the vibration lines into each other by a zigzag kind of trace. The most plausible explanation appears to be the following.

Assuming the normal curve to be the zero for the time, ordinary vibrations caused by a momentary disturbance would take place above and below this zero with *equal amplitudes*. But from the nature of the appearance of the zigzag curve, it seems as if the advancing seismic wave of condensation caused a decrement in the declination, and compelled vibrations about a displaced zero *higher* up the curve; while during the receding wave, as it passed away, the reverse and opposite effect followed, and the declination increasing, the vibrations were caused about a lowered zero, each phenomenon, of course, having its maximum and minimum effect. Fourteen such waves in all can be detected on the trace, the second or third of which—more possibly the second—was the most intense, and each complete wave was followed by another after an average interval of about two minutes,—the maximum effect of the advancing and the retreating parts of the wave being recorded at the average interval of about one minute.

Shock.	Maximum Effect showing Decreased Declination.	Maximum Effect showing Increased Declination.	Disturbance.
1	16 h. 6′	16 h. 7′	Commenced at 16 h. 5′
2	8·5	9·5	
3	10·5	11·5	
4	12·5	13·5	
5	15	16	
6	17	18	
7	19·5	20·5	
8	21·5	22·5	
9	24	25	
10	26	27	
11	28	29	
12	30	31	
13	31·5	32·5	Ended at 16 h. 34′
14	33·5	...	

It will be seen that the beginning of the seismic disturbance, as recorded by this instrument, shows that it was somewhat sudden and abrupt, and the rise to the maximum after the commencement was within three minutes to four minutes, while the falling off, as shown by the clear trace, appears to have taken place gradually and steadily till normal conditions were reached. From the very regular trace it appears that the time of vibration of the magnet must bear a certain relation to the period of disturbance, the exact time of vibration being some even sub-multiple of the period, otherwise the regularity of the vibrations would be disturbed by '*interference.*'

The *average* displacement of the declination east or west during the fourth wave, which is well defined, would equal that due to a change of ·00034 C.G.S. units, in terms of force.

If by producing the curves on each side we get the *probable* amplitude of the most intense wave where the two lines meet, the *average* displacement appears to be ·15 inch, equal to ·00051 C.G.S. units, in terms of force; and this shows that the second wave was the most intense.

The magnetic disturbance of the kind registered on the trace appears suggestive of some kind of fluid action or of some kind of undulatory action. The phenomenon appears as if due to an agitation of a fluid endowed with magnetic properties in some large subterranean cavity, upon which the indications of the Colaba magnets at least partly depended, and as if the seismic convulsions produced waves in this fluid which, approaching the instruments at Colaba and passing away from them, caused the alternate fall and rise of declination, these waves following each other regularly at an interval of two minutes. And any such wave must travel east to west at right angles to the declination magnet to affect it with maximum effect—passage of such waves due north to south having no influence on the declination.

The trace here (fig. 2) shows the disturbance in the horizontal intensity, which appears to have commenced early, about 2½ minutes after four, while the instrument shows that the disturbance did not end before 36 minutes past four. The sudden displacement of the curve at the beginning shows that the vibrations must have been performed about a displaced zero above the curve, showing increased

intensity to about ·00004 C.G.S. units, and the action appe
therefore more magnetic than mechanical. At 5½ minutes p
four the vibrations pass the limit of photographic action, and t
perhaps marks the time of commencement of the more intense os
lations. The trace appears faintly at about 19½ minutes past t
hour, less faintly at the extreme ends of the amplitudes than
the middle of the curve, and it continues faint till about 26 minu
past, when the trace darkens, but continues disturbed till
minutes past four, when the normal curve is resumed. The ti

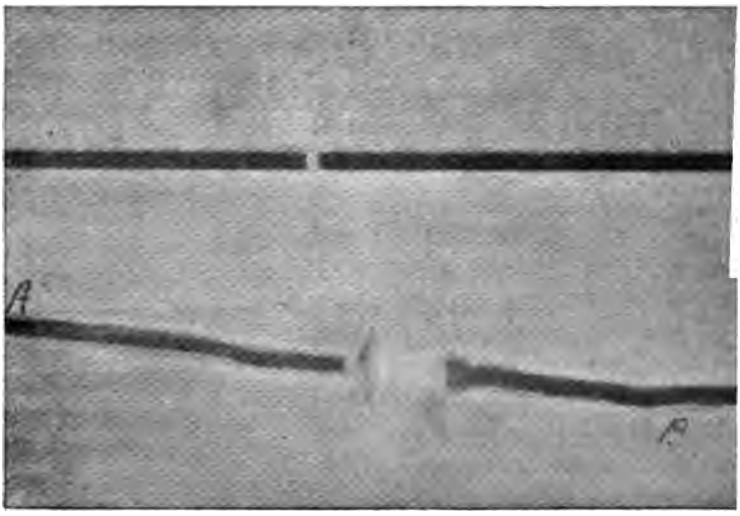

Fig. 2.

of vibration of the magnet has been ascertained to be 8 secon
and obviously, as this factor, together with the period of c
turbance, must influence the motion of the magnet, the irreg
larity in the trace is perhaps to a considerable extent due to tl
cause.

The trace shown here (fig. 3) is that of the vertical force magne
graph. As pointed out elsewhere, this instrument is very sensit
to shocks, resulting in dislocations of the curve. No such dislo
tion is detected during the earthquake. The unusual thickness
the vibration trace shows that the cause of the disturbance co

not have been instantaneous, but must have acted for some considerable time; for vibration of the above amplitude, if caused by, say, momentary action of a deflector, would die out within half a minute, while the movements shown in the trace lasted for over three minutes. The first wave appears to have commenced the disturbance at 6′·5, attained the maximum at about 8′, and ended at about 10′ past four. The second commenced immediately after this, reached its maximum effect at about 12′, and ended at 14′ past four.

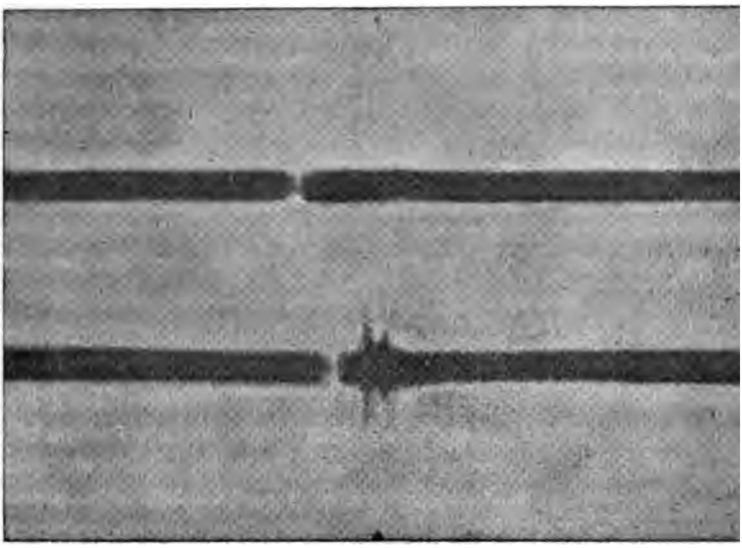

Fig. 3.

The Observatory has two vertical force magnetographs, both of which show the two waves clearly. The maximum effect of the first disturbance coincides in time with the greatest disturbance in the declination magnetograph. The amplitudes of the vibrations above the curve are somewhat smaller than those below the curve (this is more clearly shown in vertical force No. 2), which shows a slight decrease in vertical force during the time of disturbance. It may be noted that the time of vibration of this magnet is 5·35 seconds—about the same as that of declination magnet.

Fig. 4 shows the disturbance in the barograph curve, wl appears to have commenced at about 5·5 minutes after f attained its maximum effect at about 9′·5, and ended at al 12′·5 past four. The oscillations in the column of mercury have been caused by more than one distortional wave; and

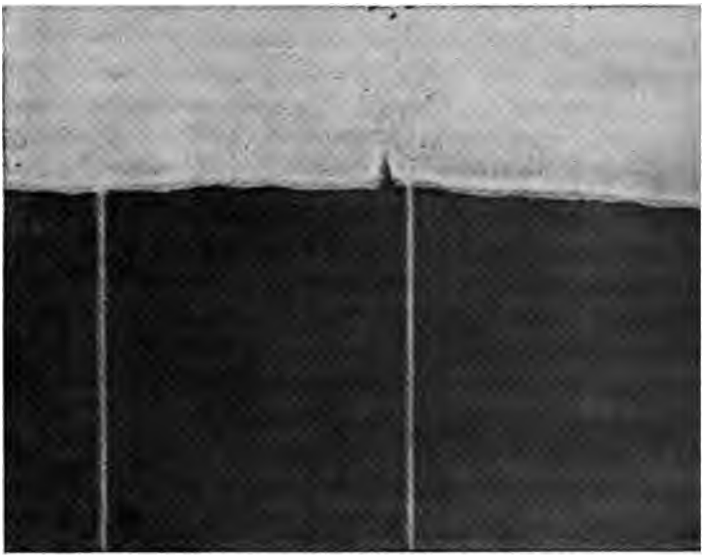

FIG. 4.

interesting to note how the disturbance, which commenced in instrument about half a minute behind that of the declina magnetograph, reached its maximum effect fully one minute a the maximum effect is recorded in declination magnetograph. oscillations of the column as photographed would correspond fall of mercury to about ·07 inch.

[SUMMAR

SUMMARY.

Disturbance of Magnetical and Meteorological Instrum
Government Observatory, Colaba, Bombay, during the
quake of 12*th June* 1897.

Declination,	1st shock,	Commencement,
		End,
Most intense,	2nd shock,	Maximum effect,
End of disturbance,		
Horizontal force,	1st shock,	Commencement,
		End,
End of disturbance,		
Vertical force,	1st shock,	Commencement,
		End,
	2nd shock,	Commencement,
		End,
Barograph,	1st shock,	Commencement,
		Maximum effect,
		End,

On the Velocity of Graded Actions. By James Walke Ph.D., D.Sc., University College, Dundee.

(Read December 6, 1897.)

The theory of mass action leads to very simple consequenc when applied to the determination of the velocity of chemic processes which take place in homogeneous solution. If the action is pure and irreversible, the initial concentrations of t reacting substances have only to be known in order to obtain expression which shall give for every value of the time (measur from the beginning of the reaction) the corresponding extent which the action has taken place, one constant being involve This constant, which is characteristic of the reaction, is called it velocity constant, and for a given medium and a given tempera ture is invariable. Reactions have been classified by Van't Ho into unimolecular, bimolecular, trimolecular, etc., reactions, accord ing to the number of molecules which interact; and to every clas there corresponds an equation by means of which the velocit constant is determined, the expressions for the constants differin from each other in form from class to class. For example, if th initial concentration of each of the reacting substances be A, th time t, and z the extent to which the action has progressed at th end of the time t, then the following expressions will be co stant:—

$$\frac{1}{t}\log_e\frac{A}{A-z} \quad \text{for unimolecular actions.}$$

$$\frac{1}{t}\cdot\frac{z}{A-z} \quad \text{for bimolecular actions.}$$

$$\frac{1}{t}\cdot\frac{z(2A-z)}{2A^2(A-z)^2} \quad \text{for trimolecular actions.}$$

Now a pure reaction must be unimolecular, bimolecular, general n-molecular, and we should therefore expect in a particular case that one of the above series of expressions sho remain constant. In actual chemical processes, however, it is only rarely that we find strict accordance with the theory. Most frequently we can account for the divergences by the disturbing effect of secondary reactions, change in the medium as the reaction progresses, etc., and can occasionally exclude these disturbing factors by suitably selecting the experimental conditions. A great many reactions, nevertheless, give numbers which cannot be brought into harmony with the theory given above, unless they are considered from a standpoint essentially different from that adopted in forming the ordinary chemical equations devised to express them. Thus, for example, the oxidation of ferrous sulphate by potassium chlorate in presence of excess of sulphuric acid is expressed by the equation,

$$KClO_3 + 6FeSO_4 + 3H_2SO_4 = KCl + 3Fe_2(SO_4)_3 + 3H_2O\,.$$

This equation would show that the concurrence of seven molecules, if we exclude those of the sulphuric acid, is necessary for the reaction to take place. But Hood * found that the reaction obeys the bimolecular formula under the most varied conditions. The action of bromic acid on hydriodic acid leads to a similar discrepance. The chemical equation is

$$HBrO_3 + 6HI = HBr + 3H_2O + 3I_2\,,$$

which assumes that seven molecules interact. The actual rate at which the action proceeds by no means conforms to this conception of it. If we adopt the theory of electrolytic dissociation, the chemical equation becomes still more complex, and the divergence between the theoretical and the actual course of the reaction still more marked. Van't Hoff himself studied two specially simple cases of this kind, viz., the decomposition of arseniuretted hydrogen and of phosphuretted hydrogen. We usually express these decompositions by means of the following equations—

$$\begin{aligned} 4AsH_3 &= As_4 + 6H_2\,, \\ 4PH_3 &= P_4 + 6H_2\,, \end{aligned}$$

indicating that the reactions are quadrimolecular. Van't Hoff's experiments showed, however, that both obey very exactly the

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* *Phil. Mag.*, 5th series, vi. 371; viii. 121. *Cf. also* Noyes and Wason, *Zeitschrift für physikalische Chemie*, xxii. 210.

law of unimolecular actions. He was therefore forced to conclud that in these cases the action takes place in two stages, viz.—

$$\begin{array}{ll} \text{I.} & AsH_3 = As + 3H\,, \\ \text{II.} & \begin{cases} 4As = As_4\,, \\ 2H = H_2\,, \end{cases} \end{array}$$

and that the velocity of the first stage alone determines t velocity of the whole reaction. Now that this may be so, t actions of the second stage must proceed at a rate immeasurab greater than the rate of the first stage. An analogy may se to make this point clear. The time occupied in the transmissi of a telegraphic message depends both on the rate of transmissi along the conducting wire and on the rate of the messenger w delivers the telegram; but it is obviously this last, slower r that is of really practical importance in determining the total ti of transmission. When we measure, then, the rate of a chemic change which proceeds in two stages, that stage which proceed most slowly plays the principal part in determining the rate of th whole reaction, being only more or less modified by the other mor rapid changes. If these are indefinitely faster than the slo change, their modifying influence will be so slight as to b negligible, and the really complex action will appear to obey th formula of a simple pure action within the limits of the exper mental error.

According to the hypothesis of electrolytic dissociation all r actions which take place between acids, bases, and salts in aqueo solutions at finite concentrations must be complex reactions of th kind considered above; for in these circumstances the dissociati is never complete. If the action takes place between ions, t undissociated portion must at each instant partially dissociate order to restore the dissociation equilibrium; for the same reason, the reaction involves directly the undissociated molecule, the io must reunite progressively as the reaction goes on. We kn that the processes of electrolytic dissociation and reassociation practically instantaneous, on the evidence of our ordinary chemi experience and of the rate of change in the resistance of electrolytic solution with varying conditions of dilution, etc. Th exert, therefore, little or no modifying influence on the rate actions occurring in aqueous solution, so far at least as the type of the reaction is concerned; although the degree of dissociation greatly influences the numerical value of the velocity constant. It was the study of such a transformation in aqueous solution that led to the following considerations with respect to the velocity of graded actions.

Let the chemical system A pass into the system C through the intermediate system B; and let the actions be irreversible. If the action A→B is indefinitely faster than the action B→C, then the total action A→C will proceed at a rate determined solely both as to type and numerical value by the rate of B→C; and if A→B is indefinitely slower than B→C, the rate of A→C will be that of A→B. Should the rates of the two actions be comparable with each other, the rate of A→C will fall under no simple type, and will differ essentially from that of any pure reaction. A case of this kind was studied by Harcourt and Esson, the pioneers in the field of reaction velocity, and a very complete mathematical treatment is given in an appendix to their paper by Esson.* This paper, although frequently cited, has been altogether neglected from the point of view of graded reactions. The process studied by them was the reduction of potassium permanganate by oxalic acid. In presence of manganese sulphate it takes place in two stages, which they express by the equations,

$$2KMnO_4 + 3MnSO_4 + 2H_2O = K_2SO_4 + 2H_2SO_4 + 5MnO_2\,,$$

$$MnO_2 + H_2SO_4 + H_2C_2O_4 = MnSO_4 + 2H_2O + 2CO_2\,.$$

These actions take place at comparable rates, and mathematical formulæ are given for them which accord very well with the results of experiment.

The simplest case to consider is that in which both actions are unimolecular. For a pure unimolecular reaction the expression for the velocity constant is, as given above,

$$m = \frac{1}{t}\log_e \frac{A}{A-z}\,.$$

To bring this into a form comparable with that of the graded reaction we rearrange it as follows:—

$$z = A(1 - e^{-mt})\,,$$

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* *Phil. Trans.*, clvi. 216, 1866.

e being the base of the natural logarithms. Suppose that in t graded action the total quantity of material in the given volume in chemical units, A, and that at the beginning of the reaction B or C is present. Let there be at the time t, x untransformed, in the intermediate state, and z in the final state; then

$$x+y+z=\mathrm{A}.$$

Let further the velocity constant of A→B be m, and that of B– be n. At the time t, then, we have for the rate of diminution of

$$-\frac{dx}{dt}=mx,$$

and for the rate of increase of z

$$\frac{dz}{dt}=ny.$$

Eliminating x and y from these equations by means of the relation

$$\frac{dy}{dt}=mx-ny \text{ and } \frac{d^2z}{dt^2}=n\frac{dy}{dt},$$

we obtain

$$\frac{d^2z}{dt^2}+(m+n)\frac{dz}{dt}+mn(z-\mathrm{A})=0,$$

whence, by treating $z-\mathrm{A}$ as the variable, we have

$$z-\mathrm{A}=\mathrm{C}_1e^{-mt}+\mathrm{C}_2e^{-nt}.$$

To determine the constants C_1 and C_2 we have $z=0$ when $t=$ and also $y=\frac{1}{n}\cdot\frac{dz}{dt}=0$ when $t=0$. We accordingly obtain

$$\mathrm{C}_1=\frac{n\mathrm{A}}{m-n} \text{ and } \mathrm{C}_2=\frac{m\mathrm{A}}{n-m},$$

so that

$$z=\mathrm{A}\left(1+\frac{n}{m-n}e^{-mt}+\frac{m}{n-m}e^{-nt}\right).$$

We have here, then, an expression for z in terms of the time a the velocity constants of the separate actions. Suppose one these constants, say n, to be infinitely great. The expression then reduces to

$$z = A(1 - e^{-mt}),$$

identical with the value for a pure reaction having the velocity constant m. If m becomes infinitely great, we have

$$z = A(1 - e^{-nt}).$$

When $n = m$ the above expression for z becomes indeterminate, but if we make the substitution in the differential equation, we get on solving

$$z = A(1 - e^{-mt} - mt\,e^{-mt}).$$

It will be observed that when both actions are unimolecular, the expression for z is symmetrical with respect to m and n. So far, therefore, as the effect on z is concerned, it is a matter of indifference whether the action with the velocity constant m or that with the velocity constant n takes place first.

A characteristic point of difference between reactions proceeding in one stage and reactions proceeding in more than one is the following. If all the material is in the state A at the beginning of the action, then in the first case the maximum rate of increase of z takes place when $z = 0$, whilst in the second case this only occurs when z has attained a finite value. Let us consider what happens with a reaction proceeding in two unimolecular stages. The rate at which x diminishes, viz., $-\frac{dx}{dt}$, is greatest at the beginning of the action; the rate at which z increases, viz., $\frac{dz}{dt}$, is at that time zero. We have then a steady diminution of $-\frac{dx}{dt}$ and a steady rise of $\frac{dz}{dt}$. Now $\frac{dy}{dt} = -\frac{dx}{dt} - \frac{dz}{dt}$; so that y will increase at a gradually diminishing rate and reach a maximum when $-\frac{dx}{dt} = \frac{dz}{dt}$. Since $\frac{dz}{dt}$ is proportional to y, the rate of increase of z will reach a maximum at the same time, *i.e.*, when $-\frac{dx}{dt} = \frac{dz}{dt}$, or when $mx = ny$. From the equations given above it is easy to calculate the values of x, y, and z at this point in terms of the velocity

constants. Let t_1, x_1, y_1, and z_1 be the values of t, x, y, and z wh
y is a maximum. Then

$$\begin{aligned} mx_1 &= ny_1 \\ x_1 &= \mathrm{A}e^{-mt_1} \\ z_1 &= \mathrm{A}\left(1 + \frac{n}{m-n}e^{-mt_1} + \frac{m}{n-m}e^{-nt_1}\right) \\ \mathrm{A} &= x_1 + y_1 + z_1 . \end{aligned}$$

By eliminating t_1, y_1, and z_1 from these equations we obtain

$$x_1 = \left(\frac{m}{n}\right)^{\frac{m}{n-m}}\mathrm{A},$$

and thence

$$y_1 = \left(\frac{m}{n}\right)^{\frac{n}{n-m}}\mathrm{A}.$$

When $n = m$ these expressions become indeterminate, but we ma
write

$$\left(\frac{m}{n}\right)^{\frac{m}{n-m}} = \frac{1}{\left(1 - \frac{m-n}{m}\right)^{-\frac{m}{m-n}}},$$

and the limit of this last expression for $m = n$ is $\frac{1}{e}$, so th
when the two velocity constants are equal, $x_1 = y_1 = \frac{\mathrm{A}}{e}$. Th
result may also be obtained directly from the expression for
when $m = n$. It may be noted that, whatever the values of t
velocity constants may be, when the maximum rate of increa
of z is reached, the value of z itself cannot be more tha
$\mathrm{A} - \frac{2\mathrm{A}}{e} = 0{\cdot}264\mathrm{A}$. This can be proved by showing that $x_1 + y$
or $\mathrm{A}\left\{\left(\frac{m}{n}\right)^{\frac{m}{n-m}} + \left(\frac{m}{n}\right)^{\frac{n}{n-m}}\right\}$, is a minimum when $m = n$.

It will be observed that the expression for x_1 becomes that for
when m and n are interchanged, and similarly, y_1 becomes x_1.
the velocity coefficients were thus interchanged, the maximu
value of $\frac{dz}{dt}$ would remain unaffected, the only change being th
the concentrations in the first and second stages would now
reversed.

To afford an idea of the relation between the above formula an that developed on the assumption that a really double reaction : simple, I have calculated values of z for reactions in whic $m = 0·01$ and $n = 0·01$, 0·1, and 1·0 respectively, A in each ca being 1. The values of z thus obtained have then been taken ε if they were derived from a simple reaction, and the values of tl "constant" C of this imaginary action calculated.

$m = 0·01$ $n = 0·01$

t	z	C
10	0·005	0·00056
40	0·062	0·00159
100	0·264	0·00306
200	0·595	0·00328
400	0·909	0·00598
1000	0·9995	0·0076

$m = 0·01$ $n = 0·1$

t	z	C
10	0·036	0·00366
40	0·258	0·00740
100	0·591	0·00892
200	0·850	0·00948
400	0·980	0·00978

$m = 0·01$ $n = 1·0$

t	z	C
5	0·0292	0·00593
10	0·086	0·00900
20	0·173	0·00947
40	0·323	0·00974
100	0·628	0·00987
200	0·864	0·00994

It will be seen that when $n = 100m$, which is a very moder ratio between reaction velocities, the second reaction might overlooked if observations near the beginning of the action were not considered. In this case the disturbing influence of the second reaction is briefly that z, instead of being equal to $1-e^{-mt}$ is approximately equal to $1-1{\cdot}01e^{-mt}$, in general to $1-\left(1+\frac{m}{n}\right)e^{-mt}$. The effect of rapid disturbing actions is, as may be seen from the tables, most evident near the beginning of the reaction, which only assumes an appearance of regularity as the process goes on. Now, "initial disturbances" play a large part in known cases of reaction velocity, and it seems to me not improbable that some of them are due to the actions considered not being simple actions, but graded actions, with one stage very much more rapid than the other.

When the actions are reversible and some of them other than unimolecular, the differential equations do not often permit of a simple solution. Thus, for example, the transformation of urea into ammonium cyanate is chemically

$$NH_4CNO \rightleftarrows NH_4{}^{\cdot} + CNO' \rightleftarrows CO(NH_2)_2 .$$

Let the total mass be A as before, composed of x in the first stage (undissociated cyanate), y in the second stage (dissociated cyanate), and z in the third stage (urea); and let the constants be

m for $NH_4CNO \rightarrow NH_4{}^{\cdot} + CNO'$
m' for $NH_4{}^{\cdot} + CNO' \rightarrow NH_4CNO$
n for $NH_4{}^{\cdot} + CNO' \rightarrow CO(NH_2)_2$
n' for $CO(NH_2)_2 \rightarrow NH_4{}^{\cdot} + CNO'$.

The differential equations are then

$$-\frac{dx}{dt} = mx - m'y^2$$

$$\frac{dz}{dt} = ny^2 - n'z .$$

From a knowledge of the composition of the solution when equilibrium occurs we get the ratio of m to m', and of n to n'. We are then in possession of all the information necessary to express, as before, z in terms of t, and it is only difficulties in the integration that present an obstacle.

I am indebted to my colleague, Dr John M'Cowan, for the solutions of some special cases where the integration can be performed; but, as they do not correspond to any cases hitherto realised, and, in addition, show no special features different from those exhibited by the two successive unimolecular reactions, I refrain from giving them here.

In the example given above of the transformation of ammonium cyanate into urea, the processes of electrolytic dissociation and association take place so rapidly, compared with the processes of urea-formation and decomposition, that they may be neglected, as far as the form of equation goes, for expressing the rate of the reaction. By a combination of chemical and electrical measurements (Walker and Kay, *Jour. Chem. Soc.*, 1897, 489) we can determine at any moment the value of $x+y$ and of y, and so can ascertain the value of n, an equilibrium experiment having previously given the ratio of n' to n.

It should be noted that we cannot, strictly speaking, compare the values of n and n' in the above equation for urea-formation, since they are of different dimensions, their ratio being altered by a change in the unit of volume. In general we can only compare together the constants of reactions of the same order, as reactions of different orders depend on the volume in different ways. This observation suggests a possible method for ascertaining the rate of very rapid reactions otherwise inaccessible. For instance, it would be of great interest to determine the rate of any case of electrolytic dissociation or association, but the rapidity with which these reactions proceed precludes any direct measurement. Suppose a graded action could be found in which the first stage was the union of two ions to form an undissociated molecule and the second stage a comparatively slow transformation of the undissociated molecule into something else. Here we should have a very rapid bimolecular reaction followed by a measurable unimolecular reaction. In solutions of ordinary concentrations the rate of the total transformation would be that of the pure unimolecular reaction; but, if we could follow the course of the graded action in extremely dilute solution, we might be able to discover a disturbing effect of the preliminary bimolecular reaction, and ascertain thereby approximately its rate. This is so, because the proportion of the time taken to transform a given proport of the total number of molecules is in a unimolecular react independent of the volume, whilst in a bimolecular reaction varies inversely as the volume. By dilution, then, we sho greatly diminish the actual rate of the bimolecular reaction compared with that of the unimolecular reaction, and its distu ing influence would become apparent. Even if it did not, should at least be able to assign an inferior limit to the rate of rapid reaction. No graded action of this kind has hitherto b investigated, but a careful search may reveal one.

Preliminary Note on a Characteristic of Certain Chemical Reactions. By Prof. John Gibson, Ph.D.

(Read December 6, 1897.)

In a paper, communicated to the Society in February 1897, the author drew attention to increase in electrical conductivity as a characteristic of photo-chemical action.

This increase in electric conductivity may be explained by assuming that light acts either in the direction of increased conductivity, or as a disturbing agent on molecular systems having an inherent tendency towards rearrangements involving increased conductivity.

It may be that both assumptions are correct.

Since making this communication the author has been engaged in experimental investigation, undertaken in order to elucidate this subject further—more particularly in its chemical aspect.

The behaviour of nitric acid with respect to light appeared especially suggestive. Nitric acid on progressive dilution with water becomes less and less subject to decomposition by light, until, as that dilution which corresponds to maximum conductivity is approached, the action of light seems to cease altogether, so that nitric acid at all dilutions greater than that corresponding to its maximum electrolytic conductivity appears to be unaltered by light. It seemed, therefore, desirable to investigate experimentally the purely chemical behaviour of nitric, hydrochloric, and sulphuric acids at different degrees of dilution with water. The investigation is now sufficiently far advanced to justify the communication to the Society, in this preliminary note, of some of the conclusions which have been arrived at.

It would appear that the chemical behaviour of the acids just mentioned depends, in many of their reactions, on whether their concentration is above or below that corresponding to their maximum electrolytic conductivity.

At a temperature of 18° C. the concentrations of aqueo solutions of these acids, which correspond to their respecti maximum conductivities as determined by Kohlrausch, are:—

Hydrochloric Acid,	. . .	18·3 per cent.
Nitric ,,	. . .	29·7 ,,
Sulphuric ,,	. . .	30·4 ,,

The following reactions have been investigated:—

The de-hydration by hydrochloric acid of hydrated cobalt chloride.—The deep blue colour of anhydrous cobaltous chlorid immediately produced on adding a few drops of the pink solut of the hydrated salt to an excess of hydrochloric acid solution 28 per cent. HCl and upwards. With acid of 24 per cent. blue coloration is still distinct. With acid of 22 per cent. i very slight. With acids below 18 per cent. it is imperceptible.

The de-hydration of sugar by sulphuric acid.—If a sma quantity of cane-sugar syrup be mixed with solutions of sulphuri acid of 36 per cent. H_2SO_4 and upwards, and if the mixture b raised momentarily to the boiling point, proper precaution bein taken to avoid local over-heating, a deep brown coloration i rapidly produced. Solutions of 30 per cent. H_2SO_4 and lowe treated in the same manner, either remain colourless, or onl slowly acquire a faint yellow tinge.

Reduction of chromic anhydride by hydrochloric acid.—Concer trated solutions of this acid rapidly reduce chromic anhydride a ordinary temperature, the orange red colour of chromic anhydrid giving place to the pure green colour of solutions characteristic c chromic chloride within a few hours. As the dilution of the aci is increased towards maximum conductivity, the velocity of th reaction diminishes. In one experiment with 24 per cent. aci the reduction was complete in about 100 hours; while in anoth experiment with 18 per cent. acid, which was begun on July 2 1897, the reduction is still incomplete, in spite of the fact that t test-tube was only loosely closed by an ordinary cork, the low end of which has been bleached by the escaping chlorine gas.

Oxidation of hydrogen iodide by sulphuric acid.—On addi about 2 centigrams of potassium iodide in dilute solution to cubic cms. of sulphuric acid of 45 per cent. and upwards, the libe ation of iodine at ordinary temperature became perceptible with ten seconds. With 36 per cent. the time required was about 4 minutes, and with 30·5 per cent. acid, 15–20 minutes.

Oxidation of nitric oxide by nitric acid.—Passing nitric oxide into nitric acid of 36 per cent., the blue colour of nitrous acid rapidly made its appearance, while nitric acid of 28 per cent. remained colourless.

These examples are sufficient to indicate the nature of the reasons for assuming an increase in electrolytic conductivity as a common characteristic of all these reactions. Whenever the dilution of the acid is carried beyond that corresponding to maximum conductivity, the velocity of the reaction is reduced to a minimum, or, as in the case of sulphuric acid and cane sugar, the velocity of reaction in an opposite direction becomes the greater. It will be remembered that dilute sulphuric acid so far from de-hydrating or charring sugar, adds water on to it, or hydrolises it.

It may be pointed out here as highly significant that those salts which are markedly deliquescent are also those whose saturated solutions gain considerably in electrolytic conductivity by dilution, and that those gases which form a cloud in ordinary air are also those whose saturated solutions have their conductivity greatly increased by dilution.

One result of this investigation is the correlation of those reactions which result in an increase of electrolytic conductivity and their differentiation from reactions in which there is a decrease of electrolytic conductivity.

Chemical decompositions dependent on absorption of energy are not generally associated with increase of electric conductivity, but the reverse. Instances of this class of chemical change are:—

(*a*) The separation of solvent from solute by evaporation.

(*b*) The decomposition of compound bodies by electrolysis.

(*c*) The decomposition of compounds by the action of heat.

The great class of reactions included under the phrase "the double decomposition of salts" frequently do not result in a finally increased electrolytic conductivity, because of the precipitation of insoluble salts.

In the reactions referred to above, as characterised by increase in electrolytic conductivity, the velocity of reaction is not proportional to the ionisation. Only so long as a marked increase of conductivi results from the dilution of the acid brought about by the reacti is the velocity of reaction considerable.

As soon as further dilution, although it may increase ionisati ceases to involve marked increase in conductivity, the velocity these reactions rapidly diminishes. In other words, the veloc of the reaction is proportional to the rate of increase in the num of carriers of electricity per unit volume.

One class of reactions may be referred to here, because of tl general importance, and because they seem to present spe difficulties from the point of view taken in this paper.

When metals combine directly with oxygen, metallic ductivity is lost, and the oxides formed are themselves poor ductors, and are frequently insoluble in water.

A closer consideration of the facts shows, however, that sol metals do not readily combine with pure oxygen. Even metal potassium throughout a wide range of temperature does not co bine directly with pure dry oxygen, and may, in fact, be distill unchanged in it. The means taken to induce combination, namel to apply heat, tends in the first place to diminish the metallic cc ductivity, and secondly to increase the vapour pressure of t metal. It seems more than probable that the chemical union tal place between gaseous oxygen and *gaseous*, not solid, me But metallic vapours are dielectrics.

The following is a preliminary list of reactions which the aut has been led to consider tentatively as characterised by increas electrolytic conductivity :—

(*a*) Photo-chemical reactions generally.

(*b*) Reactions characteristic of concentrated solutions of a and bases.

(*c*) Reversible actions generally.

(*d*) Many reactions employed in the synthesis of organic stances such as:

Etherification.
Esterification.
Saponification.
Hydrolytic actions.
The reduction of nitro bodies by alcoholic potash.

(*e*) Reactions of technical importance.
The inversion of cane-sugar by weak acids.
The alcoholic fermentation.
The acetic fermentation.
Nitrification.

(*f*) Many reactions relating to the physiology of plants and animals.
Certain chemical changes involved in carbon assimilation, in the germination of seeds, and in the ripening of fruits.
The conversion of urea in urine into ammonium carbonate.
The lactic fermentation.

The following proposition has been found useful as a working hypothesis. It is stated here for this reason.

The degradation of potential chemical energy is generally associated with molecular rearrangement involving diminished electrical resistance. Decrease of electrical resistance may result either from :

(*a*) Increased electrolytic conductivity.

(*b*) Increased metallic conductivity.

In the case of non-conducting substances (dielectrics), increase of specific inductive capacity is regarded as equivalent to decrease of electric resistance.

A loss in any one of these properties may be compensated by a gain in one or both the others.

A paper giving full details will shortly be laid before the Society.

Leakage from Electrified Metal Plates and Points plac above and below Uninsulated Flames. By The Rig Hon. Lord Kelvin, G.C.V.O., F.R.S., and Dr Magnu Maclean.

(Read July 5, 1897.)

§ 1. In § 10 of our paper "On Electrical Properties of Fume proceeding from Flames and Burning Charcoal," communicated t this Society on 5th April, results of observations on the leakag between two parallel metal plates with an initial difference o electric potential of 6·2 volts between them, when the fumes fron flames and burnings were allowed to pass between them and roun them, were given. The first part (§§ 1–4) of the present shor paper gives results of observations on the leakage between tw copper plates 1 centimetre apart, when one of them is kept at constant high positive or negative potential; and the other, afte being metallically connected with the electrometer-sheath, is dis connected, and left to receive electricity through fumes betwee the two.

The method of observation (see fig. 1) was as follows:—Tw copper plates were fixed in a block of paraffin at the top of a roun tinned iron funnel 96 centimetres long and 15·6 centimetres interna diameter. A spirit-lamp or a Bunsen burner, the only two flame used in these experiments, was placed at the bottom of the funnel, 8 centimetres below the two copper plates. One terminal of a voltai battery was connected to one plate, B, and the other terminal wa connected to the sheath of a Kelvin quadrant electrometer. Th other copper plate was connected to one of the pair of quadrants o the electrometer in such a way that by pulling a silk cord with hinged platinum wire at its end, this copper plate and this pair o quadrants could be insulated from the sheath of the electromete and the rest of the apparatus. On doing so with no flame at th bottom of the funnel, no deflection from metallic zero was observe even when the other plate was kept at the potential of 94 volts b the voltaic battery; this being the highest we have as yet trie

When the plate was kept at potentials of 2, 4 . . . 10 volts, deflection from metallic zero in three minutes was observed; for higher potentials, merely the times of attaining to 300 s divisions from metallic zero were observed.

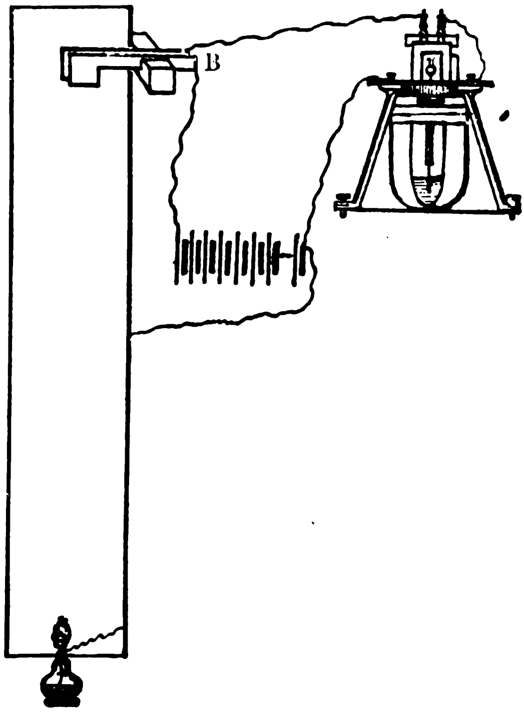

Fig. 1.

§ 2. The results obtained are summarised in the following ta In every case for potentials below 90 volts there was grea leakage when the uninsulated plate was connected to the negat terminal of the battery. This difference depended, partially all events, on the character of the inner surface of the fun which was old tarnished tin-plating.

[*Spirit Fla*

Spirit Flame.

Sensitiveness of electrometer = 60·7 scale divisions per volt. Hence 300 scale divisions corresponds approximately to 5 volts.

Difference of Potentials.		+ to plate, B, - to sheath.			- to plate, B, + to sheath.		
Volts.		Deflection.	Time.		Deflection.	Time.	
		Divisions.	Min.	Sec.	Divisions.	Min.	Sec.
2		+ 35	3	0	- 80	3	0
4		+ 92	3	0	- 133	3	0
8		+ 205	3	0	- 265	3	0
10		+ 240	3	0	- 311	1	15
Initial.	Mean.						
12	9·5	+ 300	0	53	- 300	0	38
18	15·5	+ 300	0	25	- 300	0	16
44·5	42·0	+ 300	0	4·5	- 300	0	4
89	86·5	+ 300	0	2·5	- 300	0	2·5

Bunsen Flame.

Sensitiveness of electrometer = 60·7 scale divisions per volt.

Difference of Potentials.		+ to plate, B, - to sheath.			- to plate, B, + to sheath.		
Volts.		Deflection.	Time.		Deflection.	Time.	
		Divisions.	Min.	Sec.	Divisions.	Min.	Sec.
2		+ 10	3	0	- 99	3	0
4		+ 73	3	0	- 159	3	0
8		+ 200	3	0	- 300	2	20
Initial.	Mean.						
12	9·5	+ 300	1	48	- 300	0	48
16	13·5	+ 300	1	12	- 300	0	30
19	16·5	+ 300	0	46	- 300	0	18
31	28·5	+ 300	0	15	- 300	0	13
47	44·5	+ 300	0	11	- 300	0	8
75	72·5	+ 300	0	6·5	- 300	0	5
94	91·5	+ 300	0	5	- 300	0	4

§ 3. If the leakage in these experiments were proportional to the difference of potential, then the product of mean difference of potential into time should be constant for the same deflection from metallic zero. Taking the numbers obtained for the 300 scale divisions of deflection in virtue of the Bunsen flame, we have :—

Positive Charge.					Negative Charge.				
v.		s.			v.		s.		
9·5	×	108	=	1026	9·5	×	48	=	456
13·5	×	72	=	972	13·5	×	30	=	405
16·5	×	46	=	759	16·5	×	18	=	297
28·5	×	15	=	427	28·5	×	13	=	370
44·5	×	11	=	489	44·5	×	8	=	356
72·5	×	6·5	=	471	72·5	×	5	=	362
91·5	×	5	=	457	91·5	×	4	=	366

Thus it is proved that the leakage between two plates, each 10 square centimetres in area, 1 centimetre apart when the fumes from a Bunsen burner pass between them and round them, is approximately proportional to the difference of potential between them, when that difference is above 20 volts and up to 94 volts, the highest we have tried; but that, below 20, it diminishes with diminishing voltages more than according to simple proportion.

§ 4. To determine the currents which we had in our arrangement, we took a movable plate of a small air condenser charged to a known potential, and applied it to the insulated terminal of the quadrant electrometer. In this way we found that a quantity equal to 0·15 electrostatic unit, gave a deflection of 300 scale divisions. Hence in the experiments with the Bunsen flame and with a potential of -94 volts kept on the uninsulated copper plate, the current to the insulated copper plate opposite to it, when 300 scale divisions was reached in five seconds, was—

$$\frac{0{\cdot}15}{3\times10^9}\times\frac{1}{5}=10^{-11}\text{ ampere.}$$

$$=\frac{1}{100000}\text{ mikro-ampere.}$$

§ 5. One of us about the year 1865, when occupied in experimenting with the latest form of portable electrometer, found that if it was held with the top of its insulated wire (which was about 33 centimetres long) a few inches below a gas-burner, a charge of electricity, whether positive or negative, given to this wire was very rapidly lost. The disinsulating power of flames and of hot fumes from flames was well known at that time, but it was surprising to find that cold air flowing up towards the flame did somehow acquire the property of carrying away electricity from a piece of electrified metal immersed in the cold air.* Circumstances prevented further observations on this very interesting result at that time, but the experiment was repeated with a portable electrometer in December of 1896, and we were made quite sure of the result by searching tests. During April and May of the present year observations were again made by means of (1) a multicellular electrometer reading up to 240 volts, and (2) a vertical electrostatic voltmeter (fig. 3, below) reading up to 12,000 volts. A pointed

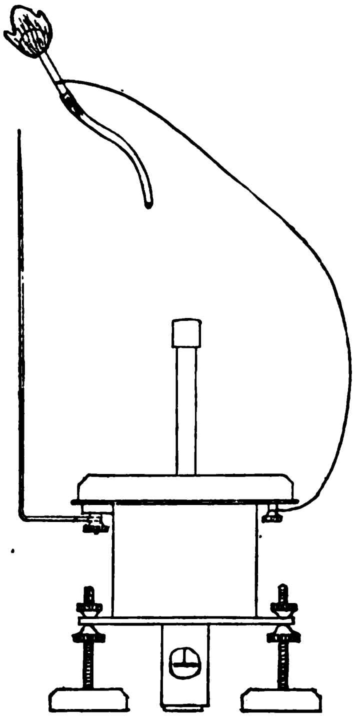

FIG. 2.

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* We have recently (June 1897) found the following statement, in Worthington's communication to the British Association (1889, Report, pp. 225, 227) "on the Discharge of Electrification by Flames":—"The observation seems to have been made by Priestley, that the discharge takes place *with apparently equal rapidity*, if the rod be held at the side of, or even below, the flame at the distance of, say, five centimetres." The four words which we have italicised are not verified with the forms and arrangements which we have used, as we find *enormously greater leakage* five centimetres above a flame than five centimetres below it; but it is very interesting to learn that Priestley had found any leakage at all through air five centimetres below a flame.

steel wire 43 centimetres long was fixed to the insulated terminal of the multicellular electrometer, with its point vertically below an ordinary gas-burner, as shown in fig. 2.

§ 6. By means of a small carrier metal plate (a Coulomb's proof plane) a positive or negative charge was given to this wire and the quadrants of the multicellular till the reading on the scale was 240 volts. The leakage was then observed (*a*) with gas not lit, (*b*) with gas lit at different vertical distances above the point of the wire. We found that there was rapid leakage when the flame was one centimetre above the wire; and the times of leakage from 240 volts to about 100 volts increased as the flame was raised to greater distances above the point; or, otherwise, the rate of fall of potential in one minute from 240 volts diminished as the distance of the flame above the point was increased. When the vertical distance of the flame above the point was 15 centimetres, or more, the time of leakage from 240 volts was practically the same as if the flame was not lit at all. A plate of metal, glass, paraffin, or mica, put between the point and the flame, diminished the rate of leakage. The leakage from 200 volts during the first minute is given in the following table, for different distances of the flame, with no intervening plate.

Distance of flame above point	Leakage during one minute	Remarks.
Centimetre	Volts	
1·0	200 to 60 = 140	
1·5	200 to 92 = 108	
3·0	200 to 179 = 21	
6·0	200 to 196 = 4	
	200 to 197 = 3	No gas lit, but wire on the electrometer as in the other tests.*

§ 7. Similar experiments were made with higher voltages measured by the vertical electrostatic voltmeter, and we found that when the flame was three or four centimetres above the point, there was very rapid discharge; but when the flame was 60 centimetres or more above the point, the leakage from 3500 volts was practically the same as if the flame was not lit.

* We sometimes found the multicellular electrometer to insulate so well that in five minutes there was no readable leakage from 240 volts.

In place of the metal point, a round disc of zinc, 8 centimetres in diameter, was fixed, as shown in fig. 3, to the end of another steel wire of the same length; and leakage from it to the flame above it, observed. For the same distance between the flame and either the point or the metal disc, the rate of leakage through the same difference of potential, was *less for the point than for the disc.* Thus with the flame 25 centimetres above the point the time of drop from 3000 volts to 2000 volts was 1 min. 53 secs., and with the flame the same distance above the disc the time of drop from 3000 volts to 2000 volts was 1 min. 14 secs. *This is a very important result.*

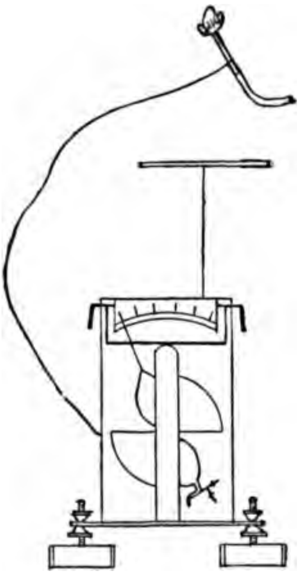


FIG. 3.

§ 8. Experiments were next made to find if; and if so, how much; the leakage is diminished by putting non-conducting plates of glass, paraffin, mica, between the point or disc and the flame. At a corner of each plate was pasted a little square of tinfoil, so as to prevent any electrification of the non-conducting substance by handling. These pieces of tinfoil were always kept metallically connected with the sheath of the electrometer. Each plate was fixed with its under surface 1 cm. above the steel point. In preliminary experiments (of which a continuation is deferred until

the insulation of the electrometer is made practically perfect b
coating its vulcanite insulators with paraffin) the following number
were obtained:—

I. *Glass Plate* 18 *cms. by* 19 *cms. by* 0·3 *cm.*

Distance of flame above point	Time of fall from 3000 to 2000 volts		Remarks.
Cms.	Min.	Sec.	
—	5	30	Insulation test, with no flame.
12	2	5	Flame lit: no intervening plate.
,,	4	7	,, ,, glass plate between.

II. *Mica Sheet* 18 *cms. by* 9 *cms. by* 0·1 *cm.*

—	6	46	Insulation test, with no flame.
12	1	56	Flame lit: no intervening plate.
,,	3	50	,, ,, mica sheet between.

III. *Paraffin Plate* 11 *cms. by* 11 *cms. and* 0·75 *cm. thick.*

—	6	40	No flame. Insulation test.
12	1	53	Flame lit: no intervening plate.
,,	2	20	,, ,, paraffin plate between.

We hope to return to the investigation with the insulation
the electrometer perfected; and to determine by special exper
ment, how much of the fall of potential in the electrometer in eac
case is due to the electricity of opposite kind induced on the uppe
most surface of the non-conducting plate, and how much, if any,
due to leakage through the air to the metal disc or point below.

§ 9. To test the quality of the electrification of both sides of th
non-conducting plates of glass and paraffin, a thin copper sheet,
was fixed to one of the terminals of a quadrant electrometer,
represented in fig. 4, where A is the plan of the plate C, and
is the plate of paraffin or glass under test.

In the primary experiment (fig. 3) the non-conducting plate w
fixed in a horizontal position one centimetre above the electrifie
metal (point or disc), and eleven centimetres below the flame.
charge was given to the metal, to raise its potential to about 35

volts. After some minutes, generally till the potential of the metal fell to 2000 volts, the non-conducting plate was removed and placed, as shown in fig. 4, above the metal plate C attached to the quadrant electrometer, and the deflection was observed. For a thin piece of glass (0·3 cm. thick) the whole effect of the two sides was negative when the electrified metal point or disc had been charged positively and *vice versâ.* But on putting two plates of glass above the electrified metal, we found the top plate to be oppositely charged ; and the under plate to be charged similarly to the point or disc, but not so highly. We found corresponding results with a plate of paraffin 0·75 cm. thick, and with two plates of paraffin, 0·5 cm. and 0·75 cm. thick. When a plate of paraffin 3·25 cms. thick was used, we always found the top face charged oppositely to the charge of the metal, whether disc or needle-point, and the under face charged similarly to the metal below. Thus the apparent total charge of the two faces of a thin non-conducting plate is due to the fact that the face of the plate away from the electrified metal is more highly charged oppositely than the face next the metal is charged similarly.

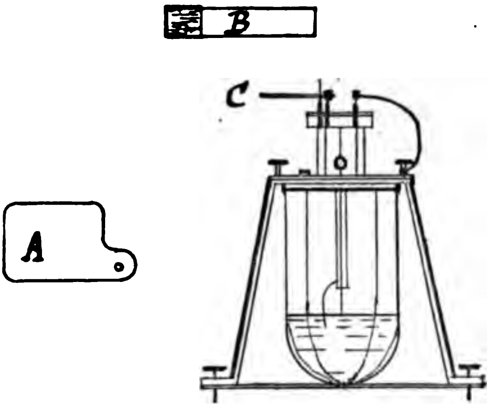


FIG. 4.

Observations on the Migratory Movements of Salmonidæ during the Spawning Season. By W. L. Calderwood.

(Read January 31, 1898.)

The fact that the salmon comes from the sea and spawns in the river has induced the supposition that this fish ascends the river only for the purpose of spawning. That the supposition is not always justifiable seems, however, sufficiently shown by the fact that clean-run salmon with undeveloped reproductive organs may be found in fresh water during the spawning season, and indeed at any season of the year, and also by the fact that a spring run of fish is usual. When we regard the head waters of many of our rivers as localities for the natural propagation of the salmon, we find, however, that, except at the spawning season, adult fish are never present. When, at the same time, we remark that the fish which ascend to those waters during the winter months are all sexually ripe, we may fairly conclude that the fish are impelled to migrate to those head waters for the express purpose of spawning.

It has been said that the shads (*Clupea allosa* and *C. finta*) and the sea lamprey (*Petromyzon marinus*) are examples of fishes which have a spawning habit analogous to that of the salmon, since they also ascend rivers for the purpose of propagating their species. Their habit is, however, more analogous to that of the comparatively few salmon which penetrate at once to head streams and tributaries, than to the fishes which inhabit the lower reaches of a salmon river. They ascend for a limited period only, and seek again the salt water whenever the operation of spawning is completed. They are marine fishes which spawn in fresh water. The common eel may be taken as an example of a fresh water fish which spawns in sea water.

We have as yet very little knowledge about the movements of adult salmon in the sea: we know that they are to be found many miles from land, and we know that they congregate in great numbers in the estuaries of rivers, presumably the particular rivers from which they have originally come to the sea; that in the estuaries they move to and fro with the flood and ebb tides; that with the ebb they seek the shallower waters at the margins of the estuaries, where they are taken in the nets; but we need to turn to our knowledge of the salmon in fresh water, and the time it remains there, in order to form an estimate of how long the fish spends in salt water. It seems to me that, even by this process, a very uncertain margin is left, but that in all probability a fish of full adult growth, say a fish of four years old, has spent an equal period of time in fresh and in salt water.

It is further noticed that a run of fish commences when the river is swollen, or during a spring tide. At such times estuarine netting is reported as at its best.

In the same way, a flood in a particular tributary is believed to induce a run of fish, either during the flood or as the water is falling after the highest of the freshet is past. If we accept in its entirety the theory that salmon return to the particular river where they were spawned, we must necessarily believe that the fish which take advantage of such a flood are some of those that were spawned in the particular tributary; yet proof is easily found that salmon will take advantage of newly accessible spawning grounds, and that such fluctuations take place in the conditions of the various tributaries of a large river as to point to the conclusion that salmon, when once in the main river of their birth, enter the particular tributaries which are instinctively known to be suitable, or which present suitable conditions for ascent at the proper time. What these conditions are seem less known to man than to the salmon, yet not infrequently fish pass favourite spawning grounds, and, impelled it seems by the instinct for ascent, travel on to ground where the ova have certainly much less chance of hatching out in numbers.

Last winter I was fortunate enough to have the opportunity of procuring data from the head waters of Tweed which, although not perfect enough to afford answers to all the questions which surround the spawning of migratory Salmonidæ, give us, nevertheless, some information of a more definite kind than can be commonly procured, and which on this account are, I think, worthy of mention. During the entire spawning season daily records were taken by three watchers as to number of fish seen, their position, the state of the water, etc.

The Talla is a comparatively small stream about 6 miles length, which joins the Tweed at Tweedsmuir. Ascending main river it is, however, the last tributary of importance. drainage area is 6180 acres, being one-fourth of the drainage a of the Tweed above Talla-mouth. The observations as to st of the water are distinguished under the headings of "norm "¼-flood," "½-flood," "¾ and full flood." In recording fish, saln and sea trout are alone mentioned, no distinction being dra between salmon and grilse. In all probability, however, few gr reach these head waters, situated as they are, 90 miles from sea, and visited by late-running fish only at the spawning season

Except when the stream was in full flood no difficulty experienced in counting the fish, since the channel is beautifu clean, and the stream comparatively small. Even in half-flood water was, as a rule, only slightly coloured. In counting, howe it is quite certain that in many instances the same fish was recor twice or three times over. This could not be avoided by watchers, and in dealing with the returns I have thought it be not to attempt any separation.

Fish were recorded as in Talla Water from 4th November 1 to 16th February 1897, that is, during a period of 105 days, fish were not observed, and were not present, on every individ day. The state of the water during those 105 days was follows:—

Normal flow	during	75	days, including 8 days "frozen."
¼-flood	"	8	"
½-flood	"	16	"
¾ or full flood	"	6	"
		105	

The number of days, included in the above list, on which fish were recorded was 47. The state of the water during th days was as follows:—

Normal flow, ...	34
¼-flood, ...	1
½-flood, ...	7
¾ or full flood, ...	5
	47

I may remark, in passing, that from 21st Jan. to 9th Feb., a period of nineteen days, no fish were observed in Talla Water, and that during this time the water was in its normal condition, but that during eight days of the time the margins of the stream were covered by ice and snow.

Deducting the 47 days, therefore, we have 58 days out of the 105 during which fish were noticed in the river.

The total number of fish recorded is 320, there being 172 salmon and 148 sea trout.

The following table gives the detailed record; all days on which no fish were recorded being omitted:—

TALLA WATER SPAWNING SEASON, 1896–97.

Date.	No. of Fish.		State of Water.	Daily Totals of Fish.	Date.	No. of Fish.		State of Water.	Daily Totals of Fish.
	Salmon.	Sea Trout.				Salmon.	Sea Trout.		
Nov. 4	1	...	Normal	1	Dec. 14	1	2	Normal	3
,, 7	1	...	,,	1	,, 15	1	6	,,	7
,, 10	2	...	,,	2	,, 16	3	5	,,	8
,, 13	1	...	,,	1	,, 17	4	3	,,	7
,, 14	3	...	¼-Flood	3	,, 18	1	3	,,	4
,, 15	4	...	½-Flood	4	,, 19	3	2	,,	5
,, 16	4	1	,,	5	,, 20	1	...	,,	1
,, 17	...	1	Normal	1	,, 23	1	1	,,	2
,, 18	3	...	,,	3	,, 24	...	1	Full fl'd	1
,, 19	...	1	,,	1	,, 28	14	2	½-Flood	16
,, 20	2	...	,,	2	,, 29	7	1	¼-Flood	8
,, 21	4	1	,,	5	Jan. 1	7	...	½-Flood	7
,, 22	4	2	,,	6	,, 2	14	1	Normal	15
,, 23	...	1	,,	1	,, 3	7	...	,,	7
,, 24	1	2	,,	3	,, 5	7	1	½-Flood	8
,, 25	1	1	,,	2	,, 6	9	2	,,	11
,, 26	...	3	,,	3	,, 8	15	...	¼-Flood	15
,, 27	...	3	,,	3	,, 9	6	...	Normal	6
,, 28	...	3	,,	3	,, 10	6	...	,,	6
,, 29	...	1	,,	1	,, 11	1	1	,,	2
Dec. 1	...	4	,,	4	,, 12	2	1	,,	3
,, 3	...	1	,,	1	,, 13	2	...	,,	2
,, 6	2	6	¼-Flood	8	,, 14	2	...	,,	2
,, 7	...	3	,,	3	,, 17	1	...	,,	1
,, 8	4	16	,,	20	,, 18	1	...	,,	1
,, 9	5	21	Normal	26	,, 20	1	...	,,	1
,, 10	3	22	,,	25	Feb. 10	3	...	½-Flood	3
,, 12	4	10	¼-Flood	14	,, 14	1	...	,,	1
,, 13	1	13	½-Flood	14	,, 16	1	...	¼-Flood	1

Salmon, 172; Sea trout, 148; a total of 320 fish.

Isolating together the state of water and the totals as to numbers of fish, we find that—

During 40 normal days, there were 178 fish.
„ 8 ¼-flood days, „ 72 „
„ 9 ½-flood days, „ 69 „
„ 1 full-flood day, there was 1 „
58 320

This result shows the greatest ascent of fish while the river is in quarter-flood, there being then 9 fish a day, as compared with 7⅔ fish a day during half-flood, and scarcely 4½ fish a day during normal flow. The result as to full flood is necessarily of small value, since, during a flood of full force, observations are impossible. Reference to the table will show, however, that the two highest totals, viz., 26 fish and 25 fish (9th and 10th Dec.) occur during normal flow; but this normal flow succeeds three days of quarter-flood, during which we may presume the fish were working up the main river. No marked rise in the number of fish recorded is noticeable during any prolonged period of normal flow, yet it is evident, by the figures which occur between two rises of the water, that with only the average flow (12,000,000 gals. in twenty-four hours) both salmon and sea trout had no difficulty in entering the stream. Only one other point may be noticed before leaving this subject. A period of normal flow existed for thirty-two consecutive days from 9th Jan. to 9th Feb. During the nine first days fish occurred, but a gradual diminution in number is noticeable. From 20th Jan. to 10th Feb. no fish are recorded, then immediately, with a rise to half-flood, we have the three last records of salmon in Talla.

From a consideration of these data, therefore, it seems allowable to infer that the condition of water-flow preferred by fish when ascending from a main river to a tributary, for the purpose of spawning, is a moderate rise rather than a flood; and that a succession of moderate rises would fulfil the conditions under which most fish would be enabled to reach the upper spawning grounds of a large river such as the Tweed. Further, it seems clear that fish ascend in limited numbers when the water is in its normal condition.

A record of water temperatures of Tweed and of Talla was also kept. The readings were not taken with any precise accuracy, degrees only being given. It may, perhaps, be of interest to note that during at least the months of January and February the temperature of Talla Water seems to be higher than that of Tweed. The same thermometer was used in both cases, the observer walking from one stream to the other, yet, as a rule, the water of Talla is recorded as the higher by 2°. Only on one occasion do I find the reverse to be the case, the readings being, Tweed 40°, Talla 39° (14th Feb.). The few instances of the readings being the same occur chiefly when the air temperature is at or below freezing point.

In no case, when comparing temperatures with records of ascending fish, do I find evidence which leads me to suppose that particular fluctuations of temperature have any effect upon the movements of the fish. This negative result is, I think, of some interest, since the effect of a change of temperature in a tributary, as compared to a main stream, has been considered by some as of great importance.

The following is a record of the temperatures, which may be compared with the above table as to arrival of fish :—

WATER TEMPERATURES OF TWEED AND TALLA, TAKEN DAILY FROM 17TH JANUARY TO 15TH FEBRUARY 1897.

Tweed.	Talla.
32	34
32	33
33	35
33	35
32	34
32	34
32	32
32	34
32	32
32	32
32	32
32	32
32	34
32	35
32	36
33	36
33	36
33	35
33	35
33	35
33	35
35	37
35	37
35	35
36	38
36	38
33	36
37	39
40	39
40	40

If, now, we compare the salmon and sea trout columns of the above table (p. 4), we find that, of 320 fish, 172 were salmon and 148 sea trout. The 58 days' observations shown in the table begin and end with records of *salmon*, which are, however, noted on only 47 days of this period. The arrival of the sea trout is later by 6 days, being on 16th Nov. Their disappearance, on the other hand, is earlier by 8 days, being on 13th Jan. They are recorded on 36 days; and both species together are recorded on 25 days. During the four weeks (32 days) from 8th Dec. to 8th Jan. the figures become higher than at any other time during the season. Not only the greatest number of fish, but the heaviest fish appeared at this time.

It may be remarked, however, that the majority of the sea trout arrived before the largest runs of salmon, for if we compare the first 16 days of this period with the second 16 days, we find that the totals are as follows:—In the first 16 days, 104 sea trout and 32 salmon; in the second 16 days, 80 salmon and 8 sea trout.

The very great amount of poaching which is practised almost openly in Tweedsmuir district makes it impossible to say how long kelts remain after spawning. As a matter of fact, I believe very few fish are allowed time even to deposit a limited number of their ova. On one of my visits to the district, an old and experienced poacher expressed the opinion that not more than one fish in twenty escapes the leister.

Turning now to the consideration of the river Tweed, in the neighbourhood of Tweedsmuir, particulars were collected similar to those described for Talla Water. The part of the river Tweed under observation was about six miles in length, viz., from Stanhope up past Talla-mouth to Tweedhope. As compared with the 320 fish in Talla, the total number of fish in this part of Tweed during the same time was 684, being 512 salmon and 172 sea trout.

The majority of those fish were noticed in Tweed below Talla in 4 miles of water. In the 2 miles above Talla only 59 fish in all were counted.

As might be expected, the order in which the fish arrived, and the relative numbers, bear a more or less close resemblance to the conditions already described for Talla.

The largest runs of sea trout occur at an earlier date than the largest runs of salmon, and the figures showing this are opposite dates a little earlier than the dates of the largest runs in Talla. This gives us, then, a natural preliminary condition to that already described. We have the fish increasing in numbers, first in Tweed, then we find them in Talla.

The condition is sufficiently obvious from the following fortnightly results:—

	Salmon.	Sea Trout.
1st fortnight	23	1
2nd "	41	35
3rd "	26	91
4th "	35	38
5th "	144	4
6th "	201	2
7th "	19	0
8th "	23	1
	512	172

As also was the case in Talla, the heaviest fish were observed when the greatest number were recorded. Till the sixth week very few salmon over 12 lbs. in weight were noticed; on the 8th Jan., however, heavy fish appeared in numbers, and the figure, 201, which represents the total number of salmon during the sixth fortnight, is made up of 152 fish, estimated as of more than 12 lbs. weight.

Turning now to the number and position of salmon and sea trout *redds*, we find that the total for the area under consideration is 71, 11 being in Talla and 60 in Tweed. The majority of redds were noticed only after the fish had moved away, or been taken away by poachers, but on 24 of them fish were seen in the act of spawning. Of the fish on these redds, 16 were salmon and 8 were sea trout.

I have the dates of the 24 redds, but with such a small number out of 71 we can hardly draw any very reliable conclusions. The first salmon redd seems to have been noticed on 20th Nov. in Talla, the last on 15th Feb. in Tweed. The first sea trout redd was on 26th Nov. far up Talla, the last on 13th Dec., *i.e.*, two months earlier than the last salmon redd. We have already noticed, however, that after this date very few sea trout were present in the river. With the exception of the first date mentioned for sea trout, the other redds made by this species were noticed on 12th and 13th Dec. This indeed marks the height of the sea trout period in Tweedsmuir for 1896, the salmon being recorded as spawning during a much more extended period.

The total number of fish counted in the area under consideration was 1004, or 502 pairs. It may be matter of remark that, with so many fish in the water, there should have been only 71 redds. It should be recollected first, however, that amongst the thousand and four fish are very many enumerated more than once, whereas the 71 is the actual number of redds present in the river so far as the watchers could ascertain. To determine the difference in size between a salmon and a sea trout redd, 20 redds of the former and 10 of the latter were measured. The average length of the salmon redd, measuring from the upper end of the trough to the centre of the tail gravel thrown up below, was found to be 4 feet 6 in., and the breadth about 3 feet. The sea trout redd averaged 30 inches long by about 20 inches broad.

I believe I am correct in saying that every adult fish seen in the district was a fish in spawning condition, that no unripe fish is ever found so high up the river, and I believe that the difference between the 71 redds and the 502 pairs of fish is almost entirely accounted for by the diligent practice of local and other poachers.

Note on the Action of Hydroxylamine Hydrochlorate.

By **W. Brodie Brodie**, M.B., C.M., Physiological Laboratory, University, Glasgow. (*Communicated by* Dr M'Kendrick.)

(Read February 7, 1898.)

Hydroxylamine, NH_2OH (oxyammonia), is an unstable substance only to be had in weak solutions. It forms many salts, two of which, the sulphate, $2(NH_2OH.)H_2SO_4$, and the hydrochloride, $NH_2OH.HCl$, are to be found in commerce.

My attention was first directed to this substance, hydroxylamine, by seeing a reference to the work of Loew and Bokorny on the chemical constitution of protoplasm, and to their theory that there is present in it a group of atoms of the nature of an aldehyde. To this group of atoms they attribute the energy of living protoplasm, and state, in support of this, that hydroxylamine, which is said to have a strong affinity for aldehyde, rapidly kills protoplasm in algæ and the young shoots of plants.

My intention was to use both hydroxylamine and its salts upon some forms of animal protoplasm, but so far the hydrochlorate is the only preparation that has been available. This salt is easily soluble in water and is extremely acid. It is difficult to neutralise it exactly, as at a certain point it reacts to both red and blue litmus simultaneously. Solutions of 1 per cent. strength were employed almost entirely.

Ciliated epithelium, scraped from the mouths of frogs, was examined in ·75 per cent. salt solution, and a drop of the solution of the hydroxylamine salt was then run in. In a minute or so the ciliary movements became somewhat slower and continued to be so until they finally ceased. At the same time, the cell-body gradually cleared up, and the nucleus came more prominently into view.

Ciliated epithelial cells, from the gills of mussels, behaved exactly in the same way.

Nerve-muscle preparations gave negative results, both when irrigated for a long time with the fluid, and when the solution had been previously injected into a pithed frog. If a crystal of the salt, however, were laid upon the muscle, its power of contraction diminished rapidly, and disappeared in about five or six minutes.

Frogs' hearts showed the most marked results. About a minute after the solution was first applied, the frequency of the beats was distinctly less, and this was followed very shortly by diminution in the amount of contraction. Both the frequency and amount of the contractions continued to fall until the heart ceased beating entirely. Usually the auricles continued to beat for a short time after the ventricle had ceased.

In the case of the heart and nerve muscle preparations the muscle substance, after the application of the solution, was soft and gelatinous and very pale in colour, especially in the case of the heart.

It seems clear that this salt of hydroxylamine acts as a poison to protoplasmic structures, but how it acts is not explained. Nothing at present can be said regarding the aldehyde theory. It seemed possible to me, however, that its action might be due to some reducing or deoxidising action. Hydroxylamine is said to be a powerful reducing agent, and the solution of the hydrochlorate certainly reduces a solution of chloride of silver in ammonia.

Its action on blood-pigment is somewhat peculiar, and is the same whether the pigment be treated with a neutralised solution or a crystal of the salt, which is acid. The two dark bands of oxyhæmoglobin gradually fade and broaden out, but never absolutely disappear, whilst the part of the spectrum, from the commencement of the green to the violet, is darkened. It resembles the spectrum of alkaline hæmatin, with the addition of these two faint bands. The single band of reduced hæmoglobin is never seen.

Notes on some Additional Fossils collected at Seymour Island, Graham's Land, by Dr Donald and Captain Larsen. By George Sharman, Esq., and E. T. Newton, F.R.S. (*Communicated by* Sir ARCHIBALD GEIKIE, F.R.S.) (With a Plate.)

(Read February 7, 1898.)

Dr Donald, who sailed on board the "Active" in the voyage of 1892–3, has given a short account of the discovery of a number of fossils by Captain Larsen on Seymour Island, to the north of Snow Hill, in January 1893 (*Geographical Journal*, vol. ii. p. 438, 1893). Some of these fossils were sent to Professor James Geikie, and at his request we gave an account of them to the Royal Society of Edinburgh, June 4, 1894 (*Trans. Roy. Soc. Edin.*, vol. xxxvii. p. 707; see also Dr Murray, *Geog. Journ.*, vol. iii. p. 11, note, 1894). The fossils do not appear to have been found *in situ*. Captain Larsen reports of the locality "that he found no traces of vegetation there, the surface being formed of volcanic débris and numbers of these fossils" (Dr Donald's notice, p. 438, *loc. cit.*).

The condition of the fossils, however, would seem to indicate that the mother rock from which they were derived could not be at any great distance from the spot where they were picked up.

As the few species of Mollusca represented by these fossils seemed to us to find their nearest allies among Lower Tertiary forms, and to be near to certain species known from Patagonia (Darwin, *Geol. Obs. Sth. Am.*, 1846), we were led to conclude that these new discoveries indicated the occurrence of Lower Tertiary rocks in Seymour Island.

Two of the fossils examined by us were sufficiently distinct from known species to justify their receiving new specific names. One of these was a large *Cucullæa* resembling *Cucullæa alta*, Sow., from tertiary beds of Santa Cruz, and *C. decussata*, Sow., from the Lower Eocene of Britain, but, as it differed in certain points, it has been named *C. Donaldi.*

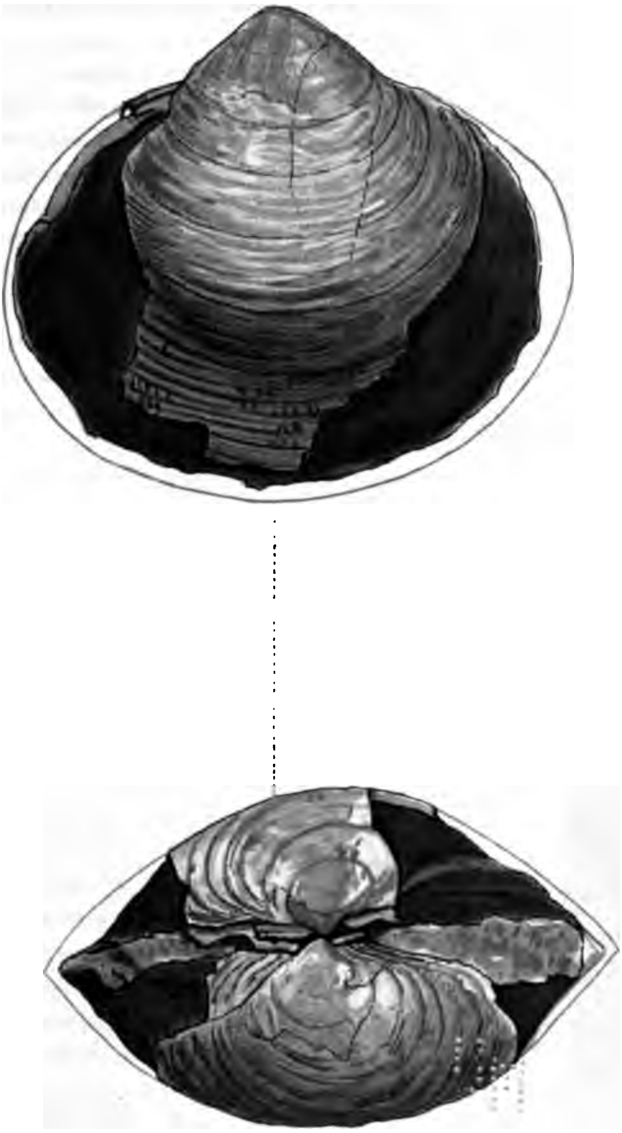
The second new form, called by us *Cytherea antarctica*, has some resemblance to the *C. orbicularis*, Edw., from the Lower Eocene of Britain, and the *C. bellovacina*, Desh., from the Sables inférieurs of France.

Besides these two shells we were able, with some doubt, to identify generically a *Crassatella* (?), *Donax* (?), and a *Natica* (?). With these shells we also received fragments of fossilised wood, which proved to be coniferous, and there is no reason for thinking that they are of different age from the fossils, as similar wood is commonly found in eocene deposits.

In 1893–4, Captain C. A. Larsen again sailed to the Antarctic regions (*Geographical Journal*, 1894, vol. iv. p. 333), and on the 18th of November 1893 landed at about the middle of Seymour Island. He says: "The land is hilly and intersected by deep valleys; some of the hills are conical and consist of sand, small gravel, and cement; here and there is some petrified wood." "When we were a quarter of a Norwegian mile from shore, and stood about 300 feet above the sea, the petrified wood became more and more frequent, and we took several specimens which looked as if they were of deciduous trees; the bark and branches, as also the year rings, were seen on the logs which lay slantingly in the soil." Captain Larsen also speaks of "petrified worms" with this wood, and these appear to be the filled-in borings in the wood, which we believe to have been made by *Teredo*. Several specimens of this wood, in the same condition as that previously described, have now reached us through Dr Murray, who, at the same time, sent another fossil shell which had been obtained on the previous voyage, on which is written "Graham's Land, 3.12.92. Cap. Seymour." This specimen is of interest as adding another genus and species to the list of fossils from these inhospitable Antarctic regions. Neither the hinge-teeth nor the pallial line can be seen, and we have, with some hesitation, therefore, referred this fossil to the genus *Cyprina*. Enough of the original shell remains to show its characters and the internal cast, which is almost perfect, gives evidence of the form of the entire shell; this, when perfect, must have been 3 inches long, 2·5 inches high, and 1·75 inch thick. The shell is regularly oval and nearly equilateral; the umbones, which are large, are nearly central, with only the slightest inclination forwards; there has been a strong external ligament. Although much denuded, it is evident that the exterior was marked by lines of growth, which are much stronger at regular intervals of about ¼ inch, giving a ringed appearance to the shell. Below some of these lines, especially towards the margin, the surface is puckered by vertical ridges, which give the shell an indistinct frilled appearance, like that seen on some examples of *C. planata* from the Thanet Sands. The almost equilateral form and nearly central umbones serve to distinguish this fossil from any shell which has been referred to the genus *Cyprina*; and the form which comes nearest to it is one from Navidad, Chili, described by Sowerby in Darwin's *Geological Observations in South America* (1846, p. 250, Pl. II. fig. 11) as *Corbis* (?) *lævigata*. The original specimen of this species is preserved in the Natural History Museum at South Kensington, and has been compared with the present fossil. It seems highly probable that the two shells may belong to the same genus, and this, as we think, is most likely to be *Cyprina*. Although the two shells are in many respects very similar, our Antarctic specimen has less inflated umbones; its greatest thickness is about the middle of the valves, and when complete was a rounder shell. In *C. lævigata* the valves are more inflated, and this chiefly towards the umbonal region, while it is a wider and more oval shell. These differences seem too great for specific identity, and, seeing how much we are indebted to Captain Larsen for the preservation of these fossils, we suggest that his name be associated with it and that it be called *Cyprina Larseni*.

The specimens of wood are very similar to that noticed in our previous paper, and sections prepared from some of them show that they also are coniferous; indeed, there is no reason why they should not all have been taken from one log.

Examples of the matrix found attached to the outside of this wood have also been received, and are found to be a calcareous grit, similar to that filling the mollusc shells, and it is highly probable that they come from the same formation. Mr J. J. H. Teall has examined this matrix, and says: "It is composed of angular fragments of quartz with minute particles of garnets, muscovite, and other minerals, cemented by carbonate of lime." Several pieces of this wood have been riddled by some boring animal, probably *Teredo*, or an allied form; sometimes the tubes are isolated, but in



G. Sharman, Del.

Cyprina Larseni, n.sp. from Graham Land.

other cases the wood has been nearly all fretted away, and the tubes filled in by a gritty matrix like that found on the exterior. In one instance the tube has been filled in by carbonate of lime, and here the shelly lining laid down by the *Teredo* may still be seen.

The following are the fossil forms now known from Seymour Island:—*Cyprina Larseni* and *Teredo* (?), here noticed for the first time; *Cucullæa Donaldi*, *Cytherea antarctica*, *Crassatella* (?), *Donax* (?), *Natica* (?), and coniferous wood. Although the number of forms known is at present few, yet these give promise of a large field of fossiliferous rocks as yet unexplored in the Antarctic regions, and it is of extreme interest to find that in this distant southern area Lower Tertiary deposits occur with forms of life so very similar to those with which we are familiar in our own northern country.

Note on the Axes of Symmetry which are Crystallographically possible. By Hugh Marshall, D.Sc.

(Read January 17, 1898.)

In a "Mémoire sur la déduction, d'un seul principe, de tous les systèmes crystallographiques avec leurs subdivisions," by Axel Gadolin,* there is given a proof that, if we assume the law of rational indices, only digonal, trigonal, tetragonal, and hexagonal axes are possible with crystals. This proof is adopted by Groth in the last edition of his *Physikalische Krystallographie*, 1895. The proof which I shall now give, although somewhat similar in general principle, is decidedly simpler than that of Gadolin. It is assumed in the former that an axis of symmetry is necessarily a possible edge or zone axis, and that there are possible edges perpendicular to any axis of symmetry, *i.e.*, that the plane to which it is normal is a possible face. These general propositions are not, so far as I am aware, to be found in text-books, although tacitly assumed in certain cases, and a proof of them, as also of two similar ones concerning planes of symmetry, is therefore indicated here before treating the main problem.

1. *Every plane of symmetry is a possible face.*—Any face A, inclined to the plane of symmetry S, gives by reflection in S a similar face A' lying on the other side of S. These two faces necessarily intersect in a line lying in S. Similarly, any other face B will intersect its image B' along another line lying in S. Consequently, S is parallel to two possible edges, and is therefore a possible face.

2. *The normal to every plane of symmetry is a possible zone axis.*—Let OX be any edge inclined to the plane of symmetry S. Its reflection OX' is a similar edge. The plane XOX' is therefore a possible face, and, from the symmetry, it is perpendicular to S.

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* *Acta Soc. Scient. Fennicæ*, ix. p. 1 (1871). A German translation is published as No. 75 of Ostwald's *Klassiker der exacten Wissenschaften*.

Similarly, any other edge OY will, with its image OY', determine a possible face YOY', also normal to S. S is therefore a zone plane, being perpendicular to two possible faces, and its normal is a possible zone axis.

3. *Every axis of symmetry is normal to a possible face.*—(*a*) Let the axis A be of the second (or any even) order. Any face B inclined to A will, on rotation round the axis through an angle π, give a similar face B', and these two faces necessarily intersect along a line perpendicular to A. Similarly, any other face C, inclined to A, has a corresponding face C' opposite it and equally inclined to A, and these two faces also intersect in a line perpendicular to A. Consequently, a plane parallel to these two edges, and therefore perpendicular to A, is a possible face. (*b*) If A is an axis of uneven order it must be at least of the third order, and any edge OB inclined to it necessitates at least two others, OB' and OB'', crystallographically identical with OB and equally inclined to A. If these three edges are taken as axes of reference, it follows, from the identity of the edges, that the axial ratios must be $b = b' = b'' = 1$. The face 1 1 1 would therefore cut OB, OB', and OB'' at equal distances from O; but such a plane is evidently the base of a right regular n-sided pyramid, and the axis A is normal to it. A is therefore normal to a possible face.

4. *Every axis of symmetry is a possible zone axis.*—(*a*) Let the axis A be of the second (or any even) order. Any edge OB inclined to A gives, by rotation through π round the axis, a similar edge CB' lying in the plane determined by A and OB. Similarly, any other edge OC necessitates one OC' symmetrical to it, lying in the plane determined by A and OC. But BOB' and COC' are possible faces, being parallel to pairs of possible edges, and A, their mutual intersection, is a possible edge or zone axis. (*b*) If A is of uneven order, it must be at least of the third order. Any face B inclined to it necessitates at least two others, B' and B'', crystallographically identical with it. These faces together form the sides of a regular n sided pyramid, whose base, perpendicular to A, is also a possible face, so that the basal edges are possible zone axes. There is, therefore, a whole series of pyramids, having the same base as the first, whose sides are also possible faces, such as C, C', and C''. The corresponding edges of two such pyramids determine a set of

possible faces normal to the common base and all intersecting in A. The base is therefore a possible zone plane, and A is a possible zone axis (see fig. 1.)

An axis of symmetry of the nth order is crystallographically possible only when $\cos\frac{2\pi}{n}$ *is rational.*—Let A be a crystallographical axis of symmetry of the *nth* order. An edge OB normal to it (possible as shown above) necessitates $n-1$ other edges crystallographically identical with it and all making equal angles with thei neighbours. Take two such neighbouring edges, OB and OB', alon with A, as axes of reference. The face 1 1 1 has equal parameter

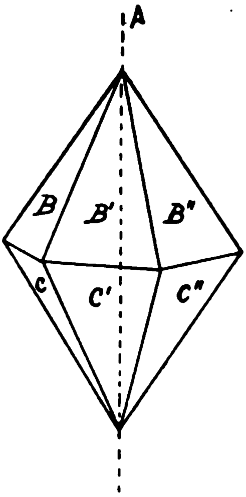

Fig. 1.

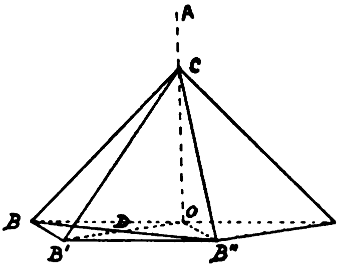

Fig. 2.

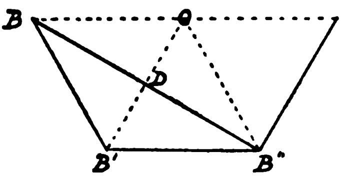

Fig. 3.

on the two equivalent axes, and gives rise, by the symmetry, to a regular n-sided pyramid. The plane passing through two alternate pyramidal edges, as CB and CB'', will also be a possible face, and must therefore have rational indices on the axes of reference. As it passes through the two points C and B it can have a rational index on the remaining axis only if the ratio OD/OB' is rational, OD being the parameter of BCB'' on the axis OB'. But $OB = OB'$, therefore OD/OB must be rational; since BB'' is perpendicular to OB', OD/OB is cos BOB', *i.e.*, $\cos\frac{2\pi}{n}$.

The law of rational indices, therefore, limits the value of n to those cases where $\cos\frac{2\pi}{n}$ is rational. Further, from the nature of an axis of symmetry, n must be a whole number. It is shown by N. Boudaief, in an appendix to Gadolin's paper, that the only values of n which satisfy these two conditions are 2, 3, 4, and 6. Consequently, only digonal, trigonal, tetragonal, and hexagonal axes of symmetry are possible with crystals.

The construction employed above is possible only when n is not less than 3. That digonal axes are possible follows at once, however, from the possibility of tetragonal and hexagonal axes, so that the case of $n = 2$ does not require to be specially considered.

Notes on some Specimens of Rocks from the Antarctic Regions, by Sir Archibald Geikie, F.R.S. With Petrographical Notes, by J. J. H. Teall, F.R.S.

(Read February 7, 1898.)

Towards the end of the year 1893, I received from Captain Thomas Robertson of Dundee, master of the steamship "Active," a number of specimens of rocks which he had collected in the Antarctic regions during a sealing expedition. In letters to me he gave the following particulars regarding these specimens:—"They were all taken from one place in Dundee Island. We had not time to land and take specimens from other parts. Dundee Island, so named by me, is a separate piece of land, close to Joinville Island, and all the stones I sent you were picked up on a beach at its south-west end. I had a boat sealing on Joinville Island, but they brought no specimens of rock on board. The round flat piece of light-coloured granite I broke off a piece the size of a man's head, the day I sent off the box to you. The small piece of blue basaltic rock I broke off at the same time, not to make the box too heavy. You have all the others as they were found. The south-west end of Dundee Island is a long low point which I named 'Welchness.' There are high cliffs up from the ness, from which the pieces of granite could have come, or they might have been carried by the ice.

"When going through 'Active Sound' I saw on the opposite shore, on the top of a high glacier, a number of large pieces of the red granite, and about five miles farther along the same shore there are steep basaltic cliffs. Close to the east end of Dundee Island there is a small island, named by Captain Ross Paulet Island, the peak of which is an extinct volcano."

The specimens sent by Captain Robertson had not the usual rounded and smoothed character of beach stones, but it was impossible to tell whether they had been derived from some neighbouring rock *in situ*, or had been borne from some distance. The granites and vein-quartz may indicate the existence of these rocks in Dundee Island. There was a piece of tuff which was probably carried from some other islet or coast-line. One of the most interesting of the specimens was a smoothed pebble of red jasper. On wetting it and examining it with a lens, I detected what appeared to be traces of *Radiolaria*. The specimen was accordingly submitted to Dr G. J. Hind, F.R.S., who was so good as to supply the following description of it:—

"In thin sections of the stone the radiolaria appear as clear translucent bodies, of circular or elliptical outlines, ranging in diameter from ·11 mm. to ·33 mm. As a rule, no structural characters are preserved, and, when magnified, the margins of these bodies are seen to be only indefinitely marked off from the reddish cloudy matrix. In some instances, however, the margins are dentate or with short projecting spines, and occasionally there are traces of a wall with alternate light spaces in it representing the perforations in the test. Most of the forms appear to have been simple spheroids or ellipsoids with a single test, but in some an inner test was likewise present. Elongate spines or rays project from the surfaces of some forms, but they are very imperfectly shown. In one or two rare cases, where the section has passed near the outer surface of the test, the perforated or lattice-like structure, characteristic of these bodies, can be distinguished.

"The preservation is too poor for certain identification of genera, but the common *Cenosphæra*, *Carposphæra*, and *Cenellipsis* are probably represented, as well as others.

"In addition to radiolaria, sponge spicules are present in this rock, and some evidently belong to Hexactinellids. No other organisms can be recognised. The rock is evidently of radiolarian origin."

The similarity of this radiolarian jasper to the cherts and jaspers of radiolarian origin, now so well known from older Palæozoic rocks in different parts of the world, suggests the possibility of the existence of rocks of high antiquity either on Dundee Island or within the ice-shed of that region. But the specimen itself furnishes no satisfactory evidence of its geological age.

The other specimens were placed in Mr Teall's hands for petrographical examination. As, however, additional material was likely to be obtained, his report and that of Dr Hind were held back. In November 1895 some further specimens were supplied by Dr John Murray, which he had received from Cape Adair, having been collected there by Mr C. E. Borchgrevink. These were also submitted to Mr Teall, who examined and described them. The publication of his notes, however, was still further delayed in the expectation that they would be capable of being extended by the receipt of another collection of specimens. As this hope has not yet been fulfilled, and as so much general interest is now felt in Antarctic exploration, it seems desirable to publish the following details without further delay.

PETROGRAPHICAL NOTES ON SPECIMENS FROM DUNDEE ISLAND AND CAPE ADAIR, by J. J. H. TEALL, F.R.S.

1. *Dundee Island.*

This collection contains several specimens of granite and vein-quartz, one of a dark green tuff, largely composed of felspars, and one of jasper. The largest specimen is an angular block of granite measuring 5″ × 6″ × 2″. There are three other specimens of the same rock of considerable size, one of which has evidently beer broken from a large mass, and nine small specimens of granite an vein-quartz, measuring about an inch across, and of approximatel equal dimensions in the different directions. These, as well as tl larger specimens, are either angular or sub-angular—never perfect rounded, as they would have been if they had formed pebbles a beach. There are no ice-marks on any of the specimens.

The granites belong to two varieties—grey and pink. T essential constituents are oligoclase, orthoclase, biotite, and ho blende. Zircon and iron-ores occur as accessory, epidote a chlorite as secondary constituents. In the grey granites id morphic crystals of white felspar, often beautifully zoned, a occasionally showing the twinning and extinctions of oligocl form a considerable part of the entire mass. Biotite and hc blende, both of which are more or less idiomorphic, also enter i the composition of the rock. The hornblende occurs sparin and may be either brown or green. One cross-section, showing forms $\{110\}$, $\{010\}$, and $\{100\}$, is brown in the interior, green on the exterior, when viewed with rays vibrating parallel to the mean axis of elasticity. The spaces between the above-mentioned idiomorphic constituents are filled up with allotriomorphic quartz and orthoclase.

The pink varieties of granite owe their colour to the presence of fairly large, but for the most part allotriomorphic, individuals of pink orthoclase. The well-formed crystals of white felspar, which enter so largely into the composition of the grey granites, are comparatively scarce. Hornblende is entirely absent. Quartz is abundant, and although never perfectly idiomorphic, it frequently indents the orthoclase in such a way as to show that it was in part formed before the growth of the felspar had ceased. The most important difference between the pink and the grey granites lies in the fact that the former are more largely composed of the minerals belonging to the second period of consolidation, that is, of quartz and orthoclase.

The tuff is a fine grained, green rock, the true character of which can only be determined by microscopic examination. It consists largely of crystals and crystalline fragments of water-clear felspar. The individuals are generally untwinned, less frequently composed of binary twins, and still less frequently striated. Next in importance to the felspars are small lapilli of a volcanic rock of andesitic character. The section shows also several grains of a pale-coloured augite, one of brown hornblende, and some of iron-ore. Chlorite, partly in the form of vermicular aggregates, occurs as a secondary product, and gives the characteristic colour to the rock.

The fragments of vein-quartz call for no special description. They may very well have been derived from the sedimentary series which furnished the specimen of radiolarian jasper in which veins of white quartz occur.

2. *Cape Adair.*

Two out of the three pebbles are formed of a vesicular olivine basalt composed of magnetite, olivine, augite, a basic plagioclase, and brown glass. The magnetite occurs in well-formed octahedra and in crystalline groups. It is found as inclusions in the olivine and augite, and is also abundantly scattered through the brown glass. Olivine is present as small crystals of simple form (about ·15 mm.), but does not make up any large proportion of the rock. Augite occurs in small crystals and crystalline groups. The largest group, consisting of four or five individuals arranged in a more or less radial manner, measures about ·5 mm. across. In colour, form, and mode of grouping, the augite is somewhat similar to that of the limburgites. The plagioclase is a basic variety, probably labradorite. The sections are, as a rule, lath-shaped in form; but there is a considerable amount of variation in the dimensions of the individuals, and in the proportion of length to breadth. The sections of some of the larger individuals indicate a tabular form. The felspar is remarkably free from inclusions. It far exceeds the other crystalline constituents in size, the individuals often measuring more than a millimetre in length.

The brown glass forms a considerable portion of the entire mass. The colour is not uniformly distributed, being less marked in the vicinity of augite and magnetite.

The third pebble is a more compact basalt in which olivine, if it occurs at all, is only represented by a few very minute grains. The rock is composed of acicular microlites of felspar, augite (scarce), magnetite, and brown glass. The microscopic section of this rock contains one large phenocryst of plagioclase. The microlitic felspars are arranged with their longer axes approximately parallel to one another, thus giving to the rock a marked fluidal structure.

Observations on the Theories of Vowel Sounds. By John G. M'Kendrick, Professor of Physiology in the University of Glasgow.

(Read February 7, 1898.)

The quality of the human voice depends on the same laws as those determining the quality, klang-tint, or timbre of the tones produced by any musical instrument. Tones of a mixed character, that is to say, composed of a fundamental and partials, are produced by the vibrations of the true vocal cords, and certain of those partials are strengthened by the resonance of the air in the air-passages, and in the pharyngeal and oral cavities.

So strongly may certain of these partials be reinforced, as to obscure or hide the fundamental tone, and give a peculiar character to the sound. These, however, are only general statements, and there are still many difficulties in the way of a true interpretation of voice-tones. In the first place, we observe that we may sing a scale, using one sound for each note, such as *la*, *la*, *la*, etc. Or, by putting the mouth in a certain position, we can pronounce the so-called vowels, *a*, *e*, *i*, *o*, *u* (*ou* as the *u* in *prune*), uttering the sounds ah, ā, ē, o, ou. As we do so, we notice that each sound appears to the ear to have a pitch of its own, different from that of the others. Thus, Helmholtz * gave the pitch of the vowel-tones as follows:—

Vowels, . .	OU	O	A	AI	E	I	EU	U
Tone, . .	fa$_2$	si$\flat_3$	si$\flat_4$	sol$_5$ or re$_4$	si$\flat_5$ or fa$_3$	re$_6$ or fa$_2$	do$_5$ or fa$_3$	sol$_5$ or fa$_2$
No. of vibs.,	170	470	940	1536 or 576	1920 or 341	2304 or 170	1024 or 341	1536 or 170

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* Helmholtz, *Sensations of Tone*, trans. by Ellis, 1875, p. 165; see also same work, p. 163, footnote 1, for result obtained by Merkel, Reyher, Hellwag, Flörcke, and Donders.

But Koenig* has given a different valuation of the pitch, thus:—

Vowels, . . .	OU	O	A	E	I
Tone, ...	si$\flat_2$	si$\flat_3$	si$\flat_4$	si$\flat_5$	si$\flat_6$
No. of vibs., . .	235	470	940	1880	3760

Donders,† who was the first to observe that the cavity of the mouth for different vowels is tuned to different pitches, gave U as f, O as d, A as b′♭, ŏ (like *oo* in *too*) as g^2, ŭ (like *aw* in *knaw*) as a″, E as c‴, and I as f‴.

Again, we may sing a vowel on a scale, and still one can recognise the vowel in each note. Thus, if we sing *a*, or *o*, or *i*, on a scale beginning with *c*, the ear catches the sound of *a*, or *o*, or *i* in each note. Such tones are termed vowels, or we might call them vowel-tones.

Theories of Vowel-Tones.—The investigation of vowel-tones may be considered to date from the experiments of Willis,‡ about 1829, who imitated vowels by means of a reed, above which he placed a resonating cavity; and his conclusions are very similar to those put forward by Hermann at the present day. About 1837, Wheatstone § made some observations, and gave a theory. In later times, the subject has been studied by Donders, Helmholtz, Koenig, Hermann, and many others.

The invention of the tinfoil phonograph by Edison in 1877, and the improvement of the instrument by the labours of Edison, Graham Bell, and others in more recent years, until we now possess, in the wax-cylinder phonograph, an almost perfect mechanism, have led to the reinvestigation of the whole question of vowel-tone by Fleeming Jenkin and Ewing, Hermann, Pipping, Boeke, Lloyd, M'Kendrick, and others. The difficulties in the way of accounting for some of the phenomena of vowel-tones will be appreciated when we state that competent observers, such as those above named and many others, are ranged in two camps,—those who uphold the theory of relative as opposed to those who contend for the theory of fixed pitch. Assuming that a vowel is always a compound tone, composed of a fundamental and partials, those who uphold the relative pitch theory state that if the pitch of the fundamental tone is changed, the pitch of the partials must undergo a relative change, while their opponents contend that whatever may be the pitch of the tone produced by the larynx, the pitch of the partial that gives quality or character to a vowel is always the same, or, in other words, vowel-tones have a fixed pitch.

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* Koenig, *Comptes rendus des séances de l'Académie des Sciences*, 1870, t. lxx. p. 931; also *Quelques expériences d'Acoustique*, 1882, p. 47.

† Donders, *De physiologie der Spraakklanken*, 1870, p. 9.

‡ Willis, *Cambridge Philosophical Transactions*, 1829, vol. iii. p. 231; also *Poggendorff's Annalen der Physik*, vol. xxiv. p. 397.

§ Wheatstone, *Westminster Review*, Oct. 1837.

There are many methods of investigating this problem, but these may be grouped in two divisions:—1*st*, Experimental methods by which the pitch of the oral cavity, in the position suitable for the production of any given vowel, may be determined; and 2*nd*, mathematical methods by which the curve, or wave-form, representing a certain quality of vowel-tone, may be analysed into its components, in accordance with Fourier's theorem.

One of the early experiments of Willis* favoured the fixed pitch theory. A piece of watch spring was held by forceps against a revolving toothed wheel. A compound tone was produced which, of course, retained the same pitch so long as the wheel revolved uniformly. Now, by keeping the wheel steadily revolving at a uniform rate, and at the same time changing the length of the portion of the spring which was allowed to vibrate, Willis found that the qualities of various vowels were obtained with considerable distinctness. The objection to this experiment obviously is that it had nothing to do with resonating cavities.

Willis also used reed pipes attached to cylindrical chambers of variable length, and altered the quality of tone by increasing or diminishing the length of the resonant chamber. The shortest tubes gave I, then E, A, O, to U. In this way he determined the pitch of the vowels, as they sound in words.†

As already stated, Donders‡ was the first to show that the cavity of the mouth, as arranged for the giving forth of a vowel, was tuned as a resonator for a tone of a certain pitch, and that different pitches corresponded to the forms of the cavity for the different vowels. This he discovered not by the use of tuning-forks, but by the peculiar noise produced in the mouth when the different vowels are whispered. The cavity of the mouth is then blown like an organ pipe, and by its resonance reinforces the corresponding partials in the rushing wind-like noise. The question was then taken up by Helmholtz,* and was treated in his usual masterly fashion. To determine the pitch of the cavity of the mouth, considered as a resonance cavity, he struck tuning-forks of different pitches, and held them before the opening of the mouth, widely opened. Then the louder the proper tone of the fork was heard, the nearer "it corresponded with one of the proper tones of the included mass of air." As the shape of the mouth could be altered at pleasure, according to the vowel to be emitted, it was easy to discover the pitch of the included mass of air for each vowel. He came to the conclusion that "the pitch of the strongest resonance of the oral cavity depends solely upon the vowel for pronouncing which the mouth has been arranged." He also found the same resonances for men as for women and children. He then carefully examined the form of the oral cavity for each vowel, and showed how very slight changes would account for the quality being somewhat altered for different dialects.

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* Willis, *op. cit.*, p. 231.

† Ellis, see footnote in *Sensations of Tone* by Helmholtz, p. 170; also "Sound," in *Encyc. Metropol.*, p. 375, by Sir John Herschell.

‡ Donders, *op. cit.*; also *Archiv. für die holländischen Beiträge*, vol. i.; see also references to older observers in Helmholtz's *Sensations of Tone*, p. 162 (footnote).

Helmholtz also showed that the tones of the human voice have peculiar relation to the human ear. Thus the upper partials of I are in the neighbourhood of e'''' up to g'''', and the human ear itself is tuned to one of these pitches, that is to say, by its resonance it favours the perception of these tones. Powerful male voices produce these partials strongly. Thus, a bass voice singing e' produced the 7th partial d'''', 8th e'''', 9th f'''', and the 10th g''''. Some of these partials, such as e'''' and f'''', may clash, and, by producing dissonance, give harshness to the voice. This illustration will show how Helmholtz, following out the theory applicable to all musical instruments, endeavoured to explain the quality of different voices. His theory as to vowel-tone is summed up in the following sentence:—"Vowel qualities of tone consequently are essentially distinguished from the tones of most other musical instruments by the fact that the loudness of their partial tones *does not depend upon the numerical order but upon the absolute pitch of those partials.* Thus, when I sing the vowel A to the note E♭, the reinforced tone b''♭ is the 12th partial tone of the compound; and when I sing the same vowel A to the note b'♭, the reinforced tone is still b''♭, but is now the 2nd partial of the compound tone sung." *

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* Helmholtz, "*Ueber die Vokale*," *Archiv. f. d. holländ. Beiträge z. Natur. v.Heilkunde*, Bd. i. pp. 354–355, 1857; also "*Ueber die Klangfarbe der Vocale*," *Gel. Ang. d. k. bayr. akad. d. Wissensch.*, 1859, pp. 537–531; *Poggendorffs Ann.*, Bd. 108, p. 180; also in *Wissenschaftl. Abhandlungen*, Bd. I. pp. 395, 397; also *Sensations of Tone*, chap. v. p. 153.

Further, Helmholtz endeavoured to demonstrate the correctness of his view by synthetically combining the tones of certain tuning-forks in his well-known vowel-tone apparatus.† He in early experiments used eight forks, the first being the fundamental tone si$_1$♭, and the others the first seven partials. Thus:—

si$_1$♭ si$_2$♭ fa$_3$ si$_3$♭ re$_4$ fa$_4$ la$_4$♭ si$_4$♭.

The vowel O is well sounded with this apparatus when we sound si$_3$♭ (characteristic of O) strongly, more feebly si$_2$♭, fa$_3$, re$_3$, and the fundamental softly. OU is good with si$_1$♭ strong, and the partials feeble. Using, in another apparatus,

si$_2$♭ si$_3$♭ fa$_4$ si$_4$♭ re$_5$ fa$_5$ la$_5$♭ si$_5$♭,

OU was given with si$_2$♭ (the fundamental) alone. O was sounded by fundamental si$_2$♭ moderate, si$_3$♭ strong, and fa$_4$ weak. If we sound si$_2$♭ (fundamental) along with si$_3$♭, and fa moderate, and si$_4$♭ and re$_5$ strongly, we obtain A. This vowel is characterised by si$_4$♭, along with the partials si$_3$♭ and si$_2$♭. To obtain E, give si$_2$♭ and si$_3$♭ moderate, and fa$_3$, la$_3$♭, and si$_3$♭ as strong as possible. The characteristic partial of this vowel is si$_3$♭. I have performed many experiments with this apparatus, and find the results obtained by Helmholtz to be consistent with experience. Much depends, in the appreciation of this experiment, on careful attention, practice, and a good ear.

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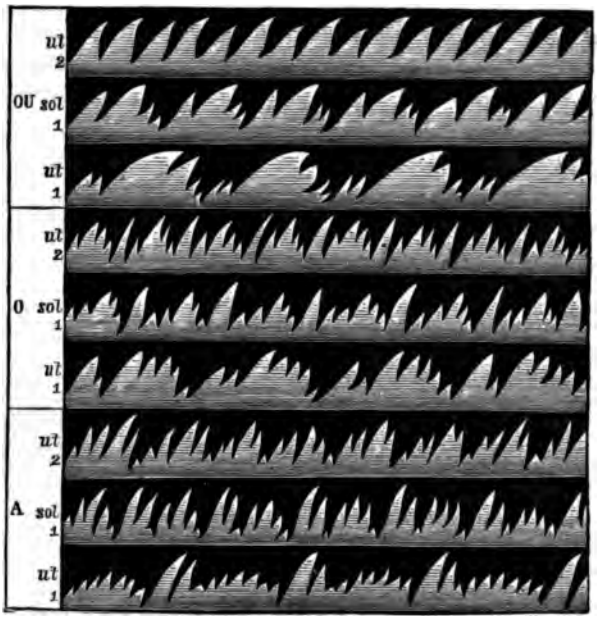
* Helmholtz, *Sensations of Tone*, p. 172.

† For figure and description of this famous apparatus, see M'Kendrick's *Physiology*, vol. ii. p. 691.

Koenig investigated the subject with the aid of his manometric flame apparatus. This method is useful, because it shows the different forms of the sound wave without change of period, corresponding to a tone of determinate pitch when the vowels are sung upon that tone. The flame picture shows the forms taken by the flame when the three notes ut$_1$, sol$_1$, and ut$_2$ are sung with the vowels A, O, and OU.



Each of these pictures is composed of two groups of teeth reproduced again and again, and thus showing a regular periodic vibratory movement. The three pictures of ut contain eight groups of teeth, while those of sol contain twelve, and those of ut$_2$ sixteen. As the length of all these pictures correspond to the same duration of a vibratory state, it is evident that the period of the vibratory movement, or the length of the wave characteristic of a tone of determinate pitch, is independent of the vowel upon which the sound is emitted. But the form of the wave characteristic of the tone of given pitch varies much with the vowel upon which it is sung. This alteration of the form of the wave, while the period is constant, must be due to the super-position of a tone developed in the mouth, characteristic of the vowel, upon the tone emitted by the larynx.*

According to Koenig, in man, OU is always easily emitted with si$_2$♭, and the neighbouring tones la$_2$, sol$_2$ with the lower partials si♭, and mi♭; O is emitted with si$_3$♭, the partials la$_3$, sol$_3$, and the lower partials si$_2$♭, mi$_2$♭, si♭; A comes with si$_4$♭, with la$_4$, with sol$_4$, along with the lower partials si$_3$♭, mi$_3$♭, si$_2$♭; E and I are not easily sounded on low tones, as their characteristic partials are very high. On tones lower than ut$_3$, the female voice turns involuntarily to O or OU, which have si$_3$♭ and si$_2$♭ as their characteristic partials. Above fa$_4$ it is A, of which si$_4$♭ is characteristic, which is most readily given. Above si$_4$ the voice passes into E and I. These observations of Koenig are harmonious with the conclusions of Helmholtz, and favour the fixed pitch theory.

Hallock† has recently employed a device founded on that of Koenig. Eight resonators in harmonic series were each connected with a manometric capsule, and photographs were taken of the eight bands of flame pictures, reflected in a mirror, when the vowels were sung before the resonators.‡ From these photographs the partials present in any vowel-tone within the range of the resonators could be determined. Of course the higher partials, on which, as pointed out by Helmholtz, much depends, were not detected.

Wave forms of Vowel-tones.—We must now turn to the evidence adduced by an analysis of the wave forms of vowel-tones. To appreciate this evidence the following statements must be kept in view:—

1. Pitch depends on the length of time in which each single vibration is executed, or, in other words, on the vibrational number of the tone.

2. Musical tones are higher the greater their vibrational number that is to say, the shorter the vibrational period.

3. The sensation excited by a periodic vibration is a musical

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* Gavarett, *Phénomenes Physique de la Phonation et de l'Audition,* Paris, 1877, p. 394.

† Hallock, *American Annual of Photography,* 1896, p. 21.

‡ For a figure of such an analyser, see M'Kendrick's *Physiology,* vol. ii. fig. 418, p. 686.

tone. This tone is usually compound, the constituents being partial tones.

4. Only one form of vibration, like that of a pendulum, or the limbs of a tuning-fork, can give rise to a simple tone, destitute of partials. This is a simple pendular vibration, and the sensation is a simple tone.

5. A compound tone is the sensation produced by the simultaneous action of several simple tones, with definite ratio of pitch. Such a compound tone corresponds physically to a wave of more or less complex form.

6. Such a compound wave is capable of being analysed into a number of simple pendular vibrations, and each pendular vibration corresponds to a simple tone, having a pitch determined by the periodic time of the corresponding motion of the air (Ohm's law).

7. It is evident that such combinations of simple waves may give rise to an infinite variety of wave forms; but, according to Fourier's theorem,* "Any given regular periodic form of vibration can always be produced by the addition of simple vibrations, having vibrational numbers, which are once, twice, thrice, four times, etc., as great as the vibrational number of the given motion."

8. If we know the *amplitudes* of the simple vibrations and the *difference of these*, then any regularly periodic motion can be shown to be the sum of a certain number of pendular vibrations; in other words, the compound wave may be analysed into its constituents.

[Dr M'Kendrick then described the method of applying the Fourierian analysis, as given in the paper by Dr R. J. Lloyd in the *Proceedings of the Royal Society of Edinburgh*, 1898.]

Phonograms.—We are now in a position to examine the methods that have been adapted to obtain graphic tracings of the wave forms, or "phonograms" corresponding to vowel-tones, so as to submit these to the Fourierian analysis.

Donders,† in 1870, was the first to apply the phonautograph of Leon Scott‡ (invented in 1856) to the investigation of the curves produced by the sounds of vowels. In 1870, Fleeming Jenkin and Ewing* succeeded in obtaining tracings of the records of vowel sounds on the tinfoil phonograph, and the curves were submitted to harmonic analysis. This was the beginning of the present discussion. These two observers obtained good curves, even with the imperfect instrument, and the curves were submitted to analysis, so as to determine the amplitude of their constituents up to the sixth partial, and the process of measurement and calculation was applied to more than 100 curves. The sixth partial was not a high limit to reach, but the phonograph at that date did not record the higher partials. For example, it could not reproduce the sound of I in machine nor the French or German Ū. Twelve values for y (the lengths of the co-ordinates for one period chosen) gave the data for calculating the amplitude and phases of the first six partials. Professor Tait supplied the authors with the solutions of the simultaneous equations for twelve values of y, and the results are given in a series of tables, in which the predominance of certain partials is unmistakable. In this way each vowel was examined. The conclusions arrived at were, on the whole, favourable to the constant pitch theory, and were thus summed up:—"It is clear that the quality of a vowel sound does not depend either on the absolute pitch of reinforcement of the constituent tones alone, or on the simple grouping of relative partials independently of pitch. Before the constituents for a vowel can be assigned, the pitch of the prime must be given, and, on the other hand, the pitch of the most strongly reinforced partial is not alone sufficient to allow us to name the vowel. To do this, we must also know the relation of the constituent partials to one another."† Again, "the ear is guided by two factors, one depending on the harmony or group of relative partials, and the other on the absolute pitch of the reinforced constituents." The ear recognises, as it were, when it hears a vowel sung *at any pitch*, the kind of oral cavity causing the reinforcement. As to the question, "is the resonance cavity for a vowel sound constant at all pitches"? the authors are cautious, and their opinion appears to be that, whilst the resonance cavity for a vowel sound has an absolute pitch, it may have a certain effect in reinforcing other subordinate tones or partials. While experimentally it may be shown that a *constant* cavity may produce a vowel-like tone, say O, over a wide range of pitch, it is probable that the resonant cavities of the human being are slightly adjustable, so as to be, as it were, tuned to the pitch on which the vowel is sung. In other words, Fleeming Jenkin and Ewing hold that both the relative and the absolute factors enter into the composition of a vowel, a conclusion not far from the truth.

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* Donkin, *Acoustics*, "Fourier's Theorem proved," pp. 65 to 71; "illustrated," pp. 56 to 65; see also Everett, *Vibratory Motion and Sound*.

† Donders, *De Physiologie der sprachklanken in het bijzonder van die der nederlandische taal*, Utrecht, 1870.

‡ E. Leon Scott, *Compt. rendus*, t. 53, p. 108.

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* Fleeming Jenkin and Ewing, "On the Harmonic Analysis of certain Vowel Sounds," *Trans. Roy. Soc Edin.*, vol. xxviii. p. 745.

† Jenkin and Ewing, *op. cit.*, p. 770.

The subject was taken up by Hermann * about 1890, and he used the much improved wax-cylinder phonograph. He succeeded in obtaining photographs of the curves on the wax cylinder, a beam of light reflected from a small mirror attached to the vibrating disk of the phonograph being allowed to fall on a sensitive plate, while the phonograph was slowly travelling. The curves thus obtained, representing the wave-forms of the vowel-tones, were very beautiful. They were submitted to analysis, with the view of estimating the pitch of the mean partial or the "formant," as it is called by Hermann, according to the method described in Dr Lloyd's paper already referred to. Hermann also pointed out that the quality of a vowel-tone varies considerably, according to the rate at which the cylinder was rotated. This should obviously be not the case were the relative pitch theory correct. It is curious that, even with competent observers, there should be such difficulty in deciding this apparently simple question of fact, some asserting that there is no difference in quality, and others as positively stating that there is a difference when the cylinder is caused to move slowly. After many experiments, I have come to the conclusion that there is a difference but not so great as to disguise the quality of the vowel. The ear can always distinguish O, or A, or E at different rates, but the quality is undoubtedly altered. The sound of a vowel never passes, in my judgment, into the sound of another vowel. Professor Hermann maintains the reverse.

Hermann presents the fixed pitch theory in a modified form, and states that there is for each vowel a characteristic tone—a formant. The pitch of the formant, however, may vary considerably; indeed, with the same prime, it may vary as much in certain cases as several semitones. The pitches of the formants according to Hermann are represented as follows *:—

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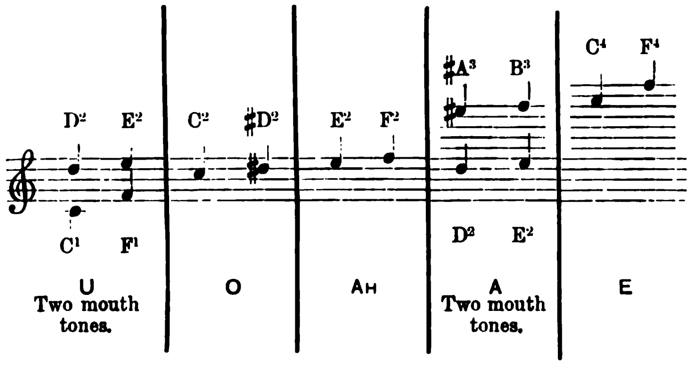
* Hermann, *op. cit.*



Hermann maintains that the "formant" need not necessarily be a partial of the fundamental. Sometimes it is such a note, but most often not.†

Sauberschwartz ‡ has investigated the subject by an ingenious application of the laws of the interference of sound. Certain vowels were sung into the mouthpiece of a long tube, to which other short tubes of definite length were attached. By closing the outer ends of certain of these tubes, various partials could be extinguished by interference, and the listener at the other end of the tube observed an alteration in the quality of the vowel. Thus, by extinguishing the formant of A, the sound approximated to that of O, and became somewhat nasal. With E there are two characteristic formants, and Sauberschwartz found that if one of these was extinguished, the quality of the vowel was not much altered, but that when both were shut out, the vowel was much changed in quality, and approximated to U or O. Sauberschwartz in general supports Hermann.

Dr Boeke § of Alkmaar has not only devised an ingenious and

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* Hermann, *Pflüger's Archiv.*, Bd. 35, p. 42, 1893. Professor Rutherford kindly directed my attention to this diagram.

† Hermann, *Pflüger's Archiv.*, vol. 61, 1895, p. 178.

‡ Sauberschwartz, *Pflüger's Archiv.*, Bd. 61, 1895.

§ J. D. Boeke, "Microskopische Phonogramstudien," *Pflüger's Archiv.*, Bd. 50, p. 297.

most accurate method of obtaining curves from the wax cylinder of the phonograph, but he has applied the Fourierian analysis with striking results in general support of the fixed pitch theory.[*] The method consists in measuring microscopically the transverse diameter of the impressions on the surface of the phonograph cylinder, on different (generally equidistant) parts of the period, and in inferring from these measurements the depth of the impressions on the same spot, or, in other words, deriving from them the curve of the tone which produced the impression.

[Dr M'Kendrick here alluded to Dr Boeke's paper, describing his method, in the *Proceedings of the Royal Society of Edinburgh*, 1898.]

Curves of the vowel-tones may also be obtained on a larger scale by M'Kendrick's[†] phonograph recorder.

The following analyses, supplied by Boeke, are very instructive. The first is the analysis of the tones of a cornet. Observe how the intensity of the partials gradually diminishes:—

Tone.	1	2	3	4	5	6	7	8	9	10	Partials.
f = 170 v.d.,	1	1·05	1·22	1·15	0·01	0·80	0·53	0·28	0·13	0·10	,,
c′ = 256 ,,	1	0·92	0·81	0·53	0·39	0·20	0·07	0·04	0·06	0·04	,,
g′ = 384 ,,	1	0·76	0·46	0·14	0·09	0·06	0·07	0·02	0·01	0·01	,,
c′ = 512 ,,	1	0·92 (?)	0·30	0·14	0·15	0·09	0·07	0·06	0·03	0·02	,,

Contrast this analysis with that of the vowel A, sung by Boeke (aged 50) on the notes *f* and *c′*, and the same vowel on the notes of *g′* and *c″*, sung by his son, aged 12, both like the vowel in the word "heart." It will be seen, from the analytical numbers, that the formant of the son's vowel A was almost the same as that of the father, although the pitch of his voice was exactly an octave higher.

Man, aged 50, *singing* āā—

Pitch.	1	2	3	4	5	6	7	8	9	10	Partials.
f 170 ♭ v.d.,	1	0·86	0·46	*1·74*	*1·90*	1·55	0·51	0·54	0·43	0·44	,,
c′ 256 ,,	1	0·49	*1·96*	*1·25*	0·60	0·56	0·23	0·05	0·06	0·10	,,

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[*] I have to thank Dr Boeke for unpublished notes on the subject, and also for many interesting analyses.

[†] M'Kendrick, "Observations on the Phonograph," *Trans. Roy. Soc. Edin.*, vol. xxxviii. part iv. No. 22; also *Proc. Roy. Soc. Edin.*, 1896–97, "Opening Address"; also *Sound and Speech Waves as revealed by the Phonograph*, London, 1897.

Boy, aged 12, *singing* āā—

Pitch.	1	2	3	4	5	6	7	8	9	10	Partials.
g′ 384 v.d.,	1	*1·22*	*2·67*	0·45	0·17	0·06	...	...	...	...	,,
e″ 640 ,,	1	*8·09*	1·45	0·53	...	...	...	...	...	...	,,

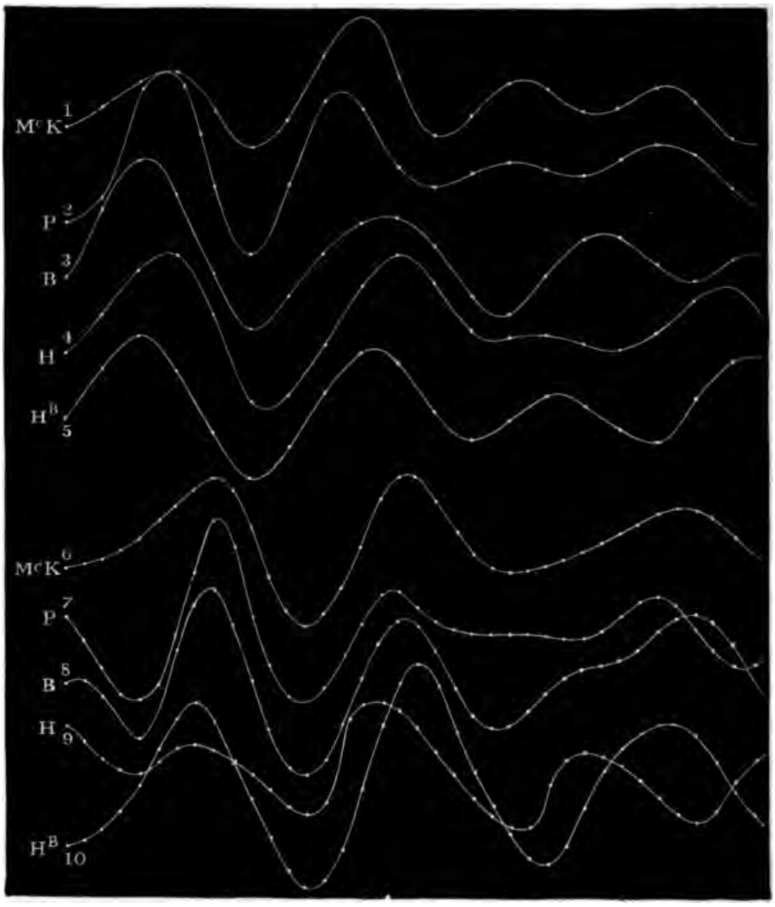
Dr Lloyd has examined these figures, and supplied me with the following very instructive table:—

Man's AA.	Partials reinforced.	Evaluated by Hermann's method.		Evaluated by Pipping's method (= Lloyd's formula).	
		Mean partial.	V.D.	Mean partial.	V.D.
f 170 v.d.,	. 4–6	4·96	846	4·90	836
c′ 256 ,,	. 3–4	3·39	868	3·35	859
Boy's AA.					
g′ 384 v.d.,	. 2–4	2·82	1084	2·75	1057
e″ 640 ,,	. 1–3	2·04	1307	1·98	1267

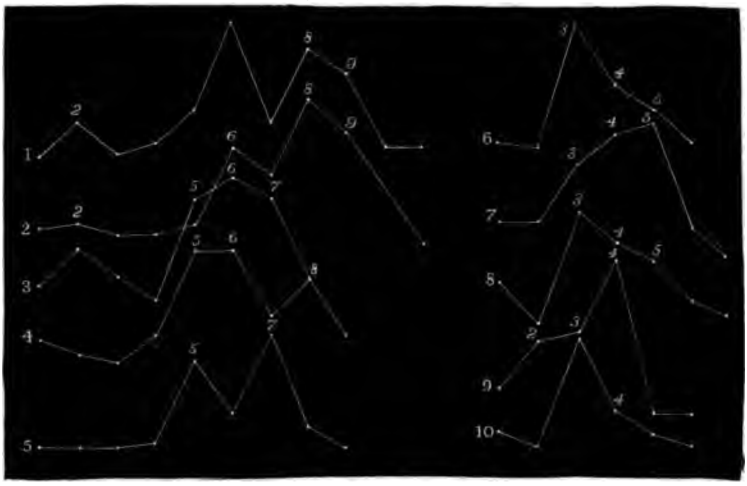
A study of these figures will show (1) that the man's resonance rises slightly ($\frac{1}{2}$ semitone) in ascending 7 semitones in the middle of his register; (2) that the boy's resonance rises 3 semitones in ascending 9 semitones in the upper half of his register; and in the mid-register the boy's resonance is to the man's as 5 : 4. This indicates that, as we ascend a scale in singing a vowel, the pitch of the oral cavity slightly changes.

Dr Lloyd holds a view differing from those already described. Vowels, according to him, owe their character, not to the resonance of a partial or partials of a certain fixed pitch, but to the relative pitch of two or more partials. This view accounts for a fact which is not explained by the other theories, namely, that the articulation of a vowel seems to be the same for a child as for an adult. Thus, in the vowel A, as in "fat," sung on a note having a pitch of 134 v.d., Lloyd finds two partials, the lower of which he terms the pharyngeal or α-resonance, and the other the oral or β-resonance. Of these the lower for A in "fat" has a vibrational number of 736 v., while the upper has 1121. The ratio of these two partials is, therefore, $\frac{1121}{763} = 1{\cdot}47$. This is termed by Lloyd the radical ratio, and it determines the nature of the vowel. Again, the radical ratio of the Swedish long A is 1·35.

The following instructive curves were sent to me by Dr Boeke, They are the curves, taken by Boeke's method, of the vowel A (" as in fat") sung by M‘Kendrick, Boeke, Pipping, and Hermann on a pitch of $ut_2 = 128$ v.d. and $ut_3 = 256$ v.d. The curves are instructive as showing the same vowel sung on the same pitch by men of different nationalities. The results of harmonic analysis, giving, in plotted lines, the value of the partials, are placed below the diagram. There is a likeness between the A^c of M‘Kendrick and Pipping, and also between M‘Kendrick's A$^{c'}$ and Boeke's A$^{c'}$, both in form and analysis.



Dr Lloyd, in support of his view of a cleavage in the reinforcements, which is the sign of two separate resonances, has reported upon two examples of o sung by Hermann, both at 132 v.d., and analysed by Dr Boeke. The amplitudes were as follows:—



V.D.,	1	2	3	4	5	6	7	8	9	10
132,	. 6·7	10·7	18·4	14·6	18·1	4·7	2·5	1·3	1·0	0·5
132,	. 7·8	23·4	11·2	7·0	7·0	3·6	2·1	1·2	0·6	1·0

To these he adds two other examples of the same vowel, one sung at 148 v.d. by M'Kendrick, and the other at 128 v.d. by Boeke—

V.D.,	1	2	3	4	5	6	7	8
148, . . .	1	1·12	1·73	0·19	1·90	0·97	0·35	...
128, . . .	1	2·32	5·81	2·10	5·48	0·32	0·55	0·76

The 4th partial in each of these four analyses shows a palpable falling off in strength, as compared with its neighbours on either side, and this falling off marks in each case the gap between an *α*-resonance of 300–400 v.d. and a *β*-resonance of 600–800 v.d. The vowel o, however low it may be sung, can hardly have more than two partials intermediate to the two culminations. In three out of the four cases above given, it has only one intermediate partial. This common partial must be subject to a strong influence from both resonances at one time. All the other partials are situated so much nearer to the one resonance than to the other that they may be regarded as being under the sole influence of the former. The common partial, on the other hand, receives a stimulus from each, and it must be remembered that these stimuli are mutually independent, as the one operates in the oral and the other in the pharyngeal cavity, and that it is a matter of chance whether, in any phonogram analysed, these two stimulations operate to exaggerate or to conceal each other in the tabulated numerical strength of the partial. Thus it comes to be a question of phase.

Dr Lloyd has evaluated the α- and β-resonances of the vowel O, as above given, and stated the result in the following table :—

Sung by	V.D.	Partials Reinforced. α	Partials Reinforced. β	Mean Resonance. α	Mean Resonance. β	Radical Ratio.
Hermann, .	132	2–4	4–7	421	653	1·553
Hermann, .	132	1–3	4–7	290	672	2·318
M'Kendrick,	148	1–4	4–7	365	812	2·221
Boeke, . .	128	2–4	4–6	391	615	1·573

These four available instances give an average of 368 v.d. for α-resonance, of 688 v.d. for β-resonance, and of 1·916 for the radical ratio. A sounder method would be to take from one voice a sufficient number of O analyses to constitute an average, thus eliminating the chances which cause the common partial to vary to such a noticeable degree.

It would appear, also, on some vowel-tones that the intensity of the fundamental or prime tone is weaker than one of the upper partials. Helmholtz laid special emphasis on this observation, and he put the statement conversely, namely, that vowel-tones differ from those of ordinary musical instruments in that one of the upper partials is more marked than the prime. Hermann also supports this view; and, in a communication to myself, Boeke expressly states that his analyses bring out the same fact. On the other hand, Auerbach * maintains that the prime tone is always the strongest. Lord Rayleigh † is also of this opinion, while he admits that in the vowel A the fundamental is not heard so loudly as in other vowels. Hermann suggests that when the prime tone is heard of very weak intensity, it may exist only in the ear, but

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* Auerbach, *Poggendorff's Annal.*, Bd. viii., 1876, p. 177.

† Rayleigh, *Theory of Sound*, 1896, vol. ii. p. 477.

Lord Rayleigh shows that even then it does exist in the external air.

General Conclusion.—It would appear, on the whole, that the truth does not rest entirely on the side of one theory, but that both are partially true. The view of Donders, that each vowel has a mouth cavity of unchangeable and fixed pitch, is too exclusive, and, on the other hand, it cannot be denied (as is virtually done by those who uphold the relative pitch theory) that each vowel has a predominant partial or predominant partials which give it a definite character, and which must be produced by the mouth cavity as a whole, or by the double resonance of portions of the cavity, as contended by Lloyd. When a vowel is spoken either separately or as it exists in a word, the complex tone is produced by the vocal cords. The resonance chambers above assume a certain form, and one or more partials peculiar to the vowel are so strengthened as to give such character to the vowel-tone as to enable the ear at once to identify it. As the form of the mouth cavity varies in individuals and in different races, the partials may not be identical in each case, but they will be so near a common pitch as to enable the ear at once to identify the vowel. If, however, men of different nationalities attempt to fix the absolute pitch of the partial they will not agree, as has been found to be the case. Again, in singing a vowel in a scale, very slight alterations in the form of the resonance cavities take place. In passing from the lower to the higher notes the larynx rises, and more or less of a muscular strain acts on the walls of the resonance cavities. Thus there must be a slight change in the volume of these cavities leading to the development of partials different from those formed by the cavity at rest or in speech, but still so sufficiently near as no materially to alter the quality of the tone. Consequently the ear still recognises the tone of the vowel, even when sung at a high pitch, and it may even recognise the special quality of a particular voice. Sometimes, though rarely, the quality may become richer as the voice rises in pitch; usually for each voice there is a register in which the voice has its maximum of good quality. It is not surprising, therefore, that a singer almost instinctively chooses such vowels as best suit the resonating arrangements of his or her voice, and avoids tones, vowels, and words containing vowels that would force the production of notes of inferior quality.

On the Derivation of the Curves of Vowel Sounds by means of Microscopical Research of their Phonograms. By J. D. Boeke, Alkmaar, Holland. (*Presented by* Professor M'Kendrick.)

(Read February 7, 1898.)

For about seven years I have been engaged upon the investigation of the impressions made by vowel sounds on phonograph cylinders.

The close accordance of my results with those of Prof. L. Hermann of Königsberg * and Dr Hugo Pipping of Helsingfors,† though obtained by widely different methods, showed, 1st, that the method by which I obtained my results, although much simpler and easier to manage than those of the above-mentioned investigators, yields, nevertheless, very satisfactory results; and 2nd, that the same vowel sounds are formed almost exactly in the same manner by persons of different age, sex, and nationality.

In the only scientific paper which, till now, I have published on this subject,‡ in 1891, only the vowel **a** (as in hard) was treated in a somewhat elaborate manner, but since then I have applied my method to other vowels as well.

As I have altered my method of research a little in the course of time, it may be useful to give a somewhat detailed description of my present mode of procedure.

It consists in measuring, microscopically, the transverse diameter of the impressions on the surface of the phonograph cylinder on different (generally equidistant) parts of the period, and in inferring from my measurements the depth of the impression on the same spot; or, in other words, in derivating from them the curve of the sound which produced the impression.

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* L. Hermann, "Phonophotografische Untersuchunge," *Pflüger's Archiv.*, 45, p. 582; 47, pp. 42, 47; 53, p. 1; 58, p. 255; 61, p. 169.

† H. Pipping, "Zur Lehre von d. Vocal Kl.," *Zeitschr. für Biologie*, 27 N.F. Bdg., p. 1; 9, 524; "Ueber die theorie der Vocale," *Acta Societatis Scientiorum fennicæ*, T. xx. No. 11.

‡ J. D. Boeke, "Mikroskopische Phonogramm-Studien," *Pflüger's Archiv.*, Bd. 50, p. 297.

This derivation does not give strictly accurate results, but the faults are too minute seriously to influence the character of the curve.

Simple reasoning will show that the depth, d, of the impression on a certain point may be easily deduced from its breadth, b, on the same point. Suppose EGDF (fig. 1) to be the transverse section of the cylindrical marker; HK the longitudinal section of the surface of the wax cylinder; ACBD the transverse section of the groove which the marker ploughs in the surface of the cylinder, $AB = b$ being its breadth; $CD = d$ being its depth; and $FG = ED = 2r$ being the diameter of the marker.

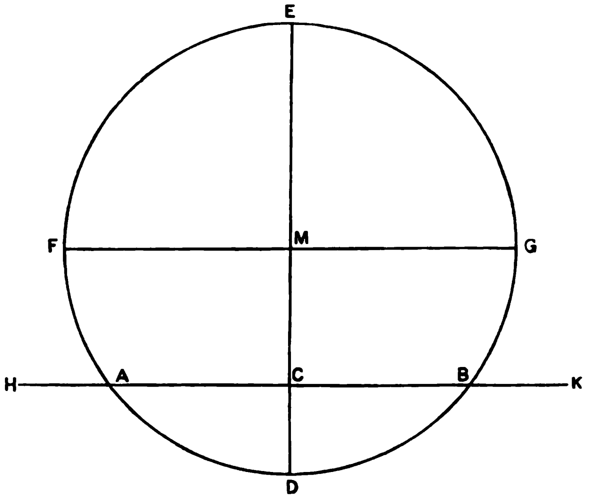

Fig. 1.

Obviously, if the axis of the recording marker were tangent to the surface of the cylinder, so that the prolongation of its edge would cut the axis of the cylinder, we should have the equation:—

$$(\tfrac{1}{2}b)^2 = d(2r - d)$$

or

$$d^2 - 2dr + \tfrac{1}{4}b^2 = 0 \quad . \quad . \quad . \quad (1)$$

from which we deduce:—

$$d = r \pm \sqrt{(r + \tfrac{1}{2}b)(r - \tfrac{1}{2}b)} \quad . \quad . \quad . \quad (2)$$

As the equation (1) is that of an ellipse, the axes of which are

$2r$ and r, we see that the breadth and the depth of the impression have the same ratio to each other as an abscissa to its ordinate in the above-mentioned ellipse.

It is true that the supposition about the axis of the marker does not hold good in using the phonograph, as, in that case, it makes a certain angle, α, with the tangent; but the only difference which this point occasions in the results is this, that they would have to be multiplied by the constant term $\cos \alpha$ to obtain their real value, which, of course, is not necessary in the case of sound curves. Since the marker is constantly moving up and down on account of the vibration of the glass membrane, the value of the angle α changes every moment, but these changes are so minute (since the greatest depth of the impressions does not exceed 0·02 mm.) that they may be neglected without any objection.

Once for all, a list of the depths (d) was calculated by means of the formula (2), answering to every transverse diameter b which might be expected.

For this purpose the diameter of the marker was cut out in a thin layer of paraffin on a glass plate, and was measured out by means of the same ocular micrometer which was to be used for measuring the transverse diameter of the impressions on the cylinder. The diameter of the recording marker proved to be fifty-six divisions of the ocular micrometer, or 1·0318 mm. Generally the largest transverse diameter of the impressions did not exceed twenty divisions of the micrometer, which corresponds to a depth of 1·847 divisions, or 0·017 mm.

Fig. 2 shows a photograph of the apparatus as I use it now for my researches. It is placed, when used, upon a firmly fixed table before a window with a northern exposure. The axis of the mandrill, G, on which the cylinder of the phonograph, H, is placed, carries a drum, P, the outside of which is divided into 360 equal parts. By means of a little pointer, W, you can read the divisions of the drum. The axis of the mandrill, G, carries also a cogged wheel, A, catching a pinion on the axis of a second drum, Q, which is equally divided into 360 parts, and may be read by means of a pointer, W′. The gearing of wheel and pinion is such that every division of the drum, Q, represents $\frac{1}{3600}$ of the circumference of the cylinder, H.

The microscope, C, used to examine the impressions on the phonograph cylinder, may slide up and down along its standard, and may be fastened at the proper height by a screw, D. The foot of the microscope is movable in a wooden sledge, E, along which it may be moved forward and backward till the proper situation for accurate examination of the impressions is found. Generally it should be moved a little beyond the axis of the cylinder, in order that sufficient light should fall upon them.

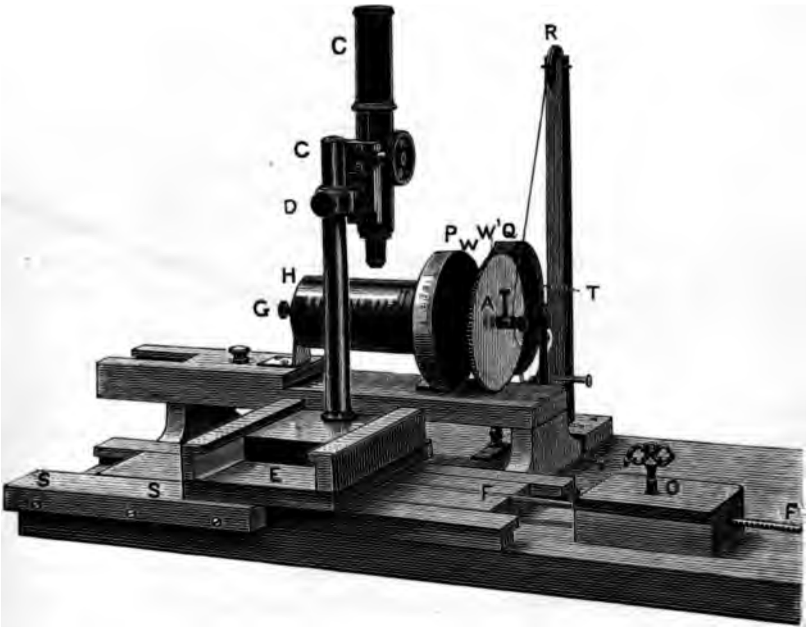

Fig. 2.

The sledge, E, together with the microscope which it carries is movable parallel to the axis of the cylinder, H, by means of a cogged bar, FF, and pinion, O, and its position may be accurately determined by a scale, SS. By this arrangement it is possible to find again any mark on the surface of the cylinder, H, the position of which has once been determined by means of the microscope. But as it happened sometimes that the cylinder moved a little on the mandrill, it was necessary to make quite sure of the identity of the observed marks; this was done by means of a cross mark, which was placed upon the surface of the cylinder, and its position accurately determined.

For the microscopical examination of the impressions a magnification of about fifty proved sufficient. This was obtained by using Zeiss' objective A.A. and ocular 2, provided with an ocular micrometer, which was divided in two directions perpendicular to each other, into fifty parts. Further magnifying proved inadvisable, as the light failed, and the curved surface of the cylinder hindered accurate results.

When a phonograph cylinder, the contents of which were exactly known, was to be subjected to measurement, the place of the cylinder on the mandrill was once for all fixed by means of a cross mark made in the surface, the position of which was accurately determined by means of the drum, P, and the scale, SS. Then the position of each vowel sound, and, if necessary, the number of its periods, were determined by turning the cylinder in the same direction in which it had been turned when taking them up. At the same time, by the microscopical examination of the periods of each vowel, their fitness to be measured could be judged of.

After this "grazing off"—so to call it—of the whole cylinder the measuring of the transverse diameter of the fit periods could be undertaken. As the latter only gives the *ordinate* of the curve, it was necessary to obtain the *abscissa* corresponding to it.

The value of the abscissae was determined in two ways.

I. First by measuring the diameter of a period only on that spot where it attained either a maximum or a minimum, and by measuring the distance from maximum to minimum by means of the perpendicular part of the ocular micrometer, the horizontal part of which had served to determine the transverse diameter.

But as the curved surface of the cylinder proved a great hindrance for accurate measurements, in my later experiments the distance of maximum to minimum, *i.e.*, the value of the abscissa, was determined by means of the drum, Q, which should indicate this distance on a tenfold magnified scale.

By-and-by, however, I came to the disagreeable conclusion that the drum, Q, did not do what it should do.

By measuring out the length of some subsequent periods of sounds, such as that of a cornet-à-piston, or that of a telephone trumpet—the length of which ought, obviously, to be constant—by means of the drum, Q, I found that the number of divisions through which the drum, Q, had to be turned for each period was by no means constant, but varied in the most capricious manner.

As I soon found out that this result was the consequence of the teeth of the wheel, A, and its pinion not always being in close contact, I tried to mend this by making the teeth cling constantly together by means of a weight, turning the axis of the cylinder in a direction opposite to that in which it was turned by means of the drum, Q. This arrangement, indeed, made the readings of the drum, Q, reliable. But only where there were a small number of underwaves did it prove practicable to measure the abscissæ as well as the ordinates of the curves. In most cases it would have been too fatiguing for the eye to read alternately the ocular micrometer and the divisions of the drum, Q.

II. For this reason, in most cases a second mode of procedure was employed to determine at the same time the abscissæ as well as the ordinates of the vowel curves under examination.

This consisted in measuring out the transverse diameter of the impressions on equidistant spots, which proceeding would obviously lead equally well to the exact form of the curve. I tried to do this by means of a clockwork, working on the axis of the mandrill on which the phonograph cylinder was placed. This clockwork was provided with a stay pressed upwards by a spring. While looking through the microscope a gentle pressure of the finger was sufficient to make the clockwork run through $\frac{1}{3000}$ of a revolution. But in this case also it was found that the apparatus did not do what it should. It was, however, a long time before I discovered that the movement of the clockwork was not at all uniform, so that the abscissæ of the curve which I derived from my measurements were not equal to each other, as I had supposed them to be.

Since I came to this conviction only gradually, after having tried, without success, to regulate by different means the movement of the clockwork, many of the numerous measurements which I have made by means of this clockwork lack the precision and reliability which vowel curves ought to have. Neither by a stronger spring in the clockwork, nor by the application of a weight pulling the cylinder either in the same direction as the clockwork did, or in the opposite one, could a thoroughly constant movement of the cylinder be obtained, and the worst of it was that the deviations from the intended course were without any regularity, and did not answer to my expectations.

I tried afterwards to correct the curves obtained by remeasuring, in the same period, the distance of maximum to minimum by means of the vertical part of the ocular micrometer referred to. But finally I dropped the clockwork altogether, and I use now the divisions of the drums, P and Q, exclusively for the measurement of the abscissæ. In order, however, to be able to measure ordinates and abscissæ of the curve at the same time, I had the inner edge of the drum, Q, made so as to be provided with shallow grooves made by a file at each of its 360 divisions. In these grooves a tooth catches, fastened to a spring, the power of which may be regulated by a screw.* By screwing the latter on, it is even possible to make the edge of the drum quite free from the tooth. If the latter is pressed by the spring against the edge it is possible, by turning the drum very carefully with the hand, while looking through the microscope, to make it stop at each groove made by the file, viz., after $\frac{1}{3600}$ of a revolution of the cylinder.

But without the application of a weight the same failure, as described above, occurred again, for instead of ten shocks, the drum, Q, ought to be turned through a varying number of shocks in order to make the drum, P, and the cylinder it carried move through exactly $\frac{1}{360}$ of its circumference.

After some trials, however, a suitable weight was found, attached to a wire running over the wheel, R, which drew the drum, P, round in a direction opposite to that in which it was turned by the hand. This arrangement caused the teeth of the wheel and pinion to cling permanently to each other, so that the movements of the cylinder were nearly equal to each other every time that the carefully-turned drum, Q, stopped by the tooth catching the groove.

The above-described mode of measuring the impressions was used exclusively in my later experiments, and it gave reliable results.

The arrangement by which it was possible to move the cylinder, while making the measurements through the microscope, by each shock through $\frac{1}{3600}$ of a circumference, was chosen because the periods of my own voice, while speaking, required about forty shocks of drum, Q, or $\frac{1}{90}$ of the circumference of the cylinder, and because forty ordinates were required to make the Fourierian analysis of the curves by means of Prof. Hermann's "Schablonen."

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* This arrangement is not to be seen on fig. 2, as it is concealed behind the drum, Q, and the wheel, A.

The numbers of the measured breadths were dictated to an assistant and written at first upon loose papers, but afterwards in a book, which proved more satisfactory, as it was sometimes difficult to follow them afterwards in the vastly increasing number of loose papers. Of course, it was exceptional when the periods required exactly 40 shocks of drum, Q, and as the number of shocks varied between 20 and 60, it was necessary to divide the curve obtained into 40 equal parts, so as to be able to submit its 40 ordinates to the harmonic analysis.

As it took a good deal of calculation to obtain this result, all periods were drawn in a constant length of 200 mm., and once for all, a number of strips of millimetre-paper were made, each of them measuring 200 mm. in length, but divided into a different number of equal parts, varying from 20 to 60 parts, with the number inscribed on them. These strips were sufficient to draw the whole curves of any pitch in a length of 200 mm.

From the measured breadths of the period were deduced the depths corresponding to them, by means of the above-mentioned list, and, by multiplying the numbers obtained by a suitable factor, all the ordinates were reduced to whole numbers not exceeding 20 to 30. The ordinates were marked on the millimetre-paper by means of dots in their proper places, by using the strips referred to, and then the curve was drawn with a finely-pointed pencil through the dots as smoothly as possible. When the curve was thus obtained, the length of its ordinates, 5 millimetres apart from each other, was determined by means of an eyeglass; and the forty ordinates so obtained were inscribed on a sheet of centimeter paper as described by Prof. Hermann.* It was found most convenient to mark the ordinates themselves with red ink and the values of their cosines with black ink, in which way mistakes are easily prevented.

* *Pflüger's Archiv.*, 47, p. 51.

An analysis may be made by Prof. Hermann's "Schablonen" in much shorter time and much more smoothly than in the ordinary manner. It does not generally take more than an hour or two. It is true that the ratio of the ordinates to their abscissæ in the curves obtained in this way, in some cases, differs widely from the real one, but this obviously does not matter for harmonic analysis. For the study of the curve itself the adopted dimensions were very convenient and useful. The curve with its real ratio of height and length would be wholly unfit for examination.

The length of the periods of vowels uttered by myself was about $\frac{1}{90}$ of the circumference of the cylinder, which generally had a value of 173–174 mm. The length of such a period being consequently about 1·93 mm., its greatest height did not even reach 0·02 mm.; so that in a curve drawn on a hundred times magnified scale, the largest underwave would not even have a height of 2 mm.

The calculations of the value of a and b, a_2 and b_2, etc., and of the amplitudes of the harmonic constituents $\sqrt{a^2+b^2}$, $\sqrt{a_1{}^2+b_1{}^2}$, etc., were not made, as Prof. Hermann advises, on the centimeter-paper itself, but on a separate sheet of paper, which was kept together with the sheet containing the numbers, so that it was possible to correct any mistakes afterwards. Since the centimeter paper (which is rather expensive) contains thirty-four vertical columns, and since every analysis requires eleven of these, one sheet of it is sufficient for three analyses, if the calculations are made on a separate paper.

Up till now I have made about 350 Fourierian analyses, most of them from vowel curves.

The values of the amplitudes of the partials obtained were generally reduced in such a way that the amplitude of the fundamental tone $\sqrt{a^1+b^1}$ was taken as unit. It is a curious fact that the amplitude of the fundamental tone, when vowel sounds are subjected to analysis, is by no means greater than that of the partials, in most cases even much smaller. Especially in spoken vowels the amplitude of the fundamental tone is relatively small. This explains the fact that it is almost impossible to estimate the pitch in which a vowel is spoken by simply hearing it.

On the Fourierian Analysis of Phonographic Tracings of Vowels. By R. J. Lloyd, Doc. Lit., etc., Liverpool.

(Read February 7, 1898.)

It is a great advantage and satisfaction to work from curves taken by the phonograph itself, because it is then possible to reproduce and verify by actual hearing the sound supposed to be represented by given curves. This ought always to be done, even with the phonograph, and the slightest change of timbre should be carefully noted; for the given curve represents not really the original sound, but the sound given back by the phonograph. Every plate or membrane vibrates more readily and strongly to some pitches than to others, and in a composite sound it is almost inevitable that some of its elements are recorded with more, and others with less, than their original relative force. Different vibrators hence yield quite different-looking phonograms for the same sound. In vowels it seems fortunately to happen that considerable changes in the relative force of the component vibrations may occur without altering the vowel. It is the pitch of the components which matters; and that is not altered by an efficient vibrator. Assuming a knowledge of the methods by which the record dug vertically into the phonographic cylinder is displayed horizontally for examination and analysis, we note that a satisfactory Fourierian analysis demands a sufficient number of measurements. A single wave is taken (AA, BB, fig. 1), and referred to rectangular axes OX, OY. Abscissæ of equal length are measured off along OX, and ordinates are drawn to meet the curve, and are very accurately measured. For a simple curve like AA a few ordinates suffice, but if the curve is complex, the number of measurements must be increased until no considerable curvature exists between the summits of any two successive ordinates. It can be seen at a glance that the curve BB, with twice the number of ordinates possessed by AA, is insufficiently registered. When a sufficient number of ordinates has been taken, an analysis carried to one-half that number of partials, or somewhat less, suffices to define the curve. A curve may be reconstituted graphically from its analysis, but the process is laborious when the partials are many. Donkin invented an instrument called the harmonograph, which suffices to describe a curve compounded of any two harmonic curves. The principle of this machine could probably be extended so as to describe the curve represented by any given Fourierian analysis, each partial being set going in the given amplitude and at the given phase. This curve could be at once compared with the original, and their difference, if any, could be seen and analysed. Broadly speaking, the loudness of subjective impression varies with the amplitude of the vibrations causing it. But there are known exceptions, *e.g.*, tone is produced by beats, though, if the beat-curve were analysed, it is manifest that this tone would come out with an amplitude of zero. Hermann affirms* that not only beats, but every thing periodic in a phonogram, is heard as tone of the given period or pitch. It is certain, at any rate, that, in the case of a vocal phonogram, the tone on which it is sung may be heard from it distinctly, and yet the amplitude of that particular tone, the fundamental, may be almost zero in the analysis. But

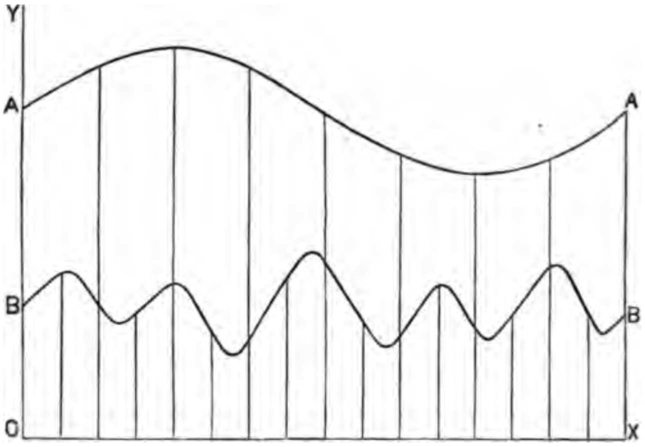

FIG. 1.

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* See list of references at end of this paper.

though it represents no amplitude, it represents a very distinct period or repetition, for the whole curve repeats itself at each vibration of the glottis, so long as the same note and the same vowel are held. But the upper partials are none of them associated with any visible period of this kind, and may be taken to be subjectively present in the degree denoted by their amplitude in the analysis, with qualifications to be hereafter mentioned.

What is it which confers upon a certain note the quality of a certain vowel? It is undoubtedly the articulation,—the shape into which the mouth and pharynx are thrown in order to produce the vowel. Vary the articulation and you vary the vowel; maintain the articulation and you maintain the vowel, *whether you maintain the same tone or not.* You may even abolish the tone, and speak in whisper, but you still have the same vowel, if the articulation is maintained. What power, then, has the articulation over the tone? None whatever, except that of resonance. The cavities of the articulation have the power to magnify certain partials of the glottal note, and to damp others. They magnify those which are nearest to the pitch of their own proper resonances, and they damp the remainder. This is best seen by examples.

It is impossible to take a phonogram from the larynx which shall be totally uninfluenced by the voice-passage, because the tone must always come through the voice-passage. But we may choose from among available phonograms that of a vowel which seems to be least influenced by the voice-passage, because it has the simplest articulation and the least distinctive quality. The ideal vowel would be a simple emission of voice, like the interjection *Uh.* Nearest to this come Hermann's phonograms of German short *u.* The analysis of one of these is here graphically displayed; it was sung on *g*, 196 v.d.:—

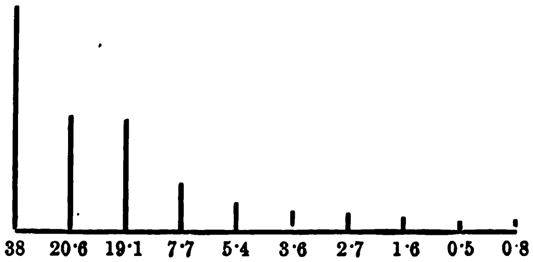

Fig. 2.

We conclude from this, what might have been surmised *a priori*, that, apart from the disturbing influence of resonance, the partials of the glottal note tend to produce themselves in a descending scale of amplitude as they rise in pitch. No doubt there are individual differences—dull voices tapering down quickly, bright voices less quickly, shrill voices still less quickly, in amplitude of partials. With this compare, first of all, an analysis of long Dutch *aa*, spoken and analysed by Dr Boeke of Alkmaar (see *Journal of Anat. and Phys.*, vol. xxxi. p. 248), pitch 175 v.d.

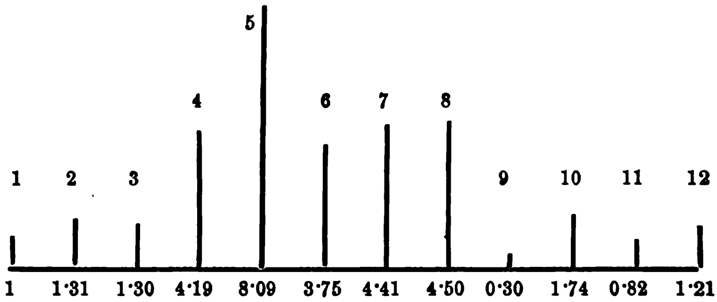

Fig. 3.

Then compare an analysis of Pipping's Swedish (resembling French) *i* at 293 v.d.

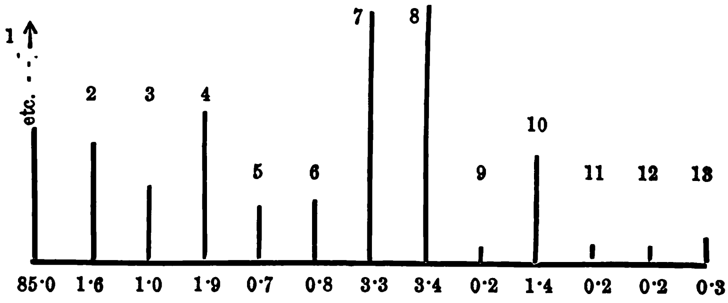

Fig. 4.

Observe how the normal declining scale of amplitudes has been distorted by the specific resonances of the vowels. In fig. 3 there are two very clear reinforcements, the one culminating on the 5th partial, the other on the 7th and 8th. In fig. 4 two high reinforcements are also very visible, the one on the 7th and 8th, and the other on the 10th partial. But observe how much narrower in range are these reinforcements of *i* than those of *aa*, for these upper resonances of *i* affect only one or two high partials; the neighbouring partials are not reinforced at all, though they are hardly two semitones removed. The two resonances are both comprised within a space of six semitones (7th to 10th partials), and yet, within that narrow space, are sharply marked off from each other by the practical absence of the 9th partial. This results from the nature of the cavity in which the resonance is generated. One of these resonances of *i* is certainly generated by the narrow tube formed between the tongue and the hard palate in the articulation of that vowel (Helmholtz, *Sensations of Tone*, p. 108). Hence its narrowness of range; for a tube will not respond to any tone which differs much from its own. But a rounded cavity has a range of resonance which increases with every increase in the size of its aperture or apertures. The articulation of *aa* consists of two such cavities, of rather wide aperture (see figures in *Journ. of Anat.*, *loc. cit.*), and I believe, after actual measurements of the articulation, that the two reinforcements displayed in the phonograms of this vowel (*e.g.*, fig. 3) can be satisfactorily traced to the proper resonances of these two cavities. Hence its two reinforcements are seen to spread over a whole octave (4th to 8th partials) and even then do not find space without some overlapping, for the 6th partial seems to be reinforced by both. The question has been warmly argued whether the Fourierian analysis accurately represents the facts of vowel-resonance or not. Does it follow, because a vibration can be analysed into a certain note and its partials, that the vowel objectively consists of those partials, and nothing more? Hensen and Pipping say yes; Hermann says that the higher-pitched elements are not partials at all, but independent resonances. A middle opinion seems best to deserve adoption. It is as difficult to conceive the voice passing through the articulation of a given vowel without arousing the proper tone of the cavities passed through, as it is to conceive the voice passing through the same articulation without having its own partials reinforced or damped. In the Fourierian analysis we get both of these elements together; they cannot be discriminated. But the proper tones of the cavities must undoubtedly be present in some degree. It seems most probable, too, that they are sufficiently strong to be recognised separately by the ear, as they certainly are in whisper, and that it is thus we arrive at the cognition of the vowel, as distinguished from that of the musical tone. We need not doubt the power of the ear to make this separate cognition; it is a much simpler matter than the separate cognition of two voices heard simultaneously; and the fact that the resonances are not, except by accident, harmonic to the glottal tone, or to any of its partials, may reasonably be thought to afford to the ear a criterion of distinction; though the auditory mechanism for such a distinction may not yet have been made clear. But if the phonogram contains inharmonic elements, how is it possible that it can be completely analysed into harmonic elements? Simply because the inharmonic elements, that is to say the resonances, arise and fall within the limits of each wave of the phonogram; they take a fresh start at each opening of the glottis. Hermann has illustrated this very fully (*op. cit.*). They are thus periodic, though not harmonic; and anything periodic can be analysed approximately into harmonic partials of a tone of that period.

As a matter of fact the inharmonic elements come out in the analysis mainly as apparent reinforcements of those harmonic elements which they most closely resemble, *i.e.*, of those very partials which they really reinforce. Hence, for the purpose of locating and evaluating the proper pitch of a resonance, it is practically immaterial whether we adopt Hensen's view or Hermann's, or an intermediate one. In any case, the locality of any reinforcement of the partials indicates in a general way the pitch of the resonance which has caused it. We may proceed, in fact, to use the data of the reinforcements to calculate the independent pitch of the resonance. But on doing so, certain preliminary questions present themselves for solution.

Before trying to deal with these, a class of facts may be noted which go to show that the reinforcements seen in vowel-phonograms are also, in some considerable measure, real partials of the glottal note. Let a good male voice sing *o*, *a*, *i* in their Italian values, at a pitch about 165 V.D.—the middle *e* of a bass voice. Not only does the vowel change, but the musical quality of the tone changes also. The brightest vowel is *a*; the *o* is agreeable, but softer and duller; the *i* is much thinner in quality, and sometimes incisive rather than strong. Compare this with what happens to the partials of each of these vowels in their phonograms. In the *a* phonograms we have a richly harmonic set of strong partials from the 4th to the 8th (fig. 2); in the *o* phonograms it is the lower and softer harmonics which come strongly into evidence, from the 2nd to the 4th; whilst in the *i* phonograms of this pitch (fig. 4 is not an exact illustration, because it is sung at a much higher pitch, and, as the resonance remains unchanged, the reinforcements naturally appear on much lower partials) there is strong reinforcement of the 2nd partial, and again of the 13th and 14th, or 16th and 17th, —high partials of a penetrating dissonance, but all the richer and higher harmonics are only feebly present. If the reinforcements were not to a large extent true partials,—not merely concomitant inharmonic resonances,—these effects could scarcely follow.

The evaluation of the pitch of any resonance which only reinforces a single partial is exceedingly simple. It is expressed by the equation

$$n = \mathrm{N}p.$$

Where N is the pitch-number of the fundamental tone, p is the number of the partial, and n is the pitch-number of the required resonance. In fig. 4, for example, the isolated reinforcement of the 10th partial in a vowel sung at 293 v.d. proves the existence of a resonance of $293 \times 10 = 2930$ v.d. The case in which two partials are equally reinforced is not quite so simple. We may say approximately that

$$n = \mathrm{N}\frac{p + p'}{2},$$

e.g., in fig. 4 the resonance which reinforces the 7th and 8th partials is about $293 \times \frac{15}{2} = 2198$ v.d. But even here there are minor considerations involved, which can only be seen clearly after examining one or two more general and more complex cases.

Let us first take an instance of German long *a*, sung and analysed by Hermann (*Pfl. Arch.*, xlvii. 355). The pitch is 98 v.d., and the analysis is only carried to ten partials, so that it does not display the higher resonance of this vowel, except slightly on the 10th partial, which is all the better for the present purpose.

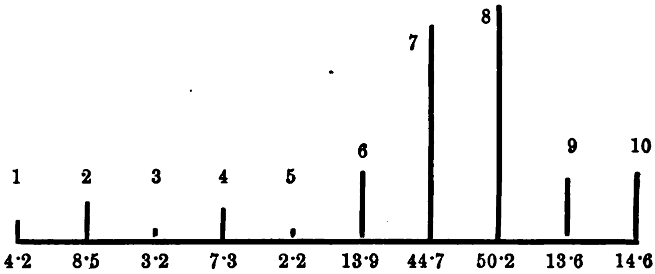

FIG. 5.

This analysis displays but one considerable reinforcement; but it extends, in varying strength, over the 6th, 7th, 8th, and 9th partials. This reinforcement probably springs, like that of the 5th partial (875 v.d.) in fig. 3, from the resonance of the intra-velar or pharyngeal cavity of these *a* vowels. But for the present it is better not to commit ourselves to any name involving a theory of origin, but to call this the *a*-resonance. How are we, from these reinforcements, to evaluate the proper pitch of that resonance in each case? Hermann used what he called a centre-of-gravity calculation, *i.e.*, he proceeds as if he were finding the centre of gravity of four heavy points situated as above, on the axis of x, and each weighing as many units as there are units in the respective amplitudes. Having thus discovered the "mean partial," he multiplies the fundamental by it to get the value of the resonance itself. Both operations are combined in the equation

$$n = \mathrm{N}\frac{p'a' + p''a'' + p'''a''' + \text{etc.}}{a' + a'' + a''' + \text{etc.}},$$

where N and n are again the pitch-numbers of the sung note and of the desired resonance, respectively; p', p'', p''' are successive integers, the numbers of the reinforced partials; and a', a'', a''' are the amplitudes found for these partials respectively. For the resonance displayed in fig. 5, the process is as follows:—

$$n = 98 \times \frac{(6 \times 13{\cdot}9) + (7 \times 44{\cdot}7) + (8 \times 50{\cdot}2) + (9 \times 13{\cdot}6)}{(13{\cdot}9 + 44{\cdot}7 + 50{\cdot}2 + 13{\cdot}6)} = 737 \text{ v.d.}$$

Pipping does not object in principle to a centre-of-gravity calculation, but he objects to the assumption, embodied in fig. 5, that the partials are equidistant from one another. It is true that they are equidistant when considered as an *arithmetical* progression, —1st partial, 98 v.d.; 2nd partial, 196 v.d.; 3rd partial, 294 v.d., etc. But musical distances are not measured in arithmetical but in geometrical progression; and the equidistant partials are not the 1st, 2nd, 3rd, 4th, etc., but the 1st, 2nd, 4th, 8th, etc. The abscissæ in fig. 5 ought therefore not to be measured off in proportion to the numbers 1, 2, 3, 4, etc., but to $\log_2 1$, $\log_2 2$, $\log_2 3$, $\log_2 4$, etc., or simply to log 1, log 2, log 3, log 4, etc., seeing that the base does not matter if the right mutual proportion is maintained. The result is shown in fig. 6.

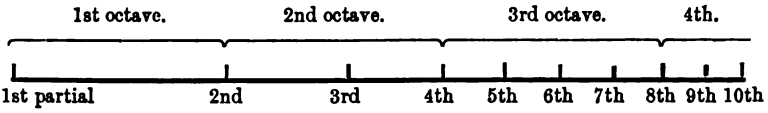

Fig. 6.

If we adopt this view, we get the following equation to find p, the mean partial:—

$$\log p = \frac{a' \log p' + a'' \log p'' + a''' \log p''' + \text{etc.}}{a' + a'' + a''' + \text{etc.}},$$

and then $n = \mathrm{N}p$ as before.

This view seems well founded as far as it goes; and I adopted it in *Journ. of Anat. and Phys.*, xxxi. 249; but there is a further consideration which it leaves out of sight. This is the consideration of frequency. Take the partials in fig. 5. The amplitudes are 4·2, 8·5, 3·2, etc.; but the first-named amplitude is traversed only once in each period, whilst the second is traversed twice, the third three times, and so on. The general expression for the whole amplitude traversed in each period is ap, *i.e.*, amplitude multiplied by frequency. It seems a right principle to estimate the strength of the reinforcement of each partial in terms of the whole amplitude traversed; and if so, the multiplied amplitude, $a'p'$, $a''p''$, etc., will have to be substituted for the single amplitudes a', a'', etc., in our last equation. Thus we get

$$\log p = \frac{a'p' \log p' + a''p'' \log p'' + a'''p''' \log p''' + \text{etc.}}{a'p' + a''p'' + a'''p''' + \text{etc.}}$$

to find the mean partial.

If this method is correct, it will practically deliver us from the consideration of another feature of the phonograms of vowels which would otherwise now demand attention. This is the normal tendency to a decline in amplitude, which was exhibited in fig. 2. The centre-of-gravity calculation assumes, in its two simpler forms, that normally, and apart from resonance, the amplitudes of successive partials would be equal; but we have seen that the series a', a'', a''', etc., is normally a rapidly declining series; and in any calculation based on them, some correction should be made for this. But when our calculation is based upon the series $a'p'$, $a''p''$, $a'''p'''$, etc. (*i.e.*, upon amplitude multiplied by frequency), there is no such tendency to rapid decline, and our calculation is very much freer from this risk of error. Compare the *a* series with the *ap* series in fig. 2, thus:—

	1	2	3	4	5	6	7	8	9	10
a series, . .	38·0	20·6	19·1	7·7	5·4	3·6	2·7	1·6	0·5	0·8
ap ,, . .	38·0	41·2	57·3	30·8	27·0	21·6	18·9	12·8	4·5	8·0

Comparing the three methods discussed, it is easy to see that Pipping's method will always give a result somewhat lower than Hermann's; and that the method just recommended will give a result somewhat higher than Hermann's. In the present case (fig. 5) the difference between the three calculations is not great. The first has given 737 V.D.; the second will be found to give 732 V.D.; and the third 741 V.D. And when the partials involved are still higher than these, the differences become more and more inconsiderable. But when the lowest partials are involved, the differences are large enough to be of vital importance. Another difficulty, which I have found in practice, in dealing with reinforcements of the lowest partials, is that when one of the reinforced partials is the fundamental, the results drawn from different phonograms of the same vowel seldom closely agree; and I have been led to conclude that the fundamental, though it may be either damped or reinforced by resonance, is much less absolutely under the control of the resonances, and therefore much less trustworthy as a criterion of resonance, than any other partial. There are good physiological reasons to account for this: for the genesis and period of the fundamental is definitely associated with the opening and shutting of the vocal chords, and it is very easy to conceive that the control of the resonances over vibrations starting from massive organic movements of this kind may be imperfect and capricious.

Better than wasting time in discussing the difficulties, theoretical and practical, thus seen to arise in connection with the lower partials, is to avoid, as far as possible, lower partials altogether. This can be done by singing the vowel low enough. The lowest vowel resonance yet observed is an *a*-resonance of *o*, 216 v.d., which I have calculated from analyses laid before the Royal Society of Edinburgh twenty years ago by Professors Fleeming Jenkin and Ewing. Next comes the *a*-resonance of *i*, about 280 v.d., and of *e*, about 352 v.d. But good vowels can be sung by male voices at 100 v.d., and sometimes much lower. Then we get a reinforcement in which the fundamental, and often the 2nd partial also, are not involved.

Another advantage of using phonograms of low pitch is that all the resonances, upper as well as lower, are displayed in greater detail. Imagine a note of 300 v.d., with its partials, entering an articulatory cavity which had a proper pitch of 400 v.d., but was capable of reinforcing any partials between 300 and 500 v.d. The only partial reinforced in the phonogram would be the fundamental 300 v.d.; and we should conclude, wrongly, that the resonance was 300 v.d., instead of 400. Under like circumstances a resonance, apparently only affecting the 2nd partial (600 v.d.), might really be due either to a resonance of 500 v.d. or to one of 700 v.d. But if the same vowel were sung at 100 v.d., the reinforcements in question would culminate exactly on the 4th, or the 5th, or the 7th partial (400, or 500, or 700 v.d.), according to the real resonance.

More important, and equally common, is the case of two independent resonances acting simultaneously. If a vowel is sung at 300 v.d. and has two resonances, differing by 300 v.d., it is never possible to discern the existence of two separate resonances in the analysis, because the partials reinforced are always consecutive. Let the resonances, for example, be 900 and 1200 v.d.; the phonograms will then, of course, show in their analysis great reinforcements of the 3rd and 4th partials. But there will be nothing to show that these two reinforcements are due to two separate resonances. The more natural thing, in the absence of other evidence, is to see in these two strong partials the evidence of a strong resonance, about half way between them (1050 V.D.), reinforcing both. Every phonographic investigator has been deceived more or less in this way. What we really want, in such a case, is to have a greater number of partials within the sphere of the resonances, to show us more clearly where each resonance culminates; and the only way to have more partials is to sing the vowels at a lower pitch. In the instance just given there is no probability of seeing any sign of the doubleness of the resonance until we sing the vowel as low as 150 V.D. Then the resonance of 900 V.D. necessarily culminates on the 6th partial, and that of 1200 V.D. on the 8th, and there is the 7th partial between, to show, by its want of reinforcement, that we have before us the effect of two reinforcements, and not of one. But in practice this is no more than a probability, for it may, and does, happen that the middle partial gets reinforced from both sides, and thus comes out with a higher strength than that of either of the real culminations. It will be best to corroborate the principles just started by actual example, and then proceed to consider this problem of the intermediate or common partial in a more general form. But enough has been already said to show that this problem becomes less and less troublesome the lower the vowel is sung. If, for example, in the instance last supposed, the vowel was sung at 100 V.D., there would not be one intermediate partial, but two; and it will be seen shortly that the chance of both of these receiving concurrent reinforcement from both resonances is slight. The resonance culminating on the 9th partial (900 V.D.) would in most cases be distinctly marked off in the analysis from that culminating on the 12th partial (1200 V.D.), by the marked weakness of the 10th or 11th partial, or both.

For actual example let us first compare with fig. 5 the analysis of the same vowel, sung at the same time by the same person, at the much higher pitch of 247 V.D. (fig. 7). For the sake of comparison the diagrams will continue to be set out on the same principles as hitherto, though in strictness the intervals ought to be set out as in fig. 6, and the perpendiculars ought to represent, not simple amplitude (*a*), but amplitude × frequency (*ap*).

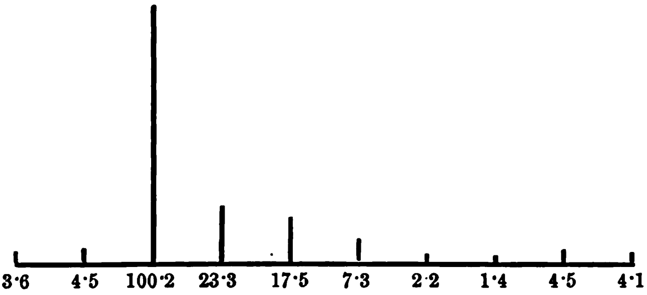

Fig. 7.

In the place of four strong high partials (the 6th, 7th, 8th, and 9th), we have the same resonance now represented only by one strong low partial, the 3rd; and it is clear, from what has been already said, that the former must afford much the more trustworthy basis for computing the resonance.

Next let us take two of Pipping's curves of Swedish *a* (not unlike Dutch *aa*), sung at two pitches, differing by an octave. The first one, fig. 8, is sung at 256 v.d.

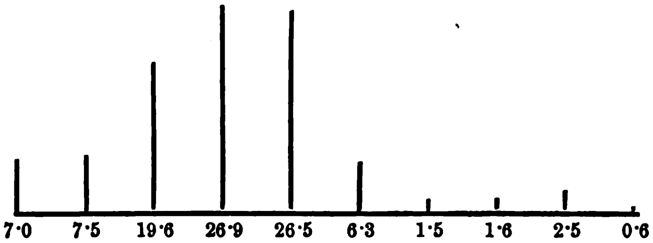

Fig. 8.

So far as this analysis shows, there is only one resonance. But the same vowel was also sung at 128 v.d. (fig. 9).

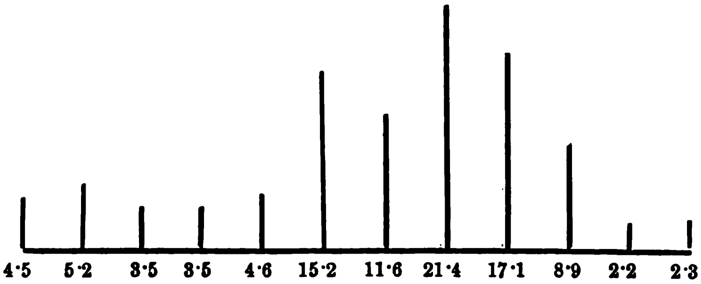

Fig. 9.

The resonance is now, of course, spread out over a wider range of partials, and is easily seen to consist of two resonances, about half an octave apart.

Next let us examine the series of nineteen analyses of Dutch *aa*, given by Dr Boeke in *Pflüger's Archiv.*, vol. i. pp. 312, 313. The analysis already shown in fig. 3 is No. 10 in that series. It is best to consider the series in two parts, viz., Nos. 1 to 9, which are *sung* by Herr von Duinen to a rising scale of notes, from *c*, 132 v.d., to *d'*, 297 v.d.; and Nos. 10 to 20 (No. 11 is missing), which were *spoken* by Dr Boeke at various (undesigned) pitches, from 175 to 214·5 v.d. The unit of amplitude in the table is, in each case, the amplitude of the first partial, or fundamental.

Herr von Duinen's sung *aa*.

V.D.	1	2	3	4	5	6	7	8	9	10	11	12
132,	1	0·63	0·36	0·40	0·81	0·36	0·67	0·23	1·07	0·16	1·06	0·67
148·5,	1	2·10	2·50	1·54	4·40	0·00	1·72	2·19	1·50	1·20	0·65	0·76
165,	1	0·81	0·88	1·30	2·26	1·14	1·00	1·32	0·61	0·16	0·05	0·13
176,	1	0·95	0·39	1·22	0·87	0·83	1·18	0·68	0·39	0·21	0·14	0·13
198,	1	0·19	0·46	0·93	0·26	0·46	0·75	0·23	0·15	0·09	0·04	0·03
220,	1	0·36	0·56	1·09	0·67	0·95	0·17	0·36	0·15	0·17	0·12	0·13
247·5,	1	0·41	4·12	0·79	1·10	0·64	0·29	0·28	0·21	0·17	0·23	0·29
264,	1	1·13	1·44	1·19	0·41	0·22	0·16	0·13	0·18	0·20	0·12	0·20
297,	1	2·68	2·96	6·50	1·20	0·56	0·38	0·36	0·03	0·24	0·37	0·24

On inspecting these nine series of partials, it is easy to discern in the first seven of them the presence of both of the resonances which made themselves felt in the analysis of Dr Boeke's *aa*, fig. 3. Proceeding to analyse these seven on the principle above recommended, we get the following table:—

α- and β-Resonances of Herr von Duinen's sung *aa*.

V.D.	Partials Reinforced.		Mean Resonances.		Radical Ratio.
	α	β	α	β	
132, . .	4–6	8–10	663	1184	1·785
148·5, . .	4–6	7–9	707	1204	1·703
165, . .	4–6	7–9	828	1304	1·574
176, . .	3–5	6–8	748	1229	1·643
198, . .	3–5	6–8	779	1361	1·747
220, . .	2–5	5–7	860	1266	1·473
247·5, . .	2–4	4–7	769	1297	1·686

The last column shows the ratio between the resonances. I call it the *radical* ratio, because it seems to me that the distinction between one vowel and another must be chiefly based upon the ratio subsisting in each vowel between its several resonances. This conclusion, at first (1890) deduced *a priori* from observation of the shape of articulations, has derived further support from every set of Fourierian analyses yet published. Hence my interest in the latter subject.

Now, let us tabulate the other ten in the same way. First, the data of amplitudes of partials :—

Dr Boeke's spoken *aa*.

V.D.	1	2	3	4	5	6	7	8	9	10	11	12
175,	1	1·31	1·30	4·19	8·09	3·73	4·40	4·50	0·30	1·74	0·72	1·21
182,	1	0·85	0·26	0·69	2·59	1·87	1·26	1·34	1·08	0·76	0·13	0·11
184,	1	0·81	0·73	0·77	2·01	0·71	1·45	0·58	0·61	0·06	0·43	0·21
190,	1	0·52	1·57	0·91	4·97	2·00	1·63	1·21	1·36	0·28	0·40	0·60
177·5,	1	1·11	1·42	1·03	2·08	1·69	0·80	0·55	0·03	0·14	0·10	0·13
190,	1	1·02	1·19	0·83	3·34	1·35	1·53	0·28	0·40	0·60	0·32	0·44
207·5,	1	1·45	1·91	1·44	7·48	2·44	3·86	0·92	0·86	0·50	0·93	0·75
214·5,	1	4·40	8·00	16·60	32·10	23·40	11·80	5·30	6·80	5·60	3·60	5·40
184,	1	0·93	0·37	0·92	4·70	1·63	1·28	2·95	1·18	1·31	0·32	0·29
195,	1	0·56	1·11	2·22	2·25	0·70	1·74	1·40	0·40	0·16	0·13	0·55

Next the evaluation of the two resonances :—

α- and *β*-Resonances of Dr Boeke's spoken *aa*.

V.D.	Partials Reinforced.		Mean Resonances.		Radical Ratio.
	α	*β*	*α*	*β*	
175, . .	4–6	7–9	879	1321	1·503
182, . .	4–6	7–10	960	1523	1·587
184, . .	3–6	6–9	873	1361	1·559
190, . .	4–7	8–10	1050	1652	1·573
177·5, . .	4–8	9–11	1022	1824	1·785
190, . .	4–7	8–11	1062	1834	1·726
207·5, . .	4–6	6–10	1063	1512	1·422
214·5, . .	4–7	8–11	1169	2018	1·727
184, . .	4–6	7–9	944	1474	1·561
195, . .	3–6	6–9	877	1440	1·641

Note in this last table that the vowel which is the highest pitched of the ten is also the one vowel whose resonances refuse to be easily evaluated, because their effects overlap, and we can discern but a vague line of demarcation in the phonogram. It was for exactly the same reason that the two highest pitches of Herr von Duinen's sung vowels, and they only, were useless for evaluation of resonances. Compare figs. 8 and 9, and observe that while the difference of these two resonances of *aa* reaches 500 to 800 V.D., these three cases occur where the sung pitch (and therefore the interval between the successive partials) is only 264, 297, and 214·5 V.D., respectively,—being in the last case certainly *less* than half the difference between the resonances (*v. supra*).

The second of these three instances (the last of Herr von Duinen's vowels) is naturally the strongest. It furnishes, indeed, a concrete instance, just as striking as the case already noted to be theoretically possible, where the intermediate partial, receiving reinforcement from both resonances, comes out stronger in the analysis than either of the two partials on which the two resonances, if their effects could be disentangled, would be respectively seen to culminate. This striking analysis is displayed graphically in fig. 10; and the relative positions of Herr von Duinen's resonances, as deduced from the other analyses, are indicated by the letters α and β.

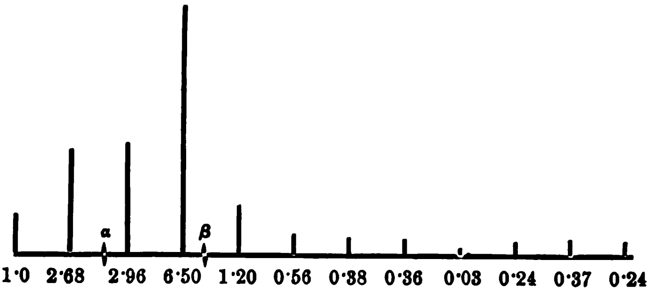

FIG. 10.

But without the lower-pitched analyses we could not have had the least suspicion that fig. 10 is the work of two resonances, much less could we have evaluated them.

If this difficulty thus arises in a vowel where there is a difference of 500 to 800 v.d. between the resonances, much more does it arise in one where the difference is only 230 to 360 v.d. Such a vowel is *o*. But before passing from *aa*, two or three minor points may be noticed. Observe in Herr von Duinen's first vowel that the same superposition of resonances is at work as in fig. 10, though it fortunately does not obscure the analysis. The "common partial" is the 7th,—equally removed from the 5th, on which culminates the *a*-resonance, and from the 9th, on which the other culminates. There is concurrent reinforcement in this case also; for there is a numerical culmination on the common partial. But this culmination is due to no independent resonance; for no such resonance shows any trace of existence in the other analyses.

Note how much less variable is the ratio between the resonances than are the resonances themselves. The highest resonances are seen in the 5th, 6th, and 8th of Boeke's *spoken* vowels, and it is important to notice that these (and the 7th) are just those which were spoken in the middle of a word, as long vowels between two closed consonants. For explanation of this coincidence, and also of the observable slight general tendency of Herr von Duinen's resonances to rise in pitch as he ascended the scale, see *Journ. of Anat. and Phys.*, vol. xxxi. pp. 235–7. Soon after that article was published, Dr Boeke favoured me with analyses of two phonograms of Dutch *aa*, sung by his son, aged 12. I was able in both cases to evaluate the *a*-resonance, and found it to be four semitones higher than Dr Boeke's. This is quite favourable, so far as it goes, to the views of vowel composition put forward in that article. But more such analyses are needed: the smaller the child, the more valuable would be the analysis.

Observations of the articulation of *o* have led me to believe that it has two resonances, differing by about an octave. An experiment of Helmholtz (*Sens. Tone*, p. 60) points strongly in the same direction. Yet no phonographic observer has noticed more than one resonance in his analysis. But the considerations contained in this paper readily explain why that is the case. In a vowel having two resonances differing only by about 300 v.d., it is useless to look for any sign of doubleness in the reinforcements evidenced by the Fourierian analysis, unless the vowel is sung below 150 V.D. Many examples are available of *o* analyses at higher pitches, but not one of them shows any sign of the existence of two separate resonances. Two analyses, however, of a lower pitch than 150 V.D. have been published, and both of them show that palpable cleavage in the reinforcements which is the constant sign of the presence of two separate resonances. They were both sung by Hermann at 132 V.D., and the two series of amplitudes are as follow:—

V.D.	1	2	3	4	5	6	7	8	9	10
132, .	6·7	10·7	18·4	14·6	18·1	4·7	2·5	1·3	1·0	0·5
132, .	7·8	23·4	11·2	7·0	17·0	3·6	2·1	1·2	0·6	1·0

To these, by the kindness of Dr Boeke, I am enabled to add two more; one sung at 148 V.D. by Professor M'Kendrick of Glasgow, and the other at 128 V.D. by himself:—

V.D.	1	2	3	4	5	6	7	8
148, .	1	1·12	1·73	0·19	1·90	0·97	0·35	...
128, .	1	2·32	5·81	2·10	5·48	0·32	0·55	0·76

The 4th partial in each of these four analyses shows a palpable falling-off in strength as compared with its neighbours on either side; and there can be no doubt that this falling-off marks in each case the gap between an α-resonance of 300–400 V.D. and a β-resonance of 600–800 V.D. But when we attempt to evaluate these resonances more closely, the problem of the common partial arises in a more imperative form than ever. The *o* vowel, however low we may sing it, can hardly have more than two partials intermediate to the two culminations. In three out of the four cases above given, it has only one intermediate partial. What is to be done with this one partial in our calculations? To what extent must we ascribe it to the α-resonance and to the β-resonance respectively? And how comes it to pass that, in the first of these four analyses, this middle partial seems to derive great support from both resonances, whilst in the third example it is supported by neither, and almost vanishes?

There is one point in which the common partial differs from all others in an analysis: it is in being subject to strong influence from both resonances at one time. All the other partials are situated so much nearer to the one resonance than to the other, that they may be regarded as being under the sole influence of the former. The common partial, on the other hand, receives a stimulus from each; and it must be clearly remembered that these stimuli are mutually and entirely independent (they are in fact successive, the one operating in the intra-velar and the other in the extra-velar cavity of *o*), and that it is a matter of pure chance whether, in any phonogram analysed, these two stimulations operate to exaggerate or to conceal each other in the tabulated numerical strength of the partial.

Briefly, it is a question of phase. The partials into which a sound is analysed by the Fourierian process are simple or pendular vibrations. If one of them enters successively into two vibrating cavities, each of which severally has power to give reinforcement, the resultant reinforcement need not be the sum of the two reinforcements: it may be their difference, or it may be anything intermediate. The first result will only follow when the two reinforcements are precisely identical in phase; the second will follow when they are precisely opposite in phase; and the third in all other (that is to say, in the great majority of) cases. On the average, it will be about one-half the sum of the two several reinforcements.

This view explains the extraordinary fluctuations in the strength of the intermediate partial, but it helps us very little in the problem of evaluation. It shows us that if the intermediate partial is very strong, and we use it for its full value in calculating both of the resonances, we shall get the α-resonance too high and the β-resonance too low, and the radical ratio $\frac{\beta}{\alpha}$ too small; whilst, if the intermediate partial is very weak, all these results will be precisely reversed.

The following table contains a rough evaluation of the four *o* analyses just given: a common partial has had to be used in three cases, and it has been used in each case for its full value in calculating both resonances; and it will be noted at once that, where the common partial is strong, the radical ratio is unduly low, and *vice versa*:—

APPROXIMATE α- AND β-RESONANCES OF SUNG *o*.

Sung by	V.D.	Partials Reinforced.		Mean Resonance.		Radical Ratio.
		α	β	α	β	
Hermann, .	132	2–4	4–7	421	653	1·553
,, .	132	1–3	4–7	290	672	2·318
M'Kendrick,	148	1–4	4–7	365	812	2·221
Boeke, .	128	2–4	4–6	391	615	1·573

For making a more exact calculation, two methods suggest themselves. When a resonance spreads over several partials, it tends to spread downwards and upwards with about equal force. Therefore, by regarding the downward falling-off of the α-resonance, and the upward falling-off of the β-resonance, we may endeavour to conjecture the real decline in the contrary direction. But there is room for much caprice in such a conjecture, and it would only lead to any certain improvement of result when the common partial is manifestly higher or lower than either of the several reinforcements can be imagined to be. A sounder, though as yet unpractised, method would be to take from one voice a sufficient number of *o* analyses to constitute an average which would eliminate the chances which cause the common partial to vary so immensely. The number need not be very large, if the instances are fairly representative of various grades of its strength and weakness. Our four available instances give an average of 368 V.D. for the α-resonance, of 688 V.D. for the β-resonance, and of 1·916 for the radical ratio. But they are hardly sufficient, either in number or in distribution, and they do not come from identical voices, nor even from identical nationalities.

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L. HERMANN.—"Phonophotographische Untersuchungen," i.–v. (*Pflügers Archiv.*, vol. xlv. p. 582; xlvii. pp. 44, 347; liii. p. 1; lviii. p. 255), contain many analyses and much controversy about Fourierian analysis; No. 2 contains a method of lightening the labour of these analyses. "Bemerkungen zur Vocalfrage" (*ib.*, vol. xlviii. p. 181); "Vocale im Telephon und Microphon" (*ib.*, vol. xlviii. p. 543); "Weitere Untersuchungen über das Wesen der Vocale" (*ib.*, vol. lx. p. 169), deals chiefly with short (German) vowels.

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On the Thermodynamics of Volta-contact Electricity.

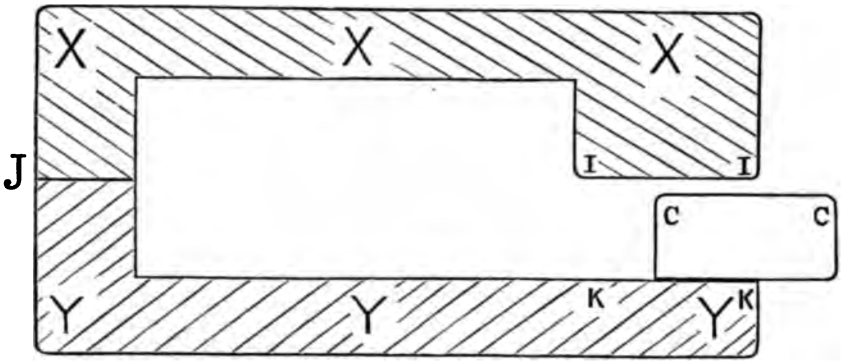
By the **Rt. Hon. Lord Kelvin**, G.C.V.O., etc.

(Read February 21, 1898.)

§ 1. Let X and Y be two metals, of which X is electrically positive to Y in the Volta-contact series for dry metals, and make an incomplete circuit of them as indicated in the diagram, with X and Y metallically connected at an interface J, and free surfaces II, KCC exposed, with ether or air, or any gas or insulating fluid, between them. Let CC be a movable slab of the Y-metal, resting frictionlessly on a fixed surface KK of the same metal. If left to itself the movable slab would, in virtue of electric attraction, as we shall see, oscillate on the two sides of the middle position in which the whole of its upper surface is opposite to II. We shall suppose it held by applied force F in any position, or moved, or allowed to move, from any one position to any other, at our pleasure.

§ 2. Suppose now our apparatus to be given with no excess of either electricity above the other, and to be insulated in air or ether at a distance from all other bodies great in comparison with its own dimensions, and with no electrified body near enough to it to produce sensible electrification through influence. Every part of the X-surface will be found positively and every part of the Y-surface negatively electrified, provided each surface is of perfectly uniform Volta-quality throughout its extent; but the electric surface-densities of the opposite electrifications will be everywhere exceedingly small, except in and near the portions of II and CC closely opposed to one another. Hence, if the slab is drawn outwards, an electric current will flow from II through the X-metal, and will cross the junction J from X to Y and flow through Y to compensate negative electricity on the portions of CC passing from close opposition to II. Our thermodynamic operations, of which we will arrange a Carnot cycle, are drawing in and out the slab CC, sometimes with the whole metal coated with an ideal varnish impermeable to heat, and sometimes with the whole surface kept at one temperature by giving heat to it, or taking heat from it, where required for the fulfilment of this condition.

§ 3. Suppose the relative thermo-electric quality of the two metals is such that, when a complete metallic circuit is made of



them, and the two junctions are kept at different temperatures, the thermo-electric current is (according to the old French rule for bismuth and antimony) "against the alphabet through hot"—that is, from Y to X through the hot junction and from X to Y through the cold junction. It is in this direction for some pairs X and Y (as, for example, X zinc, Y gold), of which X is Volta-positive to Y, and is in the opposite direction for others; but, by allowing negative values to some of the quantities, this latter case is included in our supposition which we make as a preliminary to avoid circumlocutions.

§ 4. Consider now the Peltier thermal effect of the current produced as in § 2 by drawing the movable slab of Y-metal outwards. The current crosses the junction J in the same direction as the natural thermo-electric current in a closed circuit with J the cold junction: and the thermal effect is *therefore* production of heat at J; according to a thermodynamic hypothesis which I adopted as fulfilled so far as the *sign* of the Peltier effect was concerned, in Peltier's splendid original discovery for bismuth and antimony, and verified * by myself experimentally for copper and iron below

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* Verified by Le Roux and Jahn for several other pairs of metals for which they also measured the value of the Peltier effect. Their results verify the thermodynamic hypothesis absolutely in respect to the sign, and tend to con-

280° C. (*Trans. Roy. Soc. Edin.*, May 1854, "Dynamical Theory of Heat," part vi. §§ 105–106; *Proc. Roy. Soc. Lond.*, May 1854, "Experimental Researches in Thermo-electricity," § II. Both republished in Arts. XLVIII. and LI., vol. i., *Mathematical and Physical Papers*. See pp. 239–241 and 464–465).

But we must now consider, also, a quasi-Peltier effect produced by electricity crossing the border between air or ether at the surface of either metal, and the homogeneous metal inside. We have absolutely no thermodynamic or molecular-hypothetic guide to even guess the sign or magnitude of this effect at the surface of either metal. It is conceivable that it may have opposite signs for different metals, or that it is essentially of the same sign in all; but it seems to me exceedingly improbable that it is non-existent, when I consider Pellat's and Murray's discoveries of change of Volta-surface-potential produced by scratching and by burnishing, without any change of chemical constitution of the surface layer of a metal.

§ 5. Let Q denote the total quantity of heat produced by the Peltier effect at J and the quasi-Peltier effect at the surfaces II and CC, per unit quantity of electricity flowing from II through J to CC, in virtue of motion of CC outwards. The part of Q which is produced at J is, as we have seen, positive when X and Y are in the thermo-electric order stated in § 3; but the total amount of Q may be either positive or negative.

§ 6. Our Carnot cycle will consist of the following four operations:—

I. ("Adiabatic"—according to Rankine's nomenclature.) The whole apparatus being ideally coated with varnish impermeable to heat, draw out CC so slowly that the temperature of the whole apparatus remains uniform throughout, while rising from t to t', on absolute thermodynamic scale.

II. (Isothermal.) The whole apparatus being kept by proper surface appliances at constant temperature t', let CC move inwards very slowly, until a certain quantity of heat, H′, has been taken in to the apparatus from without.

firm it in respect to the magnitude, of the Peltier effect. Le Roux (1867), *Ann. de Chim. et de Phys.* (4), vol. x. p. 201; Jahn (1888), *Wied. Ann.*, vol. xxxiv. p. 767.

III. (Adiabatic.) Let the slab move further inwards very slowly till the temperature of the whole apparatus, ideally coated with impermeable varnish, sinks to t.

IV. (Isothermal.) The whole apparatus being kept by proper surface appliances at constant temperature t, draw the slab outwards to its primitive position. Let H be the quantity of heat which must be removed to prevent lowering of temperature.

§ 7. Remark that if Q of § 5 is positive, H′ and H are both positive and $H' > H$; but if Q is negative, H′ and H are both negative and $-H > -H'$. In either case we have essentially by my definition of absolute temperature (*Math. and Phys. Papers*, vol. iii. Art. XCII. part ii. §§ 34–35)

$$\frac{H'}{H} = \frac{t'}{t};$$

in the former case $t' > t$, in the latter $t' < t$.

By working out analytically all the details of this cycle of operations, and taking into account Joule's law of equivalence between heat and work, we arrive at the result

$$JQ = dV/d(\log t)$$

given as our final result in equation (7) below.

But we arrive at it more easily, and in some respects more conveniently, by founding on the doctrine of motivity, as follows:—

§ 8. Let V be the Volta-difference of potential in air or ether between the opposed X and Y metallic surfaces.

Let ξ, ξ' be the electrostatic capacities of the variable condenser CC, II, for two positions of the movable slab CC, which for brevity we call position ξ and position ξ'.

The work required to pull out the slab from position ξ to position ξ' will be

$$\tfrac{1}{2}V^2(\xi - \xi').$$

The quantity of electricity passing from II across J to KK during this operation will be

$$V(\xi - \xi');$$

and therefore the quantity of heat which would have to be removed from the junction J and the surfaces II, CC, to prevent rise of temperature (§ 5 above), is

$$QV(\xi - \xi'), \text{ or } -QV(\xi' - \xi).$$

Hence if $e(\xi, t)$ denote the energy of the apparatus in the condition (ξ, t), we have

$$\frac{de(\xi, t)}{d\xi} = -(\tfrac{1}{2}V^2 + JQV) \quad . \quad . \quad . \qquad (1)$$

Considering now change of temperature with ξ constant, we have

$$\frac{de(\xi, t)}{dt} = JN(\xi, t) \quad . \quad . \quad . \qquad (2)$$

where $N(\xi, t)$ denotes the thermal capacity of the apparatus with ξ constant. When we consider the possible or probable quasi-Peltier productions of heat at II and CC, and the probability that these are different for the two metals, and the Peltier effect at J, we must regard N as probably varying with ξ. We must therefore regard it as a function of ξ and t.

§ 9. Consider now the motivity* of our apparatus, which we shall denote by $m(\xi, t)$, being, as is the energy, a function of ξ and t.

When the slab CC is drawn out so as to diminish the capacity from ξ to ξ', the heat $QV(\xi - \xi')$, which, to prevent temperature from rising, must be given to external matter at temperature t, contributes to the recipient an amount of motivity equal to

$$\frac{t - T}{t} JQV(\xi - \xi'),$$

where T denotes the lowest temperature of neighbouring matter, available for receiving heat. Hence we have

$$\frac{dm(\xi, t)}{d\xi} = -\left(\tfrac{1}{2}V^2 + \frac{t - T}{t} JQV\right) \quad . \quad . \qquad (3)$$

And if we raise the temperature of the whole apparatus infinitesimally from $t - \tfrac{1}{2}dt$ to $t + \tfrac{1}{2}dt$, we add to its motivity an amount

$$\frac{t - T}{t} JN(\xi, t)\, dt.$$

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* *Proc. Roy. Soc. Edin.*, 1876; *Phil. Mag.*, May 1879; *Math. and Phys. Papers*, vol. i. Art. I.; *Proc. Roy. Soc. Edin.*, March 21, 1898.

Hence

$$\frac{dm(\xi, t)}{dt} = \frac{t-\mathrm{T}}{t}\mathrm{JN}(\xi, t) \quad . \quad . \quad . \quad (4)$$

From (1) and (2) with the equation $\frac{d}{d\xi}\frac{de}{dt} = \frac{d}{dt}\frac{de}{d\xi}$; and from (3) and (4) with the equation $\frac{d}{d\xi}\frac{dm}{dt} = \frac{d}{dt}\frac{dm}{d\xi}$; we find

$$\mathrm{J}\frac{d\mathrm{N}}{d\xi} = -\left\{ \mathrm{V}\frac{d\mathrm{V}}{dt} + \mathrm{J}\frac{d(\mathrm{QV})}{dt} \right\} \quad . \quad . \quad (5)$$

and

$$\frac{t-\mathrm{T}}{t}\ \mathrm{J}\frac{d\mathrm{N}}{d\xi} = -\left\{ \mathrm{V}\frac{d\mathrm{V}}{dt} + \frac{t-\mathrm{T}}{t}\ \mathrm{J}\frac{d(\mathrm{QV})}{dt} - \frac{\mathrm{T}}{t^2}\ \mathrm{JQV} \right\} \quad (6)$$

Eliminating $\frac{d\mathrm{N}}{d\xi}$ between (5) and (6) we find finally

$$\frac{d\mathrm{V}}{dt} = \frac{\mathrm{J}}{t}\mathrm{Q} : \text{ or, } \mathrm{JQ} = \frac{d\mathrm{V}}{d(\log t)} \quad . \quad . \quad (7)$$

§ 10. The quantity of heat absorbed or produced in virtue of change of electric density on the surfaces II, CC must in all probability in every case be too small to be detected by direct observation. But the difference between the quantities produced at the two surfaces of the two different metals, per unit of electric quantity added to one surface or taken from the other, which is Q − Π if Π be the Peltier effect at the junction J, can by aid of equation (7) be readily and surely determined for any two metals for which the Peltier effect is known. In fact, it is easy to arrange apparatus for measuring V through a considerable range of temperature, say from 0° to 100° C., by the now well-known compensation method introduced independently by Pellat and myself, and thus to measure $\frac{d\mathrm{V}}{dt}$ and so find Q by equation (7). I am at present commencing experiments for this purpose with air, and with carbonic acid gas, between the two opposed metal surfaces, so that we may judge what precautions, if any, will be necessary to eliminate disturbances due to different condensations of gas on the metals at different temperatures. Very interesting and important experiments by Pellat and by Erskine Murray have shown large temperature effects on the Volta-electric-force between two plates, whether of the same or of different metal, when one of them is heated and the other kept at the ordinary atmospheric temperature; but this does not supply what we now want from experiment, which is, the variation of the Volta-electric-force between two metals at one and the same temperature, when this temperature is varied.

ADDITION, OF DATE 26TH MARCH 1898.

A number of preliminary experiments, carried on with the assistance of Mr W. Craig Henderson, have shown large but largely irregular temperature variations of the Volta E. M. F. between copper and zinc, with all parts of the Volta circuit at the same temperature. The substitution of carbonic acid gas for air, and re-introduction of air for carbonic acid, seemed to make but little difference on the results. The irregularities seemed to have been chiefly due to permanent or sub-permanent changes in the copper plate. At all events we found somewhat more nearly regular results with zinc and gold. The zinc plate used had never, so far as I know, been polished or much disturbed by touching its surface since experimented on by Mr Erskine Murray* three years ago. The "gold" was a brass plate gilded for me about 1859. It was one of the two "standard gold" plates in Erskine Murray's experiments, and, so far as I know, it has never been rubbed or polished since 1861.

We found in a range from 16° C. to 50° C. an augmentation of Volta-contact difference at rough average rate of about ·002 of a volt, or $\cdot 2 \times 10^6$C.G.S. units, per degree Centigrade. This is 800 times the thermo-electric difference of zinc and gold given as 250 C.G.S. units per degree Centigrade in Jenkins' *Electricity and Magnetism*, p. 176, and Everett's *Physical Units*, 1886, p. 173. And (§ 3 above) gold, zinc are as Y, X in respect to orders in the Volta series and in the thermo-electric series of metals. Hence, according to the secure thermo-dynamic formula (7) above, and the old probable thermo-dynamic hypothesis for thermo-electricity (§ 4 above), the quasi-Peltier effect at the interfaces gold-air and air-zinc in the Volta circuit would be 799 times the Peltier effect at the zinc-gold interface, if the preceding figure 800 were exact.

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* A description of Murray's experiments will be published probably in May in the *Phil. Mag.*, and in the *Proc. Roy. Soc. Lond.*

The *sign* of the quasi-Peltier effect for gold-zinc is such that on the whole heat is produced by vitreous electricity travelling from gold to gold-air frontier, and an equal quantity of vitreous electricity from air-zinc frontier to zinc; and, on the whole, cold is produced by equal electric motions in the opposite direction.

On Thermodynamics founded on Motivity and Energy.

By the **Rt. Hon. Lord Kelvin**, G.C.V.O., F.R.S., F.R.S.E., etc.

(Read March 21, 1898.)

§ 1. In a verbal communication to the Royal Society of Edinburgh in 1876, under the title "Thermodynamic Motivity," I suggested the name *motivity* to express energy, whether thermal or of any other kind, available to generate velocity in molar matter, or to move molar matter against resisting force. By molar matter I mean matter as we know it, consisting, as we believe, of vast numbers of atoms or molecules. Only the title of this communication was published in the *Proceedings of the Royal Society*: a short report of it was published in the *Philosophical Magazine* for May 1879, in which it was pointed out that a "very short and simple analytical method of setting forth the whole non-molecular theory of thermodynamics" might be founded on the consideration of *Motivity* and *Energy* as two functions of all the independent variables specifying a body, or a system of bodies, or some definite apparatus under consideration :—apparatus, as I shall call it for brevity, to include every case, even such as a single crystal. The object of the present communication is to carry out this proposal.

§ 2. Let the apparatus be given all at one temperature t. Denote by g_1, g_2, g_3, etc., the other variables by which its condition is specified. These will in many cases be geometrical, specifying elements or co-ordinates, such as strain-components, expressing change of bulk or shape of a piece of crystal or other elastic solid under stress; or positions of pistons in a pneumatic apparatus; or area, or curvature, of a free liquid surface in an application to theory of capillary attraction; or positions of electrified bodies, or electrostatic capacities, in an electrostatic system. Or, considered as generalised co-ordinates, the independent variables may be physical qualities, such as proportion of vapour to liquid in an enclosure, with or without a piston; or quantities of electricity on particular insulated pieces of metal in an electrical system; or proportion of salt to solvent in an osmotic application.

§ 3. Let $m_T(t, g_1, g_2, g_3, \ldots)$ and $e(t, g_1, g_2, g_3, \ldots)$ denote the motivity and the energy of the system, the latter being absolute, the former being relative to a temperature T, the lowest available for carrying off heat. These expressions written in full denote that m_T and e are functions of the independent variables t, g_1, g_2, etc.; but generally, except when it is convenient to be reminded that they are such functions, we shall for brevity denote their values simply by m and e.

§ 4. First suppose the temperature of the apparatus kept constant at T, and let the other independent variables be augmented from $g_1 - \frac{1}{2}dg_1$, $g_2 - \frac{1}{2}dg_2$, etc., to $g_1 + \frac{1}{2}dg_1$, $g_2 + \frac{1}{2}dg_2$, etc., through infinitesimal ranges dg_1, dg_2, etc. Let

$$M_1(T).dg_1 + M_2(T).dg_2 + \text{etc.} \qquad (1)$$

denote the quantity of heat (positive or negative) which must be taken in from without to keep the temperature constant at T, and let

$$P_1(T).dg_1 + P_2(T).dg_2 + \text{etc.} \qquad (2)$$

denote the mechanical work required to produce the change. This work simply contributes its own amount to the motivity of the apparatus, because, as in Carnot's theory, we have an infinite river or ocean at temperature T, always ready to give or take freely any heat to be taken in by, or rejected from, the apparatus to keep it at constant temperature T. Hence

$$dm_T(T, g_1, g_2, \ldots) = P_1(T).dg_1 + P_2(T).dg_2 + \text{etc.} \qquad (3).$$

On the other hand, the energy is augmented, not only by the mechanical work done on it, but, in addition to this, by the dynamical equivalent of the quantity (positive or negative) of heat taken in. Hence, if J denote the dynamical equivalent of the thermal unit, we have

$$d[e(T, g_1, g_2, \ldots) - m_T(T, g_1, g_2, \ldots)] = JM_1(T).dg_1 + JM_2(T).dg_2 + \text{etc.} \quad (4).$$

The second members of (3) and (4) are complete differentials of functions g_1, g_2, etc., on the supposition that T is constant. Hence

$$\frac{d}{dg_1}\mathrm{P}_2(\mathrm{T}) = \frac{d}{dg_2}\mathrm{P}_1(\mathrm{T})\,; \ \frac{d}{dg_2}\mathrm{P}_3(\mathrm{T}) = \frac{d}{dg_3}\mathrm{P}_2(\mathrm{T})\,; \text{ etc.} \qquad (5)$$

and

$$\frac{d}{dg_1}\mathrm{M}_2(\mathrm{T}) = \frac{d}{dg_2}\mathrm{M}_1(\mathrm{T})\,; \ \frac{d}{dg_2}\mathrm{M}_3(\mathrm{T}) = \frac{d}{dg_3}\mathrm{M}_2(\mathrm{T})\,; \text{ etc.} \qquad (6).$$

§ 5. Passing now from the supposition that the temperature is kept constant at T and going back to § 3, remark that whatever heat is taken in at temperature t imparts motivity to the apparatus to an amount equal to the proportion $(t - \mathrm{T})/t$ of its dynamical equivalent. Hence, if N denote the thermal capacity of the apparatus with g_1, g_2, etc., each constant; and if M_1, M_2, etc., and P_1, P_2, etc., denote the coefficients in (1) and (2) for any variable temperature t instead of the constant temperature T, we now have, instead of (3),

$$dm = \mathrm{J}\frac{t-\mathrm{T}}{t}\mathrm{N}dt + \left(\mathrm{P}_1 + \mathrm{J}\frac{t-\mathrm{T}}{t}\mathrm{M}_1\right)dg_1 + \left(\mathrm{P}_2 + \mathrm{J}\frac{t-\mathrm{T}}{t}\mathrm{M}_2\right)dg_2 + \text{etc.} \quad (7)\,;$$

and for the energy we have simply

$$de = \mathrm{JN}dt + (\mathrm{P}_1 + \mathrm{JM}_1)dg_1 + (\mathrm{P}_2 + \mathrm{JM}_2)dg_2 + \text{etc.} \qquad (8).$$

From this and (7) we have

$$d(e - m) = \frac{\mathrm{JT}}{t}(\mathrm{N}dt + \mathrm{M}_1 dg_1 + \mathrm{M}_2 dg_2 + \text{etc.}) \qquad (9).$$

§ 6. From the conditions that the second members of (7) and (9) are complete differentials of all the independent variables, we now find from (9), as formerly from (4), in respect to g_1, g_2, etc.,

$$\frac{d\mathrm{M}_2}{dg_1} = \frac{d\mathrm{M}_1}{dg_2}\,; \ \frac{d\mathrm{M}_3}{dg_2} = \frac{d\mathrm{M}_2}{dg_3}\,; \text{ etc.} \qquad (10)$$

and from these in conjunction with (7),

$$\frac{d\mathrm{P}_2}{dg_1} = \frac{d\mathrm{P}_1}{dg_2}\,; \ \frac{d\mathrm{P}_3}{dg_2} = \frac{d\mathrm{P}_2}{dg_3}\,; \text{ etc.} \qquad (11).$$

Lastly, in respect to t, g_1; t, g_2; etc., we find from (9) and (8)

$$\frac{d\mathrm{N}}{dg_1} = \frac{d\mathrm{M}_1}{dt} - \frac{\mathrm{M}_1}{t}\,; \ \frac{d\mathrm{N}}{dg_2} = \frac{d\mathrm{M}_2}{dt} - \frac{\mathrm{M}_2}{t}\,; \text{ etc.,} \qquad (12)$$

$$\text{and } \mathrm{J}\frac{d\mathrm{N}}{dg_1} = \mathrm{J}\frac{d\mathrm{M}_1}{dt} + \frac{d\mathrm{P}_1}{dt}\,; \ \mathrm{J}\frac{d\mathrm{N}}{dg_2} = \mathrm{J}\frac{d\mathrm{M}_2}{dt} + \frac{d\mathrm{P}_2}{dt}\,; \text{ etc.} \qquad (13).$$

From these, by elimination of N, we find

$$JM_1 = -t\frac{dP_1}{dt}\,;\; JM_2 = -t\frac{dP_2}{dt}\,;\; \text{etc.} \tag{14}$$

These equations (11), (12), (13), and (14) express all the knowledge regarding properties of matter which can be derived, according to suggestions of Carnot and Clapeyron, from the combined Carnot and Joule thermodynamic theory.

§ 7. For some applications the following condensation of notation and corresponding modification of formulas will be found convenient. Let w denote the mechanical work performed in altering the apparatus from any one configuration $(t, g_1', g_2', \ldots)$ to any other configuration $(t, g_1, g_2, \ldots)$, both at the same temperature t, and let H be the heat absorbed during the process. Equations (11) and (10) demonstrate that w and H are independent of the particular succession of configurations through which the apparatus is brought from the initial to the final configuration, *provided heat is given to it, or taken from it, by external agency, so as to keep it at one unchanged temperature* t *throughout the process*. With this notation (11) and (10) are equivalent to the following:—

$$w = \chi(t,g_1,g_2,\ldots) - \chi(t,g_1',g_2',\ldots) \tag{15}$$
$$H = \psi(t,g_1,g_2,\ldots) - \psi(t,g_1',g_2',\ldots) \tag{16}$$

where χ and ψ denote two functions of the variables; and we have

$$P_1 = \frac{dw}{dg_1}\,;\; P_2 = \frac{dw}{dg_2}\,;\; \text{etc.} \tag{17}$$

$$M_1 = \frac{dH}{dg_1}\,;\; M_2 = \frac{dH}{dg_2}\,;\; \text{etc.} \tag{18}$$

In terms of this condensed notation we find as an equivalent for (14) the following single equation:—

$$JH = -t\frac{dw}{dt} \tag{19}$$

and by integration of (8), (9), and (7) with reference to g_1, g_2, etc.

$$e = w + JH + e(t,g_1',g_2',\ldots) \tag{20}$$

$$e - m = \frac{JTH}{t} + e(t,g_1',g_2',\ldots) - m_T(t,g_1',g_2',\ldots) \tag{21}$$

$$m = w + J\frac{t-T}{t}H + m_T(t,g_1',g_2',\ldots) \tag{22}$$

And, eliminating H by (19), we have finally e and m in terms of w as follows:—

$$e = w - t\frac{dw}{dt} + e(t, g_1', g_2', \ldots) \qquad (23)$$

$$m = w - (t - \mathrm{T})\frac{dw}{dt} + m_{\mathrm{T}}(t, g_1', g_2', \ldots) \qquad (24).$$

§ 8. For the particular case $t = \mathrm{T}$ we fall back on the formulas of § 4, and we see that there is in that case no distinction between motivity and mechanical work done with the apparatus kept constantly at temperature T.

§ 9. Our present notation, w, H, and e, is exactly that which I used forty-three years ago in my paper "On the Thermo-elastic and Thermo-magnetic Properties of Matter," published in the first number of the *Quarterly Journal of Mathematics* (April 1855), and republished with additions in the *Philosophical Magazine* for January 1878, and in Vol. I. of my *Mathematical and Physical Papers* (Art. XLVIII. Part vii.). Equations (6) and (8) of that article, found originally without the very convenient aid to thought given by the idea of motivity, are now reproduced as equations (19) and (23) above. The application to the thermo-elastic properties of fluids, of non-crystalline elastic solids, and of crystals, and to Thermo-magnetism, and to Pyro-electricity or the Thermo-electricity of non-conducting crystals, which that article contains, and my paper on "Thermodynamics of Volta-Contact Electricity," read before the Royal Society of Edinburgh at its recent meeting of February 21, may be referred to as sufficiently illustrating the system of generalised co-ordinates and thermodynamic formulas of the present communication.

On Electric Equilibrium between Uranium and an Insulated Metal in its Neighbourhood. By the Rt. Hon. Lord Kelvin, G.C.V.O., *P*.R.S.E., &c.; J. Carruthers Beattie, D.Sc., F.R.S.E.; and M. Smoluchowski de Smolan, Ph.D.

(Read March 1, 1897.)

[Accidentally omitted from *Proc. Roy. Soc. Edin.* for Session 1896–7, p. 417, but published in *Nature* for March 11, 1897.]

The wonderful fact that uranium held in the neighbourhood of an electrified body diselectrifies it was first discovered by H. Becquerel. Through the kindness of Prof. Moissan we have had a disc of this metal, about 5 cm. diameter and ½ cm. thickness, placed at our disposal.

We made a few preliminary observations on its diselectrifying property. We observed first the rate of discharge when a body was charged to different potentials. We found that the quantity lost per half-minute was very far from increasing in simple proportion to the voltage, from 5 volts up to 2100 volts; the electrified body being at a distance of about 2 cm. from the uranium disc. [*Added* March 9, 1897.—We have to-day seen Prof. Becquerel's paper in *Comptes Rendus* for March 1. It gives us great pleasure to find that the results we have obtained on discharge by uranium at different voltages have been obtained in another way by the discoverer of the effect. A very interesting account will be found in Prof. Becquerel's paper, which was read to the French Academy of Sciences on the same evening, curiously enough, as ours was read before the Royal Society of Edinburgh.]

These first experiments were made with no screen placed between the uranium and the charged body. We afterwards found that there was also a discharging effect, though much slower, when the uranium was wrapped in tinfoil. The effect was still observable when an aluminium screen was placed between the uranium, wrapped in tinfoil, and the charged body.

To make experiments on the electric equilibrium between uranium and a metal in its neighbourhood, we connected an insulated horizontal metal disc to the insulated pair of quadrants of an electrometer. We placed the uranium opposite this disc and connected it and the other pair of quadrants of the electrometer to sheaths. The surface of the uranium was parallel to that of the insulated metal disc, and at a distance of about 1 cm. from it. It was so arranged as to allow of its easy removal.

With a polished aluminium disc as the insulated metal, and with a similar piece of aluminium placed opposite it in place of the uranium, no deviation from the metallic zero was found when the pairs of quadrants were insulated from one another. With the uranium opposite the insulated polished aluminium, a deviation of − 84 scale-divisions from the metallic zero was found in about half a minute. [Sensibility of electrometer 140 scale-divisions per volt.] After that, the electrometer-reading remained steady at this point, which we may call the uranium rays-zero for the two metals separated by air which was traversed by uranium rays. If, instead of having the uranium opposite to the aluminium, with only air between them, the uranium was wrapped in a piece taken from the same aluminium sheet, and then placed opposite to the insulated polished aluminium disc, no deviation was produced. Thus in this case the rays-zero agreed with the metallic zero.

With polished copper as the insulated metal, and the uranium separated only by air from this copper, there was a deviation of about + 10 scale-divisions. With the uranium wrapped in thin sheet aluminium, and placed in position opposite the insulated copper disc, a deviation from the metallic zero of + 43 scale-divisions was produced in two minutes, and at the end of that time a steady state had not been reached.

With oxidised copper as the insulated metal, opposed to the uranium with only air between them, a deviation from the metallic zero of about + 25 scale-divisions was produced.

When the uranium, instead of being placed at a distance of 1 cm. from the insulated metal disc, was placed at a distance of 2 or 3 mm., the deviation from the metallic zero was the same.

These experiments lead us to infer that two polished metallic surfaces connected to the sheath and the insulated electrode of an electrometer give, when the air between them is influenced by the uranium rays, a deflexion from the metallic zero the same in direction, and of about the same amount, as when the two metals are connected by a drop of water. The uranium itself may be one of the two metals.

A Relation between Permanents and Determinants.

By Thomas Muir, LL.D.

(Read December 20, 1897.)

1. If all the negative terms of the determinant $|a_1b_2c_3 \dots|$ be changed in sign, we obtain a symmetric function, dealt with by Borchardt and Cayley, known as a *Permanent* and denoted by

$$\overset{+}{|}a_1b_2c_3 \dots \overset{+}{|}.$$

The more important elementary properties of such functions are given in a paper published in the *Proc. Roy. Soc. Edin.*, xi. pp. 409–418. As might be expected, relations are found to exist between them and determinants, an important instance being the theorem of § 7 of the said paper. Another theorem, not hitherto noted, deserves now to be put on record.

2. For the case of the 2nd order it is

$$\left.\begin{array}{r} |a_1b_2| - a_1b_2 \\ + \overset{+}{|}a_1b_2\overset{+}{|} - a_1b_2 \end{array}\right\} = 0,$$

the truth of it being self-evident.

For the case of the 3rd order it is

$$\left.\begin{array}{r} - |a_1b_2c_3| + a_1|b_2c_3| + b_2|a_1c_3| + c_3|a_1b_2| \\ + \overset{+}{|}a_1b_2c_3\overset{+}{|} - a_1\overset{+}{|}b_2c_3\overset{+}{|} - b_2\overset{+}{|}a_1c_3\overset{+}{|} - c_3\overset{+}{|}a_1b_2\overset{+}{|} \end{array}\right\} = 0,$$

which is easily verified by observing that the coefficients of a_1, a_2, a_3 in the expression on the left-hand side are respectively

$$\left.\begin{array}{r} - |b_2c_3| + |b_2c_3| + b_2c_3 + b_2c_3 \\ + \overset{+}{|}b_2c_3\overset{+}{|} - \overset{+}{|}b_2c_3\overset{+}{|} - b_2c_3 - b_2c_3 \end{array}\right\},$$

$$|b_1c_3| - b_1c_3 + \overset{+}{|}b_1c_3\overset{+}{|} - b_1c_3,$$

$$|b_2c_1| - b_2c_1 + \overset{+}{|}b_1c_2\overset{+}{|} - b_2c_1,$$

and that by the previous case each of these vanishes.

For the case of the 4th order it is

$$\left.\begin{aligned}&|a_1b_2c_3d_4| - a_1|b_2c_3d_4| - b_2|a_1c_3d_4| - c_3|a_1b_2d_4| - d_4|a_1b_2c_3| + \overset{+}{|}a_1b_2\overset{+}{|}\,|c_3d_4| + \overset{+}{|}a_1c_3\overset{+}{|}\,|b_2d_4| + \overset{+}{|}a_1d_4\overset{+}{|}\,|b_2c_3|\\ &+\overset{+}{|}a_1b_2c_3d_4\overset{+}{|} - a_1\overset{+}{|}b_2c_3d_4\overset{+}{|} - b_2\overset{+}{|}a_1c_3d_4\overset{+}{|} - c_3\overset{+}{|}a_1b_2d_4\overset{+}{|} - d_4\overset{+}{|}a_1b_2c_3\overset{+}{|} + |a_1b_2|\,\overset{+}{|}c_3d_4\overset{+}{|} + |a_1c_3|\,\overset{+}{|}b_2d_4\overset{+}{|} + |a_1d_4|\,\overset{+}{|}b_2c_3\overset{+}{|}\end{aligned}\right\}=0,$$

where again the verification is easily made by observing that the coefficients of a_1, a_2, a_3, a_4 are

$$\left.\begin{aligned}&|b_2c_3d_4| - |b_2c_3d_4| - b_2|c_3d_4| - c_3|b_2d_4| - d_4|b_2c_3| + b_2|c_3d_4| + c_3|b_2d_4| + d_4|b_2c_3|\\ &+\overset{+}{|}b_2c_3d_4\overset{+}{|} - \overset{+}{|}b_2c_3d_4\overset{+}{|} - b_2\overset{+}{|}c_3d_4\overset{+}{|} - c_3\overset{+}{|}b_2d_4\overset{+}{|} - d_4\overset{+}{|}b_2c_3\overset{+}{|} + b_2\overset{+}{|}c_3d_4\overset{+}{|} + c_3\overset{+}{|}b_2d_4\overset{+}{|} + d_4\overset{+}{|}b_2c_3\overset{+}{|}\end{aligned}\right\}$$

$$\left.\begin{aligned}&-|b_1c_3d_4| + c_3|b_1d_4| + d_4|b_1c_3| + b_1|c_3d_4|\\ &+\overset{+}{|}b_1c_3d_4\overset{+}{|} - c_3\overset{+}{|}b_1d_4\overset{+}{|} - d_4\overset{+}{|}b_1c_3\overset{+}{|} - b_1\overset{+}{|}c_3d_4\overset{+}{|}\end{aligned}\right\},$$

$$\left.\begin{aligned}&|b_1c_2d_4| + b_2|c_1d_4| - d_4|b_1c_2| + c_1|b_2d_4|\\ &+\overset{+}{|}b_1c_2d_4\overset{+}{|} - b_2\overset{+}{|}c_1d_4\overset{+}{|} - d_4\overset{+}{|}b_1c_2\overset{+}{|} - c_1\overset{+}{|}b_2d_4\overset{+}{|}\end{aligned}\right\}$$

or

$$\left.\begin{aligned}&-|b_2c_1d_4| + b_2|c_1d_4| + c_1|b_2d_4| + d_4|b_2c_1|\\ &+\overset{+}{|}b_2c_1d_4\overset{+}{|} - b_2\overset{+}{|}c_1d_4\overset{+}{|} - c_1\overset{+}{|}b_2d_4\overset{+}{|} - d_4\overset{+}{|}b_2c_1\overset{+}{|}\end{aligned}\right\},$$

$$\left.\begin{aligned}&-|b_1c_2d_3| - b_2|c_1d_3| - c_3|b_1d_2| + d_1|b_2c_3|\\ &+\overset{+}{|}b_1c_2d_3\overset{+}{|} - b_2\overset{+}{|}c_1d_3\overset{+}{|} - c_3\overset{+}{|}b_1d_2\overset{+}{|} - d_1\overset{+}{|}b_2c_3\overset{+}{|}\end{aligned}\right\}$$

or

$$\left.\begin{aligned}&-|b_2c_3d_1| + b_2|c_3d_1| + c_3|b_2d_1| + d_1|b_2c_3|\\ &+\overset{+}{|}b_2c_3d_1\overset{+}{|} - b_2\overset{+}{|}c_3d_1\overset{+}{|} - c_3\overset{+}{|}b_2d_1\overset{+}{|} - d_1\overset{+}{|}b_2c_3\overset{+}{|}\end{aligned}\right\},$$

and that by the previous case each of these vanishes.

3. For convenience in making the verifications the identity in these special cases has been written in two lines. For other purposes, however, it is better not to make this bisection, but to write as follows:—

$$|a_1b_2| - \Sigma a_1|b_2| + \overset{+}{|}a_1b_2\overset{+}{|} = 0,$$

$$|a_1b_2c_3| - \Sigma a_1|b_2c_3| + \Sigma\,\overset{+}{|}a_1b_2\overset{+}{|}\,c_3 - \overset{+}{|}a_1b_2c_3\overset{+}{|} = 0,$$

$$|a_1b_2c_3d_4| - \Sigma a_1|b_2c_3d_4| + \Sigma\,\overset{+}{|}a_1b_2\overset{+}{|}\,.\,|c_3d_4| - \Sigma\,\overset{+}{|}a_1b_2c_3\overset{+}{|}\,d_4 + \overset{+}{|}a_1b_2c_3d_4\overset{+}{|} = 0.$$

The effect of the change is the same as if the two lines had been retained, but the second read from right to left. The Σ, it will be noted, does not tolerate any permutations of suffixes separately from the letters to which they are attached in the typical term, the entities which are subject to combination being a_1, b_2, c_3, d_4, ... viewed as wholes. Thus, a typical term in the next case being

$$\overset{+}{|}a_1b_2\overset{+}{|}\,.\,|c_3d_4e_5|\,,$$

the first factors of the remaining terms of the same kind are got by taking the other pairs obtainable from a_1, b_2, c_3, d_4, e_5, viz., the pairs

$$a_1c_3,\ a_1d_4,\ a_1e_5,\ b_2c_3,\ b_2d_4,\ b_2e_5,\ c_3d_4,\ c_3e_5,\ d_4e_5\,;$$

and the full aggregate of terms indicated by

$$\Sigma\,\overset{+}{|}a_1b_2\overset{+}{|}\,|c_3d_4e_5|$$

is

$$\overset{+}{|}a_1b_2\overset{+}{|}\,|c_3d_4e_5| + \overset{+}{|}a_1c_3\overset{+}{|}\,|b_2d_4e_5| + \overset{+}{|}a_1d_4\overset{+}{|}\,.\,|b_2c_3e_5| + \ldots\ldots$$

With this explanation the general theorem may be put in the form

$$\begin{aligned} |a_1b_2c_3d_4e_5\ldots| &- \Sigma a_1|b_2c_3d_4e_5\ldots| \\ &+ \Sigma\,\overset{+}{|}a_1b_2\overset{+}{|}\,|c_3d_4e_5\ldots| \\ &- \Sigma\,\overset{+}{|}a_1b_2c_3\overset{+}{|}\,.\,|d_4e_5\ldots| + \ldots\ldots = 0. \end{aligned}$$

Heat of Combination of Metals in the Formation of Alloys. By Alexander Galt, D.Sc.

(Read March 7, 1898.)

In the course of his valuable researches on the heats of combination and decomposition of bodies, the late Professor Thomas Andrews, of Belfast, arrived at the conclusion that "If three metals, A, B, C, be so related that A is capable of displacing B and C from their combinations, and also B capable of displacing C; then the heat developed in the substitution of A for C will be equal to that developed in the substitution of A for B, added to that developed in the substitution of B for C; and a similar rule may be applied to any number of metals similarly related." A tabular statement of experimental results is given, and from them illustrations of the above conclusion may be obtained. For example, let A represent zinc, B lead, and C copper; then, for equivalent quantities of the metals, it was found that the number of (gramme-water) heat units centigrade developed when zinc displaces copper is approximately equal to the number developed when zinc displaces lead, added to that obtained when lead displaces copper. But, in contact electricity, the electromotive force between zinc and copper is equal to that between zinc and lead added to that between lead and copper. These facts led Andrews to make the following important remark:—"Electromotive forces which are really due to contact of dissimilar bodies are also the very forces which cause heat when chemical combination ensues, potential energy being converted into kinetic energy by the rushing together of the particles under attracting forces."

Lord Kelvin* has investigated the question of the amount of heat equivalent to the work done by polished parallel plates of copper and zinc kept metallically connected by a flexible wire, and allowed to attract one another to very small distances, such as the one ten-millionth or the one hundred-millionth of a centimetre; and, in full agreement with Andrews, has pointed out that the heat equivalent of the work is a portion of the heat of combination which would be given out if the two metals were brought into thorough chemical combination with one another.

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* *Popular Lectures and Addresses*, vol. i., "The Size of Atoms," pp. 160–166.

Hitherto, however, few experiments and no measurements have been made to exhibit or determine the heat of combination of zinc with copper, or of other pairs of solid metals. Not only in connection with the theory of contact electricity in particular but generally in respect to chemical affinities it is important that we should have some knowledge in regard to this question, and at the request of Lord Kelvin I have carried out the following experimental investigation in the Physical Laboratory of the University of Glasgow.

The method of procedure was to dissolve a known weight of an alloy, and also, under similar conditions, the same weight of a mixture of the elements which are present in the alloy, the proportions taken being the same as those known to be in the alloy, and noting the initial and final temperature in each case.

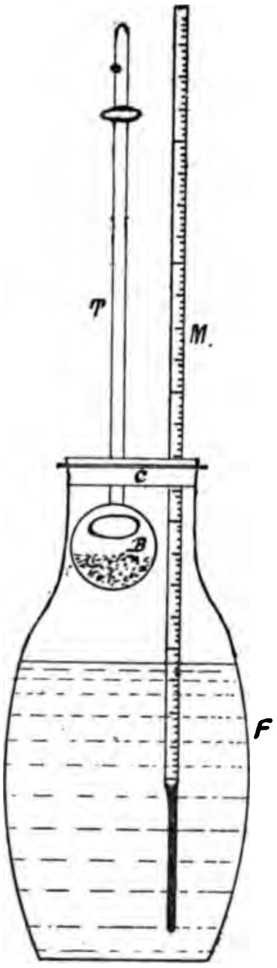
Experiments were first made on brass and on its mixed elements to ascertain if there was a difference in the heat of solution in the two cases; but before this question could be answered with certainty, a large number of preliminary experiments were made to determine the most suitable conditions for the work. The nature and strength of the solvent, and its quantity for a given mass of the metals to be treated, keeping in view the advisability of obtaining a moderate range of thermometric readings, and the necessity of minimising as far as possible the violence of the reaction between metals and solvent, had to be settled. Bromine (pure, or diluted with water or hydrochloric acid solution in various proportions), chlorine dissolved in hydrochloric acid solution, and nitric acid of different strengths, were all tried, and the nitric acid was found to be the most suitable. Excellent results were obtained from nitric acid containing 57 per cent. of pure acid; its density at 15° C. was 1·355, and its specific heat was taken as ·634. This strength of acid was that used throughout in the subsequent experiments on zinc and copper and on brass. Different kinds of apparatus were used and in different ways, but that finally adopted is shown in the figure.

With regard to the metals, no difficulty was experienced in dissolving the brass and the zinc; indeed the chemical action when the latter was being dissolved was rather violent, and the evolution of fumes was so rapid that in the early experiments considerable trouble was experienced in the efforts to control the fumes and prevent their escape and consequent loss of heat. On the other hand the free copper dissolved more slowly, otherwise a more dilute nitric acid than that mentioned would have been used. As it was, it was found necessary to have the copper always in the state of fine filings, prepared by the somewhat laborious process of filing down from the sample by means of a very fine file. The brass and the zinc were also used in the form of filings, much coarser than those of the copper. In all cases the filings of the separate elements were made from the same pieces which had been partly used in making the alloy.

The first experiments were made on a sample of ordinary brass, and I am indebted to the kindness of Mr W. R. Lang, B.Sc., and of Mr Carrick Anderson, M.A., B.Sc., of the Chemical Department of the University, for a quantitative analysis not only of this specimen but of others, metals and alloys, used in the course of this investigation. It was always necessary to know the composition of the alloy used, so that a convenient quantity of the mixed metals, in the proportions in which they exist in the brass, could be weighed out for solution, and a fair comparison made between the heat of solution of this quantity of the alloy and of the same weight of the mixed metals. The brass was found to contain 64 per cent. of copper, 32 per cent. of zinc, and 4 per cent. of lead.

The experiments were only partially satisfactory, as a residue was always obtained which took a considerable time to dissolve. It was small in amount, and was doubtless due to the lead. However, the experiments clearly showed that the heat of solution of the alloy was *less* than the heat of solution of the mixture. To eliminate the disturbing influence of the lead, a sample of brass was specially and most carefully made, from the best commercial zinc and copper, by Mr M'Phail, brassfounder, Cambridge Street, Glasgow. Its composition was about 52 per cent. copper and 48 per cent. zinc.

Each experiment was carried out in detail as follows:—One end of a short length of thin glass tube, T, was closed and then sealed to a very small glass bulb, B. Near the point of attachment there were, on opposite sides, two oval-shaped openings in the bulb. The glass tube was free to move up and down through one of two holes bored through a very short common cork, C, the downward movement being prevented from exceeding six centimetres by a knob in the upper part of the glass tube. Special care being taken to see that the bulb was clean and dry, it was drawn down from the cork about six centimetres and the cork fixed in a clamp. The filings (·5 gramme was the quantity always used in each of the brass and copper and zinc experiments) were then most carefully inserted into the bottom of the bulb by one of the openings, and the bulb was then drawn up close to the cork. Through the other hole in the cork a very thin, sensitive, short-range thermometer, M, whose marked divisions correspond to ·05° C., was passed. The cork carrying the bulb and attached tube and thermometer was then carefully fixed in the neck of a small flask, F, of thin glass containing the nitric acid, 60 cubic centimetres of which were used in every one of these experiments. The figure is a representation of the arrangements at this stage. Holding the flask by the lip with three fingers of one hand, it was gently shaken so as to give the acid a rotating motion. In this way the flask and contents soon attained a uniform temperature, which was very carefully noted, the reading of the thermometer being always



taken by the aid of a magnifying glass. The bulb was now quickly plunged to near the bottom of the flask by pushing the glass tube to which it was sealed down through the cork, its descent being limited by the knob on the tube. If the method of pouring the acid on the filings or of dropping the filings into the acid had been adopted, a violent action would have occurred and it would not have been possible to prevent the loss of heat due to escape of fumes. But the plan adopted effectually got rid of this difficulty by the almost instantaneous projection of the bulb containing the filings to the bottom of the acid.* It was very interesting to observe the scouring effect in the bulb due to the chemical action; the filings were almost instantly expelled from it by the rapid evolution of gas, the removal being facilitated by the existence of the two apertures already described. The gentle rotatory motion given to the acid was kept up while solution was going on, and when it was complete the thermometer reading was again noted. The time required to effect solution was 50 to 55 seconds, and it was observed that complete solution and maximum temperature were reached about the same time. The total weight of the whole apparatus—flask, cork, thermometer, and glass tube with attached bulb, but without metals and acid—was 20·5 grammes, and its water equivalent was taken as 3·5 grammes. The tabular statement below gives some of the latest results obtained on 29th January last.

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* Andrews' *Scientific Papers*, p. 214. "Every chemist is familiar with the violent action of nitric acid on zinc and copper, and the abundant evolution of gas which accompanies it. But the facility with which the gases may be condensed by the acid solution is probably not so generally known, and when the experiment is made for the first time it cannot fail to excite surprise."

[TABULAR STATEMENT.

Heat of Solution of ·5 gramme (·26 gramme Copper + ·24 gramme Zinc) of the Mixed Metals and of ·5 gramme of the Alloy.

MIXTURE.			ALLOY.		
Temperature of the Acid before and after Solution.			Temperature of the Acid before and after Solution.		
Before.	After.	Difference.	Before.	After.	Difference.
C.°	C.°	C.°	C.°	C.°	C.°
9·05	17·85	8·80	9·60	17·70	8·10
9·30	18·08	8·78	9·65	17·77	8·12
9·35	18·15	8·80	10·20	18·30	8·10
9·22	18·01	8·79	9·97	18·05	8·08
Mean Difference, .		8·792	Mean Difference, .		8·10

The increase of temperature of the solution of the mixed metals (*per gramme* of metal) was therefore greater than the increase of temperature of the solution of the brass by 2(8·792 − 8·10), that is, 1·384° C.; and the heat of combination of any quantity of copper and zinc in the proportions stated (52 per cent. copper, 48 per cent. zinc), expressed as a fraction of the heat developed by the solution of the same quantity of the mixed metals in like proportions in nitric acid, is

$$\frac{1\cdot384}{2\times8\cdot792}=\frac{1}{12\cdot7}\,.$$

The absolute amount of heat evolved by the combination of ·52 gramme of copper with ·48 gramme of zinc was approximately ascertained as follows:—

Let H represent, in (gramme-water) heat units Centigrade, the heat of solution of 1 gramme of metal.
,, v ,, in cubic centimetres, the quantity of acid used.
,, ρ ,, the density of the acid at 15° C.
,, s ,, the specific heat of the acid.
,, w ,, in grammes, the weight of metal dissolved.
,, c ,, the water equivalent of the glass and cork of the apparatus.
,, t ,, in degrees C., the mean difference of temperature before and after solution.

Then an approximation to the mean absolute amount of heat evolved in the solution of one gramme of the mixed metals, or of one gramme of an alloy of the same metals in the same proportions, is ascertained by the following expression:—

$$H = t[\{(v\rho) + w\}s + c].$$

For the mixture

$$H = 2 \times 8{\cdot}792[\{(60 \times 1{\cdot}355) + {\cdot}5\}{\cdot}634 + 3{\cdot}5]$$

$$= 2 \times 8{\cdot}792[55{\cdot}36] \quad . \quad . \quad . \quad . \quad = 973{\cdot}45$$

For the alloy

$$H = 2 \times 8{\cdot}1 \times 55{\cdot}36 \quad . \quad . \quad . \quad . \quad = 896{\cdot}83$$

Difference, ... 76·6

This difference approximately represents, in (gramme-water) heat units Centigrade, the heat of combination of ·52 gramme of copper with ·48 gramme of zinc. It will be noticed that ·5 has been used for w instead of 1, this being done to make some allowance for the lower specific heat of the metals when compared with that of the acid. An allowance, however, is almost unnecessary, as it has scarcely any influence on the value of the difference between the absolute amounts of heat obtained for the mixture and the alloy.

Andrews* obtained 1420 thermal units as the heat of solution of one gramme of zinc, and 650 thermal units as the heat of solution of one gramme of copper, both in diluted nitric acid. Taking 48 per cent. of the former result, and 52 per cent. of the latter, to enable a comparison to be made with my results, the amounts are, respectively, 681·6 and 338, or a total of 1019·6, which is fully four per cent. greater than my value, 973·4, for the mixture. The difference is, doubtless, largely accounted for by the presence of impurities in the metals used, and by the radiation of heat from the apparatus during the time of solution.

In chemical reactions in which copper and zinc take part these metals are usually bivalent elements, and in all such cases their relative combining weights are, therefore, as 63 to 65, or about 49·2 per cent. copper to 50·8 per cent. zinc, proportions nearly the same as those used in the experiments described, and for which the value 76·6 heat units Centigrade was obtained as the heat of formation of one gramme of the alloy. Having obtained this result, the experiments were repeated, a new alloy and a new mixture of the same metals in proportions very different from the relative combining weights referred to being used. The proportions were 70 per cent. copper and 30 per cent. zinc, and the specimens were again supplied by Mr M'Phail. Some of the latest results, obtained on 6th February last, are given in the tabular statement below.

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* *Scientific Papers*, p. 215.

Heat of Solution of ·5 gramme (·35 gramme Copper mixed with ·15 gramme Zinc) of the Mixed Metals and of ·5 gramme of the Alloy.

Mixture.			Alloy.		
Temperature of the Acid before and after Solution.			Temperature of the Acid before and after Solution.		
Before.	After.	Difference.	Before.	After.	Difference.
C.°	C.°	C.°	C.°	C.°	C.°
9·45	16·90	7·45	10·75	17·90	7·15
9·90	17·38	7·48	10·85	17·99	7·14
9·70	17·15	7·45	11·10	18·27	7·17
9·85	17·31	7·46	10·85	18·00	7·15
Mean Difference, .		7·46	Mean Difference, .		7·15

The heat of combination of any quantity of copper and zinc in the proportions stated, expressed as a fraction of the heat developed by the solution of the same quantity of the mixed metals in like proportions in nitric acid, is

$$\frac{7{\cdot}46 - 71{\cdot}5}{7{\cdot}46} = \frac{1}{24}.$$

The absolute amount of heat evolved in the solution of one gramme of the mixed metals (70 per cent. copper, 30 per cent. zinc) and of one gramme of an alloy of the same metals, in the same proportions, determined as on the previous occasion, is

For the mixture	826·0	(gramme-water)	heat units	Centigrade.
„ alloy	791·6	„	„	„
Difference,	34·4	„	„	„

Thus the heat of combination of ·7 gramme of copper with ·3 gramme of zinc is 34·4 (gramme-water) heat units Centigrade, or less than half the quantity (76·6) obtained as the heat of combination of ·52 gramme of copper with ·48 gramme of zinc, the total weight of metal in each case being one gramme.

Having obtained these interesting results with copper and zinc, Lord Kelvin advised that the experiments should be repeated with other metals. Copper and silver were fixed upon, and I am indebted to Messrs Thomas Smith & Son, silversmiths, Queen Street, Glasgow, for the care and trouble they have taken in making, from pure silver and copper, two alloys, one of which contained 216 parts by weight of silver to 63 parts by weight of copper, or 77·4 per cent. of silver to 22·4 per cent. of copper, these being the relative combining proportions of the metals usually observed in chemical reactions in which they take part. The other alloy contained about equal weights of these metals. Separate specimens of silver and copper of the same quality as those in the alloys were also supplied.

In the preliminary experiments on these four specimens, it was found that while all dissolved slowly in nitric acid, and therefore required to be in the state of fine filings so as to have solution completed in a short time, the silver was particularly slow in this respect; and it was absolutely necessary to use it in the form of exceedingly fine filings. A special set of very fine files was used in the making of the filings,—a very slow and laborious process. It was also found that the silver dissolved more readily in fairly dilute nitric acid than in stronger acid. Repeated tests showed that acid of density 1·265 at 15° C. was, perhaps, the best.

A quantitative analysis of the first mentioned alloy showed that its composition was 76·73 per cent. silver and 23·27 per cent. copper. Comparative tests were made with one gramme (·7673 gramme of silver mixed with ·2327 gramme of copper) of the mixed metals and one gramme of the alloy. Owing to the heat of solution being much less than that observed in the copper-zinc experiments only

32 cubic centimetres of nitric acid were used as the solvent; its density was 1·270 at 15° C., and its specific heat was taken as ·703. The apparatus used and the method of experimenting were exactly the same as formerly, and the results obtained in the several experiments are shown in the subjoined table.

Heat of Solution of 1 *gramme* (·7673 *gramme of Silver mixed with* ·2327 *gramme of Copper*) *of the Mixed Metals and of* 1 *gramme of the Alloy.*

MIXTURE.			ALLOY.		
Temperature of the Acid before and after Solution.			Temperature of the Acid before and after Solution.		
Before.	After.	Difference.	Before.	After.	Difference.
C.°	C.°	C.°	C.°	C.°	C.°
11·90	17·58	5·68	12·00	17·16	5·16
14·05	19·75	5·70	12·00	17·14	5·12
11·90	17·59	5·69	11·92	17·06	5·14
Mean Difference, .		5·69	Mean Difference, .		5·14

The heat of combination of any quantity of copper and silver in the proportions stated, expressed as a fraction of the heat developed by the solution of the same quantity of the mixed metals in like proportions in nitric acid, is

$$\frac{5\cdot69-5\cdot14}{5\cdot69}=\frac{1}{10\cdot3}.$$

The absolute amount of heat evolved in the solution of one gramme of the mixed metals (76·73 per cent. silver, 23·27 per cent. copper) and of one gramme of an alloy of the same metals in the same proportions is

For the mixture, 186·5 (gramme-water) heat units Centigrade.
„ alloy, 168·5 „ „ „

Difference, 18·0 „ „ „

Thus the heat of combination of ·7673 gramme of silver with ·2327 gramme of copper is 18 (gramme-water) heat units Centigrade.

But it has been shown on page 143 that the heat of combination of one gramme (·52 gramme copper and ·48 gramme of zinc) of the mixed copper and zinc filings was 76·6 (gramme-water) heat units Centigrade; therefore the amount of heat developed in the formation of the zinc-copper alloy is fully four times the amount evolved in the formation of the silver-copper alloy, the metals being present in each case nearly in the proportion of their chemical equivalents as usually observed in chemical reactions. The much higher result for the zinc-copper experiments is just what might be expected from the fact that in contact electricity the force of attraction between zinc and copper is greater than that between silver and copper.

The other alloy of silver and copper was next tested, ·8 gramme of the mixed metals or of the alloy being used. The alloy was made from exactly equal weights of the metals, but on analysis it was found to be 51·62 per cent. silver and 48·38 per cent. copper.

Heat of Solution of ·8 *gramme* (·413 *gramme of Silver mixed with* ·387 *gramme of Copper) of the Mixed Metals and of* ·8 *gramme of the Alloy.*

MIXTURE.			ALLOY.		
Temperature of the Acid before and after Solution.			Temperature of the Acid before and after Solution.		
Before.	After.	Difference.	Before.	After.	Difference.
C.°	C.°	C.°	C.°	C.°	C.°
12·10	19·15	7·05	12·30	19·20	6·90
12·00	18·97	6·97	12·05	18·95	6·90
11·60	18·70	7·1	12·20	19·10	6·90
Mean Difference, .		7·04	Mean Difference, .		6·90

The heat of combination of any quantity of copper and silver in the proportions just stated, expressed as a fraction of the heat

	ZINC-COPPER.						SILVER-COPPER.					
	48 per cent. Zinc. 52 ,, Copper.			30 per cent. Zinc. 70 ,, Copper.			76·73 per cent. Silver. 23·27 ,, Copper.			51·62 per cent. Silver. 48·38 ,, Copper.		
	Mixture.	Alloy.	Difference.	Mixture.	Alloy.	Difference.	Mixture.	Alloy.	Difference.	Mixture.	Alloy.	Difference.
Mean increase of temperature of solution per gramme of metal dissolved, expressed in degrees Centigrade, . .	17·584	16·2	1·384	14·92	14·3	0·62	5·69	5·14	0·55	8·8	8·625	0·175
Heat of combination of the metals in the formation of one gramme of the alloy, expressed as a fraction of the heat of solution of one gramme of the mixture, . . .	$\frac{1·384}{17·584}=\frac{1}{12·7}$			$\frac{0·62}{14·92}=\frac{1}{24}$			$\frac{0·55}{5·69}=\frac{1}{10·3}$			$\frac{0·175}{8·8}=\frac{1}{50}$		
Absolute amount of heat evolved by the combination of the metals in the formation of one gramme of the alloy, expressed in (gramme-water) heat units Centigrade, .	76·6			34·4			18·0			7·0		

developed by the solution of the same quantity of the mixed metals in like proportions in nitric acid, is therefore

$$\frac{7{\cdot}04 - 6{\cdot}90}{7{\cdot}04} = \frac{1}{50} .$$

The absolute amount of heat evolved in the solution of one gramme of the mixed metals (51·62 per cent. silver, 48·38 per cent. copper) and of one gramme of an alloy of the same metals in the same proportions, calculated as usual, is

For the mixture,	351·9	(gramme-water)	heat units	Centigrade.
„ alloy,	344·9	„	„	„
Difference,	7·0	„	„	„

Thus the heat of combination of ·5162 gramme of silver with ·4838 gramme of copper is 7 (gramme-water) heat units Centigrade, or less than half the quantity (18) obtained as the heat of combination of ·7673 gramme of silver with ·2327 gramme of copper.

All the results recorded in this communication are shortly summarised on the preceding page.

The experiments hitherto made show in each case greater heat of combination when the proportions of the metals in the alloy are nearly in the ratio of their usual chemical equivalents than when they are in proportions largely different from this ratio. Further experiments are needed to answer questions that occur in connection with this result. I am therefore continuing the investigation, of which a part is described in this paper.

A Problem of Sylvester's in Elimination.

By E. J. Nanson, M.A. Communicated by Professor **Chrystal.**

(Read December 6, 1897.)

1. Denoting by A, B, C, F, G, H the co-factors of a, b, c, f, g, h, in the determinant

$$\begin{vmatrix} a & h & g & x \\ h & b & f & y \\ g & f & c & z \\ x & y & z & \cdot \end{vmatrix},$$

the problem in question is to eliminate x, y, z from the three equations

$$\mathrm{A}=0\,,\ \mathrm{B}=0\,,\ \mathrm{C}=0 \quad . \quad . \quad . \qquad (1)$$

and it has been discussed by Sylvester,* Cayley,† Muir,‡ Tait,§ and Lord M'Laren.||

Sylvester deduces from (1) this new system

$$\mathrm{F}=0\,,\ \mathrm{G}=0\,,\ \mathrm{H}=0 \quad . \quad . \quad . \qquad (2)$$

and hence finds the dialytic eliminant

$$\begin{vmatrix} \cdot & c & b & -2f & \cdot & \cdot \\ c & \cdot & a & \cdot & -2g & \cdot \\ b & a & \cdot & \cdot & \cdot & -2h \\ f & \cdot & \cdot & a & -h & -g \\ \cdot & g & \cdot & -h & b & -f \\ \cdot & \cdot & h & -g & -f & c \end{vmatrix} = 0 \,. \quad . \qquad (3)$$

the expansion of which (an obvious error being corrected) he states to be

$$abc\Delta = 0$$

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* *Cam. Math. Journ.*, ii. p. 232. See also Muir, *Theory of Determinants*, part i. p. 231.

† *C. M. P.*, 198 ; *Proc. R.S.E.*, xx. p. 306,

‡ *Proc. R.S.E.*, xx. pp. 300, 371.

§ *Proc. R.S.E.*, xix. p. 131.

|| *Proc. R.S.E.*, xix. p. 264.

where

$$\Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

so that "rejecting the special (*N.B.*, not *irrelevant*) factor $a\,b\,c$, we obtain"

$$\Delta = 0.$$

2. Now, as will be shown presently, $a\,b\,c$ is a factor of the true eliminant of (1). But $a\,b\,c$ is not a factor of (3). For, if we multiply columns 4, 5, 6 by bc, ca, ab; then multiply columns 1, 2, 3 by $-af$, bf, cf, and add to column 4; multiply by ag, $-bg$, cg, and add to column 5; multiply by ah, bh, $-ch$, and add to column 6, we find the value of the determinant in (3) to be

$$\frac{1}{a^2b^2c^2}\cdot\begin{vmatrix} \cdot & c & b & \cdot & \cdot & \cdot \\ c & \cdot & a & \cdot & \cdot & \cdot \\ b & a & \cdot & \cdot & \cdot & \cdot \\ f & \cdot & \cdot & aA' & aH' & aG' \\ \cdot & g & \cdot & bH' & bB' & bF' \\ \cdot & \cdot & h & cG' & cF' & cC' \end{vmatrix}$$

where A′, B′, C′, F′, G′, H′ are the co-factors of a, b, c, f, g, h in Δ. But this is

$$\frac{1}{abc}\cdot\begin{vmatrix} \cdot & c & b \\ c & \cdot & a \\ b & a & \cdot \end{vmatrix}\cdot\begin{vmatrix} A' & H' & G' \\ H' & B' & F' \\ C' & F' & C' \end{vmatrix}$$

that is $2\Delta^2$.

3. Next, Cayley points out, first, that the equations (1), (2), are found by expressing that an arbitrary point on the line

$$(x, y, z)\,(\xi, \eta, \zeta) = 0 \quad . \quad . \quad . \quad (4)$$

is on the conic

$$(a, b, c, f, g, h)\,(\xi, \eta, \zeta)^2 = 0 \quad . \quad . \quad (5)$$

and, second, that in virtue of the identical equations

$$\left.\begin{array}{l} Ax + Hy + Gz = 0 \\ Hx + By + Fz = 0 \\ Gx + Fy + Cz = 0 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (6)$$

the equations (1), (2) are equivalent to only three independent equations which may be taken to be any three of the six: (1), (2), except the sets A, H, G'; H, B, F; G, F, C. He thence infers, first, that the eliminant of any independent set of three of the equations (1), (2) is (3), and, second, that Δ must be a factor of the eliminant. He states that by actual development the value of the determinant (3) is found to be $2\Delta^2$, and remarks that it would be interesting to show, *à priori*, that Δ^2 is a factor.

4. The *à priori* explanation is as follows:—The equation $A=0$ expresses that the line (4) passes through one or other of the points in which the conic (5) cuts the line $\xi=0$. Hence, denoting the points in which (5) cuts the lines $\xi=0$, $\eta=0$, $\zeta=0$ by P, P′; Q, Q′; R, R′, the equations $B=0$, $C=0$ determine (4) as one of the four lines QR, QR′, Q′R, Q′R′, say QR; the equation $A=0$ then makes QR pass through P or P′, and in either case the conic breaks up, and so Δ^2 is a factor of the eliminant. In this explanation it has been assumed, as is done by Cayley, that all zero values of x or y or z are excluded.

5. If we regard (3) as the eliminant of (2), a similar *à priori* explanation can be given of the occurrence of Δ^2 as a factor. The equation $F=0$ expresses that the two points in which (4) cuts the lines $\eta=0$, $\zeta=0$ are conjugate with respect to (5). Hence, denoting by L, M, N the points in which (4) cuts the lines $\xi=0$, $\eta=0$, $\zeta=0$, the equations $G=0$, $H=0$ give L the two conjugates M, N, so that (4) touches (5) at L. The equation $F=0$ then makes M, N conjugate. But of two conjugate points on a tangent one must be a point of contact. Hence M or N must be a point of contact distinct from L. In either case the conic breaks up, and so Δ^2 is a factor of the eliminant.

A similar *à priori* explanation of the occurrence of Δ^2 as a factor can be given in the case of each of the systems

$$B=0,\ C=0,\ F=0;$$
$$B=0,\ C=0,\ G=0;$$
$$A=0,\ F=0,\ G=0;$$

6. Muir, evidently unacquainted with Cayley's first paper on the subject, finds, "not without considerable trouble," that the value of this dialytic determinant (3) is $2\Delta^2$, shews that (1) are found by expressing that (4) is a factor of (5), and accounts for

the fact that the dialytic eliminant has Δ^2 as a factor by considering (1) as a special case of the more general system

$$\left.\begin{aligned} bz^2 + c'y^2 - 2xyz &= 0 \\ cx^2 + a'z^2 - 2gzx &= 0 \\ ay^2 + b'x^2 - 2hxy &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (7)$$

But in point of fact his work shows that the true eliminant of (1) is not Δ^2 but $a^2b^2c^2\Delta^2$. It is not, however, necessary to consider a system more general than (1) in order to arrive at this result.

7. In the first place, if, as explained in Salmon's *Higher Algebra*, we eliminate dialytically from (1), and from the three equations obtained by equating to zero the differential coefficients of the factors of A, B, C, we get the eliminant in the form

$$\begin{vmatrix} \cdot & c & b & -2f & \cdot & \cdot \\ c & \cdot & a & \cdot & -2g & \cdot \\ b & a & \cdot & \cdot & \cdot & -2h \\ \cdot & c\mathrm{F}' & b\mathrm{F}' & 2(abc-fgh) & 2b\mathrm{H}' & 2c\mathrm{G}' \\ c\mathrm{G}' & \cdot & a\mathrm{G}' & 2a\mathrm{H}' & 2(abc-fgh) & 2c\mathrm{F}' \\ b\mathrm{H}' & a\mathrm{H}' & \cdot & 2a\mathrm{G}' & 2b\mathrm{F}' & 2(abc-fgh) \end{vmatrix} = 0$$

and reducing the determinant exactly as in § 2 we get

$$8abc \;.\; \begin{vmatrix} \cdot & c & b \\ c & \cdot & a \\ b & a & \cdot \end{vmatrix} \begin{vmatrix} \mathrm{A}' & \mathrm{H}' & \mathrm{G}' \\ \mathrm{H}' & \mathrm{B}' & \mathrm{F}' \\ \mathrm{G}' & \mathrm{F}' & \mathrm{C}' \end{vmatrix}$$

that is $16\,a^2b^2c^2\Delta^2$.

8. In the second place, it may be shown by elementary algebra that the eliminant is $a^2b^2c^2\Delta^2$. For we have in effect to eliminate x, y, z from

$$cx^2 + az^2 - 2gzx = 0 \quad . \quad . \quad . \quad (\alpha)$$

$$ay^2 + bx^2 - 2hxy = 0 \quad . \quad . \quad . \quad (\beta)$$

$$my + nz = 0 \quad . \quad . \quad . \quad (\gamma)$$

where $my + nz$ is a factor of $bz^2 + cy^2 - 2xyz$, so that

$$bm^2 + cn^2 + 2xmn = 0 \quad . \quad . \quad . \quad (\delta)$$

Now eliminating z from (α), (γ), we have

$$cn^2x^2 + nm^2y + 2gmnxy = 0,$$

and eliminating x, y from this by (β) we get

$$4B'C'm^2n^2 = (acn^2 + abm^2 + 2ghmn)^2,$$

and therefore

$$abm^2 + acn^2 + 2(gh \pm \sqrt{B'C'})mn = 0 \quad . \quad . \qquad (\epsilon)$$

By eliminating m, n from (δ), (ϵ) we get

$$(bc - f^2)\{a^2bc - (gh \pm \sqrt{B'C'})^2\} = \{abc - f(gh \pm \sqrt{B'C'})\}^2$$

or, after a simple reduction

$$bc\{F' \pm \sqrt{B'C'}\}^2 = 0,$$

and on rationalizing we get

$$a^2b^2c^2\Delta^2 = 0.$$

9. In the third place, it may be shown by the geometrical process of § 4 that $a^2b^2c^2$ is a factor of the eliminant. In that section it was shown that either PQR, or P'QR is a straight line. As we have already seen, if zero values of x or y or z are excluded and PQR be straight, the conic (5) must break up. But if zero values are not excluded, PQR is straight if any two of the three points P, Q, R coincide. Now this may happen when $a = 0$ or $b = 0$ or $c = 0$. Thus, if PQR be straight, it follows that $abc\Delta = 0$. Exactly the same result holds if P'QR is straight. Hence $a^2b^2c^2\Delta^2$ is a factor of the eliminant, and since the eliminant must be of order 12 in the coefficients there can be no other factor.

In a similar way it can be shown that the eliminant of (2) is $abcfgh\Delta^2$.

10. The dialytic method as exhibited in §§ 1, 3 does not give the true eliminant, because, first, the system (2) does not follow from (1) when any one of the three, x, y, z, is zero; and, second, the system (1) is satisfied by $x = 0$, $bz^2 + cy^2 - 2xyz = 0$ provided the single relation $a = 0$ is satisfied. But the dialytic process as given in § (7) does give the true eliminant, because the differential coefficients of the Jacobian of A, B, C vanish when (1) are satisfied, provided x, y, z are not all zero. The elementary algebraic methods given by Tait, Lord M'Laren, and Muir do not give the complete result of elimination, because they assume that x, y, z are all different from zero.

11. The elimination may readily be effected without any preliminary transformation as follows:—

The equations (1) are

$$bz^2 + cy^2 - 2xyz = 0\,,\ cx^2 + az^2 - 2gxz = 0\,,\ ay^2 + bx^2 - 2hxy = 0,$$

and from these we have

$$\begin{aligned} 8fghx^2y^2z^2 &= (bz^2 + cy^2)(cx^2 + az^2)(ay^2 + bx^2) \\ &= ax^2(bz^2 + cy^2)^2 + by^2(cx^2 + az^2)^2 + cz^2(ay^2 + bx^2) - 4abcx^2y^2z^2 \end{aligned}$$

and therefore

$$(abc + 2fgh - af^2 - bg^2 - ch^2)x^2y^2z^2 = 0.$$

Hence, either $\Delta = 0$ or one of the three, x, y, z, is zero. Now if $x = 0$ the second and third equations give

$$ay^2 = 0 \qquad az^2 = 0,$$

and therefore either $a = 0$ or y and z are both zero. Hence, unless x, y, z are all zero we must have

$$\Delta = 0\,, \text{ or } a = 0\,, \text{ or } b = 0\,, \text{ or } c = 0.$$

Thus, the elimination of x, y, z from (1) leads to

$$abc\,\Delta = 0.$$

Now the eliminant of three ternary quadrics is of order 12 in the coefficients. Hence, as all possible alternatives have been taken into account, it follows by symmetry that the characteristic of the true eliminant must be one of the three

$$abc\,\Delta^3,\ a^2b^2c^2\,\Delta^2,\ a^3b^3c^3\,\Delta\,.$$

12. That the second of these three forms is the true eliminant is easily shown. For when the eliminant of a number of equations vanishes, the equations are satisfied by a common system of values of the variables. But in the present case there are eight such common systems. When $a = 0$ the equations are satisfied if

$$x = 0 \text{ and } bz^2 + cy^2 - 2xyz = 0\,;$$

that is, there are two systems of values of x, y, z which satisfy (1), and these values are the tangential coordinates of the lines joining the point $\eta = 0$, $\zeta = 0$ to the intersections of $\xi = 0$ with the conic (5). Similarly, when $b = 0$ or $c = 0$ we get two systems. When

$\Delta = 0$ we also get two systems, viz., the tangential coordinates of the two lines represented by (5) when $\Delta = 0$. It is thus seen that the squares of a, b, c, Δ are each of them a factor of the eliminant.

13. Muir has obtained the eliminant of the system (7) in several different forms. To these may be added the following:—

$$f\sqrt{\mathrm{QR}} + g\sqrt{\mathrm{RP}} + h\sqrt{\mathrm{PQ}} = \mu - fgh$$

$$\begin{vmatrix} \mu & hca' & gab' & 2\mu gh + faa'b'c \\ hbc' & \mu & fab' & 2\mu hf + gbb'c'a \\ gbc' & fca' & \mu & 2\mu fg + hcc'a'b \\ f & g & h & 2fgh + \mu \end{vmatrix} = 0$$

$$(\mu^2 + 2\mu fgh - aa'b'cf^2 - bb'c'ag^2 - cc'a'bh^2)^2$$

$$= 4(\mu^2 - abca'b'c')(f^2g^2h^2 + 2\mu fgh - g^2h^2bc' - h^2f^2ca' - f^2g^2ab')$$

where

$$2\mu = abc + a'b'c'$$

and

$$\mathrm{P} = bc' - f^2,\ \mathrm{Q} = ca' - g^2,\ \mathrm{R} = ab' - h^2.$$

14. The first of these forms was originally found by applying to (7) the method used in § 8. But the first and second forms are most readily found by transforming (7) to the form

$$\cos a = \frac{f}{\sqrt{bc'}},\ \cos\beta = \frac{g}{\sqrt{ca'}},\ \cos\gamma = \frac{h}{\sqrt{ab'}}$$

and then eliminating a, β, γ by means of the obvious relation

$$\cos(a + \beta + \gamma) = \frac{\mu}{\sqrt{abc\,a'b'c'}}.$$

The third form may be derived from the second by multiplying columns 1, 2, 3 by $faa'b'c$, $gbb'c'a$, $hcc'a'b$, and subtracting from μ times the fourth column, and is interesting as showing not only how the eliminant of (7) degenerates to $a^2b^2c^2\Delta^2$ when $a', b', c' = a, b, c$, but also that the eliminant of (7) is a perfect square when either $a'b'c' = abc$ or

$$\begin{vmatrix} 1 & \frac{b'}{h} & \frac{c}{g} \\ \frac{a}{h} & 1 & \frac{c'}{f} \\ \frac{a'}{g} & \frac{b}{f} & 1 \end{vmatrix} = 0.$$

15. Cayley's first paper at once suggests a generalisation of Sylvester's elimination problem. Denoting by A, B, C, D, F, G, H, L, M, N the co-factors of a, b, c, d, f, g, h, l, m, n in the determinant

$$\begin{vmatrix} a & h & g & l & x \\ h & b & f & m & y \\ g & f & c & n & z \\ l & m & n & d & w \\ x & y & z & w & . \end{vmatrix}$$

it may be proposed to eliminate x, y, z, w from any independent set of four of the ten equations

$$\mathrm{A}=0, \mathrm{B}=0, \ldots, \mathrm{N}=0 . \qquad (8)$$

Now if the conicoid

$$(a, b, c, d, f, g, h, l, m, n)(\xi, \eta, \zeta, \omega)^2=0 . \qquad (9)$$

be a cone, and

$$(x, y, z, w)(\xi, \eta, \zeta, \omega)=0 . \qquad (10)$$

be a proper tangent plane thereto, the ten equations (8) are obviously all satisfied. For these equations are found by expressing that the line of intersection of (10) with an arbitrary plane touches the conicoid (9).

Hence it may be inferred, first, that the discriminant of (9) is a factor of the eliminant of any independent set of four of the equations (8); second, that in the case of any such set of four the solution which corresponds to the discriminant factor of the eliminant is indeterminate, and, consequently, that the *square* of the discriminant must be a factor of the eliminant; third, that the discriminant is also a factor of the determinant obtained by eliminating x^2, y^2, etc., dialytically from the ten equations (8).

It would certainly be interesting to determine the remaining factors of the eliminant of any independent set of four of the equations (8), and also the remaining factors of the dialytic eliminant of the whole of the equations (8).

On the Ellipse-Glissette Elimination Problem. By E. J. Nanson. *Communicated by* Professor CHRYSTAL.

(Read January 31, 1898.)

The problem in question is to determine the equation of the curve traced out by any point of an elliptic or hyperbolic disc which touches two fixed rectangular axes. Mechanically constructed figures of different forms of the curve have been given by Tait,* who also showed that the same glissette can be traced either by means of an ellipse or a hyperbola. If p, q are the coordinates of the tracing point referred to the axes of the disc as axes of coordinates, the glissette is clearly the θ eliminant of

$$\left.\begin{aligned}(x-p\cos\theta+q\sin\theta)^2&=a^2\cos^2\theta+b^2\sin^2\theta\\(y-p\sin\theta-q\cos\theta)^2&=a^2\sin^2\theta+b^2\cos^2\theta\end{aligned}\right\} \quad . \quad (1)$$

and Cayley stated that it would be found to be of order 8 in x, y. The actual elimination was first performed by Muir,† who obtained in the first instance an equation of order 10. On dividing by an extraneous quadratic factor a lengthy equation of order 8 was obtained; and subsequently Lord M'Laren ‡ verified the accuracy of the terms of highest order in this equation.

By addition and subtraction we obtain from (1)

$$(l, m, n)(\cos\theta, \sin\theta, 1) = 0 \quad . \quad . \quad (2)$$

$$(\mathrm{A, B, C, F, G, H})(\cos\theta, \sin\theta, 1)^2 = 0 \quad . \quad . \quad (3)$$

where

$$\begin{aligned}l&=2(px+qy)\\m&=2(py-qx)\\n&=a^2+b^2-p^2-q^2-x^2-y^2\\&=c^2-d^2-r^2, \quad \text{say}\\\mathrm{A}=-\mathrm{B}&=a^2-b^2-p^2+q^2=\lambda^2, \quad \text{say}\\\mathrm{C}&=y^2-x^2\\\mathrm{F}&=-py-qx\\\mathrm{G}&=px-qy\\\mathrm{H}&=2pq\end{aligned}$$

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* *Proc. Roy. Soc. Edin.*, xvii. pp. 2-4.

† *Ibid.*, xix. pp. 25-31.

‡ *Ibid.*, xix. pp. 89-96.

so that c is the radius of the director circle of the disc and d, r are the distances of the tracing point from the centre of the disc and from the intersection of the guides.

We have also the equation

$$(1,\ 1,\ -1,\ 0,\ 0,\ 0)(\cos\theta,\ \sin\theta,\ 1)^2 = 0 \quad . \quad . \quad (4)$$

and the result of eliminating $\cos\theta$, $\sin\theta$ from (2), (3), (4) is

$$4\Delta\Delta' = \Phi^2 \quad . \quad . \quad . \quad . \quad (5)$$

where Δ, Δ' are the bordered discriminants of (3), (4), and Φ is the intermediate of Δ, Δ'. Thus we have

$$\Delta = \begin{vmatrix} A & H & G & l \\ H & B & F & m \\ G & F & C & n \\ l & m & n & \end{vmatrix}$$

$$\Delta' = l^2 + m^2 - n^2$$

$$\Phi = (B - C)l^2 + (A - C)m^2 - (A + B)n^2 + 2Fmn + 2Gnl - 2Hlm.$$

Substituting the values of the coefficients we find

$$\begin{aligned}\Delta = {} & (\lambda^4 + 4p^2q^2)\{r^4 + (c^2 - d^2)^2\} \\ & - 16(a^2 - b^2)xy(px + qy)(qx - py) \\ & - 2(c^2 - d^2)\{(a^2 - b^2)^2 - d^4\}r^2\end{aligned}$$

$$\Delta' = -\{r^2 - (c - d)^2\}\{r^2 - (c + d)^2\}$$

$$\Phi = 8(a^2q^2 + b^2p^2)(x^2 - y^2) - 16(a^2 - b^2)pqxy.$$

Thus the curve $\Delta = 0$ is a quartic which reduces to four straight lines through the origin when $c = d$, that is, when the tracing point is on the director circle of the disc. The curve $\Delta' = 0$ consists of two concentric circles with common centre at the origin, and having for radii the maximum and minimum distances $c + d$, $c \backsim d$ of the tracing point from the origin. The curve $\Phi = 0$ consists of two perpendicular straight lines through the origin. The form of (5) shows that the glissette touches the quartic Δ and the circles Δ' at the points where these curves are met by the lines Φ.

If we put

$$c + d = \alpha, \quad c - d = \beta$$
$$\lambda^4 + 4p^2q^2 = \mu^4, \quad a^2 - b^2 = k^2$$

the equation of the glissette is

$$\{\mu^4(r^4 + \alpha^2\beta^2) - 16k^2xy(px + qy)(qx - py) - 2\alpha\beta(k^4 - d^4)r^2\}(r^2 - \alpha^2)(r^2 - \beta^2) \\ + 16\{(a^2q^2 + b^2p^2)(x^2 - y^2) - 2k^2pqxy\}^2 = 0 \quad . \quad . \quad (6)$$

where $r^2 = x^2 + y^2$.

If the tracing point be on either axis of the disc, the glissette will clearly be symmetrical about the axes of reference and also about the bisectors of the angles between the axes. Taking the tracing point on the major axis, we have $q = 0$, and therefore $\mu^2 = \lambda^2 = k^2 - p^2$, so that the locus is

$$\{(k^2 - p^2)^2(r^4 + \alpha^2\beta^2) + 16k^2p^2x^2y^2 - 2\alpha\beta(k^4 - p^4)r^2\}(r^2 - \alpha^2)(r^2 - \beta^2) \\ + 16b^4p^4(x^2 - y^2)^2 = 0 \quad . \quad . \quad . \quad (7)$$

If we suppose the disc reduced to a rod, we have $b = 0$, and therefore $\alpha = k + p$, $\beta = k - p$, and (7) reduces to

$$(\alpha^2x^2 + \beta^2y - \alpha^2\beta^2)(\beta^2x^2 + \alpha^2y^2 - \alpha^2\beta^2)(r^2 - \alpha^2)(r^2 - \beta^2) = 0$$

as it ought to do.

If the tracing point be at the centre of the disc, we see from (7) that the glissette is

$$(r^2 - c^2)^4 = 0.$$

In these two cases Φ vanishes identically and the locus consists of the two curves Δ, Δ'.

If the tracing point be at a focus of the disc, we have $p = k$ and $\alpha^2 + \beta^2 = 4a^2$, $\alpha\beta = 2b^2$, so that (6) becomes

$$x^2y^2(r^2 - \alpha^2)(r^2 - \beta^2) + b^4(x^2 - y^2)^2 = 0$$

i.e.,

$$x^2y^2\{r^4 - 4a^2r^2 + 4b^4\} + b^4(r^4 - 4x^2y^2) = 0$$

or

$$r^2\{r^2(x^2y^2 + b^4) - 4a^2x^2y^2\} = 0$$

Thus the locus consists of the sextic

$$r^2(x^2y^2 + b^4) = 4a^2x^2y^2$$

and a conjugate point at the origin. In this case the curves Δ, Φ are

$$x^2y^2=0\,,\quad x^2-y^2=0\,,$$

and the contact of the glissette with Δ is at the conjugate point.

If the disc be circular the glissette is

$$4(r^2+\alpha\beta)^2(r^2-\alpha^2)(r^2-\beta^2)+(\alpha+\beta)^4(x^2-y^2)^2=0$$

and this gives the four circles

$$r^2+\alpha\beta=\pm\frac{\alpha+\beta}{\sqrt{2}}(x\pm y)$$

as it ought to do. The curve Δ is

$$(r^2+\alpha\beta)^2=0\,,$$

and is real or imaginary according as the tracing point is outside or inside the director circle of the disc. When real it represents two coincident circles through four of the intersections of the four circles which form the glissette.

If the tracing point be on the director circle, we have $\beta=0$, $\alpha^2=4(a^2+b^2)$, and writing $p=d\cos\delta$, $q=d\sin\delta$, $x=r\cos\theta$, $y=r\sin\theta$, equation (6) takes the form

$$\begin{aligned}&r^2(4a^2+4b^2-r^2)\{a^4\sin^2(\delta-2\theta)+b^4\cos^2(\delta-2\theta)\}\\&=4(a^2+b^2)^2\{a^2\sin\delta\sin(\delta-2\theta)+b^2\cos\delta\cos(\delta-2\theta)\}^2.\end{aligned}$$

The form of this equation shows that the four lines represented by $\Delta=0$ in this case are imaginary.

On the Directions which are most altered by a Homogeneous Strain. By Prof. Tait. (With a Plate.)

(Read Dec. 7, 1897.)

The cosine of the angle through which a unit vector ρ is turned by the homogeneous strain ϕ is

$$u = -\frac{S\rho\phi\rho}{T\rho . T\phi\rho}.$$

This is to be a maximum, with the sole condition

$$T\rho = 1.$$

Differentiating, &c., as usual we have

$$x\rho = -2\phi\rho \, S\rho\phi'\phi\rho + \phi'\phi\rho \, S\rho\phi\rho .$$

Operate by S.ρ and we have

$$-x = -S\rho\phi\rho \, S\rho\phi'\phi\rho ; \text{ so that}$$

$$\rho = -2\frac{\phi\rho}{S\rho\phi\rho} + \frac{\phi'\phi\rho}{S\rho\phi'\phi\rho}.$$

Hence the required vector, its positions after the strain, and after a *subsequent* application of the conjugate strain, lie in one plane; and the tangent of the angle between ρ and its first distorted position is half of the tangent of the angle between it and its doubly distorted position.

When the strain is pure, the required values of ρ are easily found. Let the chief unit vectors of ϕ be α, β, γ, and its scalars g_1, g_2, g_3. Then the equation above gives at once three of the form

$$S\alpha\rho . \left(1 + \frac{2g_1}{S\rho\phi\rho} - \frac{g_1^2}{S\rho\phi^2\rho}\right) = 0 .$$

There are two kinds of solutions of these equations.

First. Let the first factor vanish in two of them, *e.g.*,

$$S\beta\rho = 0, \quad S\gamma\rho = 0, \quad \text{or} \quad \rho = \alpha .$$

Then the remaining equation is satisfied identically, because its second factor becomes

$$1 - \frac{2g_1}{g_1} + \frac{g_1^2}{g_1^2} ; \text{ whence } u^2 = 1 .$$

Thus, as we might have seen at once, the lines of zero alteration (minima) are the axes of the strain.

Second. Let the second factor vanish in two of the equations, *e.g.*

$$1 + \frac{2g_2}{S\rho\phi\rho} - \frac{g_2^2}{S\rho\phi^2\rho} = 0 , \quad 1 + \frac{2g_3}{S\rho\phi\rho} - \frac{g_3^2}{S\rho\phi^2\rho} = 0 .$$

These give at once

$$S\rho\phi\rho = -\frac{2g_2g_3}{g_2+g_3} , \quad S\rho\phi^2\rho = -g_2g_3 ;$$

so that

$$u^2 = \frac{4g_2g_3}{(g_2+g_3)^2} .$$

In this case it is evident that we have also

$$S\alpha\rho = 0 .$$

[In fact, neither the first factors, nor the second factors, in the three equations, can simultaneously vanish:—except in the special case when two of g_1, g_2, g_3 are equal.]

Of the three values of u^2 just found, the least, which depends upon the greatest and least of the three values of g, gives the single vector of maximum displacement:—the other two are minimaxes, corresponding to *cols* where a contour line intersects itself.

(Read February 21, 1898.)

The self-intersecting contour-lines, corresponding to 3, 2, 1 as the values of the gs, were exhibited on a globe; whose surface was thus divided into regions in each of which the amount of displacement lies between definite limits. The contour $u^2 = \frac{8}{9}$ encloses the regions in which the maximum $\left(u^2 = \frac{3}{4}\right)$ is contained:—and (where its separate areas are superposed) one of the minima. This minimum is surrounded by a detached part of $u^2=\frac{24}{25}$, while the rest surrounds the other two minima ($u^2=1$); and the double points of these contours are the minimaxes.

A general idea of their forms may be gathered from their orthogonal projections on the principal planes, as shown in figs. 1, 2, 3 of the Plate. These projections are curves of the 4th order:—but $u^2=\frac{8}{9}$ (dashed) splits into two equal ellipses on the xy plane, and hyperbolas on that of xz; while $u^2=\frac{24}{25}$ (dotted) gives ellipses on yz and hyperbolas on xz. Fig. 4 gives, on a fourfold scale, the region near the z pole of the projection on yz, of which the details cannot be shown on the smaller figure.

The curves were traced from their equations. One example must suffice. Thus

$u^2=\frac{8}{9}=\frac{(3x^2+2y^2+z^2)^2}{9x^2+4y^2+z^2}$ gives, eliminating z by the condition

$$x^2+y^2+z^2=1\,,$$

$$(2x^2+y^2+1)^2=\frac{8}{9}(8x^2+3y^2+1) \quad \text{or}$$

$$\left(2x^2+y^2-\frac{1}{3}\right)^2=\frac{16}{9}x^2\,,$$

i.e.

$$2\left(x\pm\frac{1}{3}\right)^2+y^2=\frac{5}{9}\,.$$

The forms of these curves depend only on the ratios $g_1:g_2:g_3$, so that I have appended fig. 5, in which we have 5 : 4 : 3, for comparison with fig. 3 where we have 3 : 2 : 1.

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PROF. TAIT ON HOMOGENEOUS STRAIN.

Fig. 1.

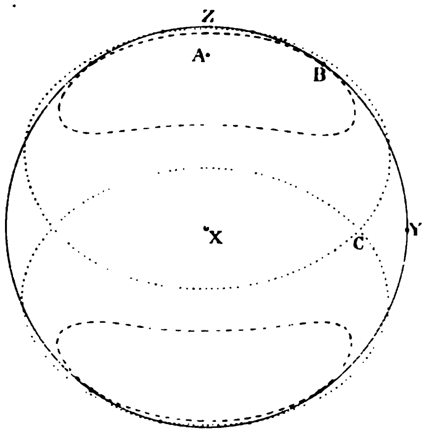
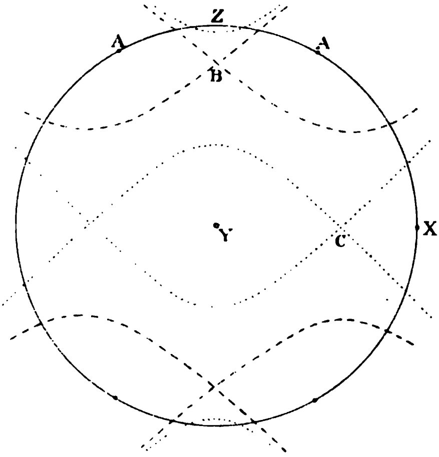


Fig. 2.



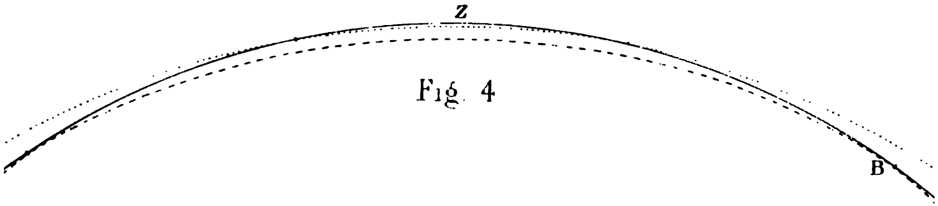


Fig. 4

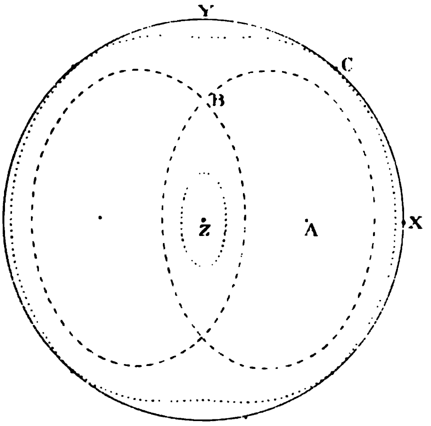


Fig. 3

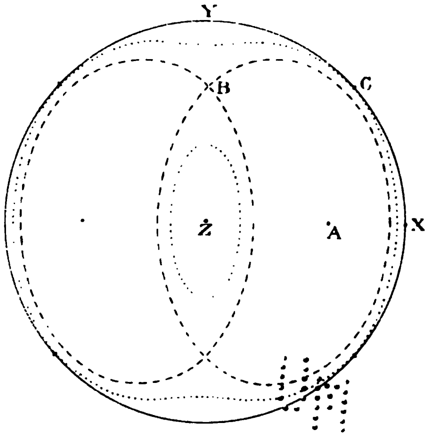


Fig. 5.

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On the Generalization of Josephus' Problem. By Prof. Tait.

(Read July 18, 1898.)

In the third Book of *The Wars of the Jews*, Chap. VIII. § 7, we are told that Josephus managed to save himself and a companion out of a total of 41 men, the majority of whom had resolved on self-extermination (to avoid falling into the hands of Vespasian) provided their leader died with them. The passage is very obscure, and in a sense self-contradictory, but it obviously suggests deliberate fraud of some kind on Josephus' part.

"And now," said he, "since it is resolved among you that you will die, come on, let us commit our mutual deaths to determination by lot. He whom the lot falls to first, let him be killed by him that hath the second lot, and thus fortune shall make its progress through us all; nor shall any of us perish by his own right hand, for it would be unfair if, when the rest are gone, somebody should repent and save himself." Whiston, *Works of Flavius Josephus*, IV. 39.

Bachet, in No. XXIII. of his *Problèmes plaisants et délectables*, makes a definite hypothesis as to the possible nature of the lot here spoken of; so that the problem, as we have it, is really his.

"Supposons qu'il ordonna que comptant de 3 en 3 on tuerait toujours le troisième, ... il faut que Josèphe se mît le trente-unième après celui par lequel on commençait à compter, au cas qu'il visât à demeurer en vie lui tout seul. Mais s'il voulut sauver un de ses compagnons, il le mit en la seizième place, et s'il en voulut sauver encore un autre, il le mit en la trente cinquième place."

Thus stated, the problem can be solved in a moment by the graphical process of striking out every third, in succession, of a set of 41 dots placed round a closed curve. When three only are left, they will be found to be the 35th, 16th, and 31st; and, if the process were continued, they would be exterminated in the order given. And any similar question, involving only moderate numbers, would probably be most easily solved in a similar fashion. But, suppose the number of companions of Josephus to have been of the order even of hundreds of thousands only, vastly more if of billions, this graphic method would involve immense risk of error, besides being toilsome in the extreme; and the *whole* process would have to be gone over again if we wished the solution for the case in which the total number of men is altered even by a single unit.

It is easy, however, to see that the following general statement gives the solution of all such problems:—

Let n men be arranged in a ring which closes up its ranks as individuals are picked out. Beginning anywhere, go continuously round, picking out each m^{th} man until r only are left. Let one of these be the man who originally occupied the p^{th} place. Then, if we had begun with $n+1$ men, one of the r left would have been the originally $(p+m)^{\text{th}}$, or (if $p+m>n+1$) the $(p+m-n-1)^{\text{th}}$.

In other words, provided there are always to be r left, their original positions are each shifted forwards along the closed ring by m places for each addition of a single man to the original group.

A third, but even more simple and suggestive, mode of statement may obviously be based on the illustrations which follow. In these the *original* number of each man is given in black type, the *order* in which he is struck off, if the process be carried out to the bitter end, in ordinary type.

By threes:—

$$\begin{array}{ccccccccc} 3 & 5 & 1 & 7 & 4 & 2 & 8 & 6 & 0 \\ \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \end{array}$$

$$\begin{array}{ccccccccc} 9 & 7 & 1 & 4 & 6 & 2 & 8 & 5 & 3 \\ \mathbf{1} & \mathbf{2} & \mathbf{3} & \mathbf{4} & \mathbf{5} & \mathbf{6} & \mathbf{7} & \mathbf{8} & \mathbf{9} \end{array}$$

Increase by unit every number in the first line (to which a 0 has been appended) and write it over the corresponding number in the third. We have the scheme

$$\begin{array}{ccccccccc} 4 & 6 & 2 & 8 & 5 & 3 & 9 & 7 & 1, \\ 9 & 7 & 1 & 4 & 6 & 2 & 8 & 5 & 3. \end{array}$$

Here the numbers, and their order, are the same, but those in the lower rank are three places in advance.

By fives:—

12	10	3	5	1	11	8	7	4	2	9	6	0
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	

5	3	10	7	1	13	11	4	6	2	12	9	8
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**

The numbers of the first line, increased by units, and those of the third, are

13	11	4	6	2	12	9	8	5	3	10	7	1
5	3	10	7	1	13	11	4	6	2	12	9	8,

again the same order, but now shifted forwards by five places.

It is easy to see that the two rows thus formed are *identical* when $m = n + 1$. Thus

By tens:—

1	4	2	8	6	3	7	9	5	0
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	

2	5	3	9	7	4	8	10	6	1
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**

and the statement above is obviously verified.

To show how rapidly the results of this process can be extended to higher numbers, I confine myself to the Josephus question, as regards himself alone, the last man. For the others, the mode of procedure is exactly the same.

Given that the final survivor in 41, told off by threes, is the 31st, we have

n	last man.
41	31

The rule just given shows that succeeding numbers in these columns are formed as follows:—taking only those which commence, as it were, a new cycle:—

$$41 + x \qquad 31 + 3x - (41 + x) = 2x - 10.$$

The value of x which makes the right hand side one or other of 1, and 2, is therefore to be chosen, so we must put $x = 6$, and the result is

47	2

Successive applications of this process give, in order

70	1	13,655	2
105	1	20,482	1
158	2	30,723	1
237	2	46,085	2
355	1	69,127	1
533	2	103,691	2
799	1	155,536	1
1,199	2	233,304	1
1,798	1	349,956	1
2,697	1	524,934	1
4,046	2	787,401	1
6,069	2	1,181,102	2
9,103	1	1,771,653	2

provided the (merely arithmetical) work is correct. And, of course, we can at once interpolate for any intermediate value of n.

Thus, in 799 men, or in 30,723, the first is safe:—in 1000 the 604th; in 100,000 the 92,620th, and in 1,000,000 the 637,798th.

The earlier steps of this process, which lead at once to Bachet's number for 41 (assumed above), are

1	1	9	1
2	2	14	2
3	2	21	2
4	1	31	1
6	1		

so that the method practically deals with millions, when we reach them, more easily than it did with tens.

Unfortunately the cycles become shorter as the radix, and with it the choice of remainders increases; so that a further improvement of process must, if possible, be introduced when every hundredth man (say) is to be knocked out.

From the data above given, it appears that up to two millions the number of cases in which the first man is safe is 19, while that in which the second is safe is only 16. (The case of one man, only, is excluded.) As these cases should, in the long run, be equally probable, I extended the calculation to 13,059,835,455,001, 1, with the result of adding 20 and 19 to these numbers respectively. But the next 15 steps appear to give only 2 cases in favour of the first man!

On the Nature of "Palladium Hydrogen." By John Shields, D.Sc., Ph.D., Davy-Faraday Laboratory, Royal Institution, London.

(Read March 7, 1898.)

Ever since the remarkable discovery by Graham in 1866 that palladium possesses the power of *occluding* hydrogen gas in large quantity, various views have from time to time been put forward to explain the true nature of the phenomenon. Whilst some observers regard hydrogenised palladium as an alloy or solid solution, others again consider it to be, or at least to contain, a definite chemical compound or hydride. Those who hold the latter opinion, however, are not agreed as to which compound or hydride is formed, as is proved by the fact that different formulæ have been ascribed to it. Graham himself recognised the possibility that a definite chemical compound might be formed, for he says (*Researches*, 287) that in fully-charged palladium there exists one equivalent of palladium to 0·772 equivalent of hydrogen, or an approximation to single equivalents which would be represented by the formula PdH. His opinion was, nevertheless, opposed to the idea of such a definite chemical combination, one of his chief objections being that no visible change is occasioned to metallic palladium by its association with hydrogen. He regarded the product simply as an alloy of the volatile metal *hydrogenium*, in which the volatility of the one element is restrained by its union with the other, and which owes its metallic aspect equally to both constituents.

Considerations of a purely chemical character have up to the present time proved insufficient to decide which of these views is correct. The aid of physical methods has also been invoked, but the results achieved are not quite so satisfactory as might be desired. On certain points, however, evidence of a fairly conclusive character has been produced, and before describing some electro-chemical attempts which were made to solve this interesting problem, a short summary of similar attempts, and of the views of other observers, may not be out of place.

What is really required is some criterion which is conclusive either for or against the idea of chemical combination or solid solution. Of course, we may possibly have to deal with both of these phenomena; but a third alternative, namely, that the occluded hydrogen is simply condensed or liquefied in the capillary pores, seems to be altogether inadmissible. In support of this conclusion, we have the fact (Mond, Ramsay, and Shields, *Phil. Trans.*, cxci. 105) that palladium in all its different states of aggregation, which presumably differ with respect to the degree of porosity, always occludes, under proper conditions, approximately the same quantity of hydrogen. Again, if the occlusion of hydrogen was merely the liquefaction of the gas in the capillary pores, we should expect the heat of occlusion to be identical with the heat of condensation of hydrogen. Although we do not know the magnitude of the heat of condensation of hydrogen, we can arrive at the required result in an indirect manner.

Assuming that the occlusion of hydrogen by finely-divided palladium is precisely the same sort of phenomenon as the occlusion of hydrogen by finely-divided platinum, then, if in both instances we were dealing simply with the liquefaction of hydrogen, we would expect the heat of occlusion of one gram of hydrogen in palladium to be identical with the corresponding heat of occlusion of one gram of hydrogen in platinum. The numbers actually found, however, under comparable conditions (*Phil. Trans.*, *loc. cit.*) were $+46{\cdot}4K$ (4640 g – cal) and $+68.8K$ (6880 g – cal) respectively. We may therefore reject the hypothesis that the phenomenon of occlusion represents the condensation of the gas in the capillary pores of the absorbing substance.

A knowledge of the dissociation pressures of a substance provides us with a valuable criterion for determining whether we are dealing with a compound or simply with a solid solution. According to the Phase Rule it can be shown that when a solid substance dissolves a gas the vapour pressure varies continuously, at one and the same temperature, with the increase of concentration of the dissolved gas. The pressure-concentration diagram will in general resemble that shown in fig. 1. If we operate at a constant temperature, and start with the solid substance containing no gas dissolved in it, then the pressure will also be zero, provided that the solid has no vapour pressure of its own at the given temperature. If the gas be now gradually dissolved in the solid, the pressure will rise continuously, as shown in the diagram, or *vice versa*; and at any other constant temperature the pressure will also vary with variations of concentration, only at a different rate.

If, on the other hand, a chemical compound is formed on bringing together the solid substance and the gas, a totally different diagram will be obtained, as shown in fig. 2. In this case the first introduction of gas produces a small quantity of the compound, which has a perfectly definite vapour pressure of its own at the given constant temperature. The pressure therefore will immediately rise to the vapour pressure (dissociation pressure) of the compound, and on admitting more gas more of the compound will be formed, but the pressure will remain *unaltered* until the whole of the solid substance has been converted into the compound. At this point, indicated by the dotted line, the direction of the isothermal, hitherto parallel with the concentration axis, abruptly changes, and if more gas is introduced into the system the pressure

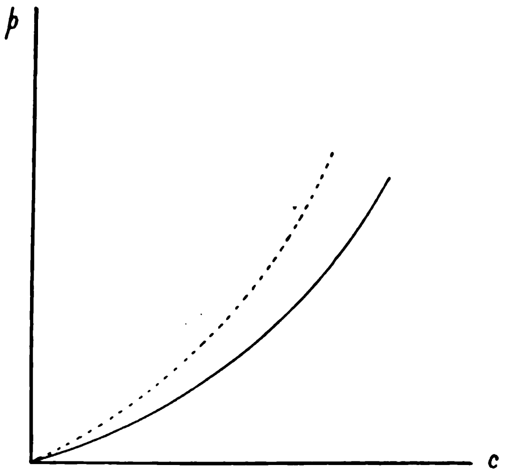

FIG. 1.

will simply obey Boyle's law, provided that no further solution of the gas takes place and that no higher compound is formed. At any other constant temperature the horizontal part of the curve will be displaced upwards or downwards, parallel to itself, and the abrupt change of curvature will again take place at the same concentration, C′, of the gas in the solid, that is, when the whole of the solid has been converted into the compound. On removing gas from the system the same diagram will be traced in the reverse order.

We need not here take account of complications which have been carefully considered by Hoitsema (*Zeits. f. physikal. Chem.*, xvii. 1). It will be sufficient to recognise the fact that chemical compounds are characterised by a constant dissociation pressure, that the pressure changes abruptly when, and only when, the maximum quantity of the compound has been formed, and that alteration of temperature does not affect the point at which the abrupt change of pressure takes place. If solution of the gas takes place before combination begins, the left-hand end of the horizontal part of the diagram may undergo modification; but this does not apply to the right-hand end, where the maximum quantity of the compound is formed.

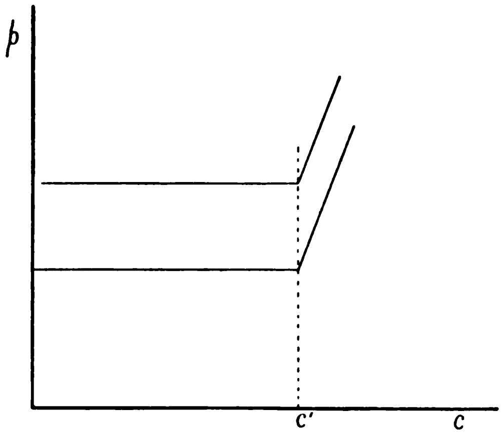

Fig. 2.

These rules have been amply verified experimentally in the case of salts containing water of crystallisation (Frowein, *Zeits. f. physikal. Chem.*, i. 5; Andreæ, *Zeits. f. physikal. Chem.*, vii.

241, etc.), in the absorption of ammonia by silver chloride (Horstmann, *Ber. deutsch. Chem. Gesell.*, ix. 749; Isambert, *Compt. rend.*, vols. lxvi. and lxx.), etc.

This criterion was first applied to hydrogenised palladium in 1874 by Troost and Hautefeuille (*Ann. chim. phys.* (5), ii. 279), who discovered that the pressure-concentration curve was horizontal or nearly horizontal—*i.e.*, there existed a constant or nearly constant dissociation pressure over a certain range, from which they concluded that the definite compound Pd_2H was formed, and that the excess of hydrogen over and above that which was required for the formation of this compound was simply occluded or dissolved in the ordinary way.

This conclusion, however, has been called in question by Hoitsema (*Zeits. f. physikal. Chem.*, xvii. 1), who, from more extended and critical investigations by himself and Rooseboom, concludes that the measurements of the dissociation pressures furnish no evidence in favour of the existence of Pd_2H, or of any definite chemical compound. The only interpretation which he can give of the results obtained, and this is given under due reserve, is that two immiscible solid solutions are formed.

As the results obtained by the Dutch investigators are extremely interesting, some of them have been reproduced graphically in fig. 3, which shows how the pressure of palladium hydrogen varies at different temperatures with the concentration of the hydrogen. In the diagram the ordinates represent the pressure in centimetres of mercury, whilst the abscissæ indicate the number of atoms of hydrogen associated with one atom of palladium.

From a careful study of the diagram, Hoitsema insists on three points,—first, the middle nearly horizontal part of the curves is not absolutely horizontal as it ought to be, and as it is in the case of other diagrams (Andreæ, *loc. cit.*); second, the change of curvature at the right-hand end of the nearly horizontal part is not abrupt but very gradual; third, the concentration of the hydrogen at the point corresponding to this change of curvature does not remain constant but varies for each isothermal. None of the criteria, therefore, which are characteristic of a chemical compound are fulfilled in the above diagram. With reference to the supposed formation of Pd_2H it was purely accidental, that in the case of the isothermal studied in greatest detail by Troost and Hautefeuille, viz., the isothermal of 100° C., the change of curvature took place at the point where half an atom of hydrogen was associated with the palladium. At other temperatures the change takes place at the following concentrations:—

Temp.	Concentration (atom of H_2 to one of Pd).
10°, ...	0·60
50°, ...	0·58
100°, ...	0·54
120°, ...	0·50
150°, ...	0·44
180°, ...	0·37

If Pd_2H were actually formed, the change of curvature ought to take place always at the concentration 0·5 of an atom of H_2, and under no circumstances at a lower concentration. It would be extremely interesting to follow the course of the isothermal at a very low temperature, say at the temperature of boiling liquid air.

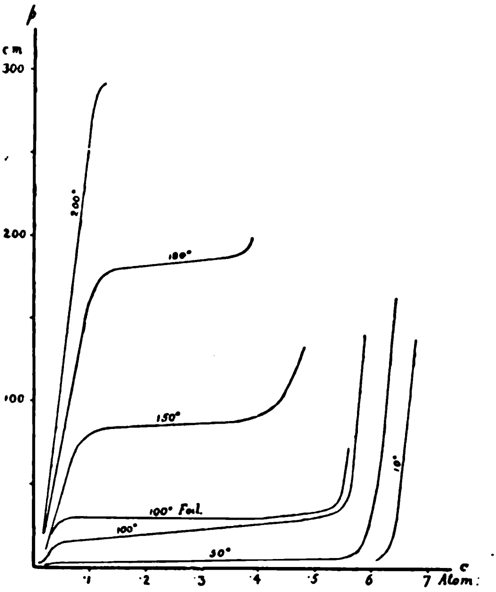

FIG. 3.

The point at which the change of curvature takes place, however, varies considerably with the nature of the palladium employed; and as it is difficult to know how far the above conclusions are influenced by this fact, and also by the fact that it is no easy matter to determine whether equilibrium has or has not actually set in, it is very desirable that these conclusions should be checked by some independent method.

It has been deemed necessary to go into some detail with regard to the history of this subject, in order to be able to explain precisely the object of the following experiments.

Before quitting this part of the subject, however, it may be mentioned that there is other independent evidence against the supposed formation of Pd_2H. Favre (*Compt. rend.*, lxxvii. 649, and lxxviii. 1257) observed that the heat of occlusion of hydrogen by palladium remains constant throughout the whole range of absorption, and this observation has been confirmed by Mond, Ramsay, and Shields (*Phil. Trans.*, cxci. 105). If a chemical compound, say Pd_2H, were first formed, we should expect to get a certain definite evolution of heat per gram of hydrogen combined for the hydrogen first admitted, and then, after sufficient hydrogen has been added to form the compound Pd_2H (about 630 volumes), we should expect to find a different value for the heat evolved per gram of hydrogen dissolved, or occluded, or absorbed. The fact that no differentiation can be observed militates against the view that Pd_2H is formed, although the possible formation of a compound containing more hydrogen—*e.g.*, Pd_3H_2 or PdH—is not excluded. Again, the same conclusion may be drawn from the observation by Dewar (*Trans. Roy. Soc. Edin.*, xxvii.), and by C. G. Knott (*Proc. Roy. Soc. Edin.*, xii. 181), that the increase in electrical resistance of a palladium wire is directly proportional to the increase in the amount of occluded hydrogen. Measurements of the specific gravity and specific heat of hydrogen occluded by palladium (Dewar, *Trans. Roy. Soc. Edin.*, xxvii. 167), likewise do not lend any support to the view that the definite compound Pd_2H is formed.

It has been suggested by Dewar (*Phil. Mag.* (4), xlvii. 334) that the composition of fully-charged palladium corresponds to the formula Pd_3H_2; and in connection with this view we have the very significant fact (Mond, Ramsay, and Shields, *Phil. Trans.*, cxci. 105), that fully charged palladium, no matter whether it exists as black, sponge, foil, wire or block metal, or whether it is charged by direct exposure to hydrogen gas (the proper conditions being observed), or charged electrolytically, has approximately the same amount of hydrogen occluded in each case.

This is well shown in the following table:—

		Atomic Ratio Pd:H.	Volumes of Hydrogen occluded.
	Palladium Black, . . .	1·42	873
	,, ,, I., . . .	1·41	881
	,, ,, II, . . .	1·40	889
	,, ,, III., . . .	1·40	889
	,, Sponge, . . .	1·46	852
	,, Foil, . . .	1·47	846
Charged electrolytically.	,, Wire, I. (Graham), .	1·37	912
	,, ,, II. ,, .	1·46	846
	,, ,, III. ,, .	1·44	859
	,, Metal (Dewar), .	1·47	847

If the compound Pd_3H_2 were formed, the theoretical value of the atomic ratio palladium: hydrogen would be 1·5. The ratio actually found varies between 1·37 and 1·47, which means that rather more than the theoretical quantity of hydrogen has been taken up. The excess, which is not large, as was the case with Troost and Hautefeuille's hypothetical compound Pd_2H, might be ascribed to a true occlusion, or perhaps a surface action after the formation of the compound Pd_3H_2.

A still greater excess, amounting to about ten per cent., may be forced into palladium, which is charged electrolytically, but this is immediately given off again on breaking the charging current. Increase of pressure up to four or five atmospheres does not influence the quantity of hydrogen occluded when the charged palladium is simply exposed to hydrogen gas.

Up to this point, therefore, it still remained uncertain whether on the occlusion of hydrogen by palladium a solid solution or alloy was formed, or whether a definite chemical compound was produced. The chief argument in favour of the formation of the hypothetical compound Pd_3H_2 is simply the approximation of the above atomic ratios to the theoretical value 1·5.

From the following considerations it appeared probable that the electro-chemical behaviour of hydrogenised palladium would throw some light on the matter.

It is well known that if a concentration cell be constructed containing silver, concentrated silver nitrate solution, dilute silver nitrate solution, silver, the current will flow in the cell from the dilute to the concentrated solution as long as a difference of concentration remains. In the light of the modern theory of the osmotic production of the current, we may regard the solution of the silver electrode in the dilute solution, and the deposition of silver on the other electrode as being due to the osmotic pressure of the silver ions (which acts against the 'solution pressure' of the silver electrodes), being greater in the concentrated solution than in the dilute solution.

The electromotive force π, in volts, of such a cell is calculable from the expression,

$$\pi = \frac{RT}{n_e \epsilon_0} \text{ log. nat. } \frac{P}{p_1} - \frac{RT}{n_e \epsilon_0} \text{ log. nat. } \frac{P}{p}$$

where R represents the gas-constant expressed in electrical units, T the absolute temperature, P the solution pressure of the silver, p and p_1 the osmotic pressures (concentrations) of the dilute and concentrated solutions of silver nitrate, ϵ_0 the electro-chemical unit of quantity of electricity—viz., 96540 coulombs, and n_e the valency of the ions. Since the solution pressures of both electrodes are equal, the above expression becomes

$$\pi = \frac{RT}{n_e \epsilon_0} \text{ log. nat. } \frac{p}{p_1}.$$

The electromotive force of the cell therefore depends only on the ratio of the osmotic pressures (concentrations) of the silver ions.

In the same way we may calculate the electromotive force of a cell, composed of weak amalgam, a solution containing the ions of the metal forming the amalgam (salt of the metal), strong amalgam. The first to call attention to concentration cells composed of amalgams was von Türin (*Zeits. f. physikal. Chem.*, v. 340), in a paper entitled "Gedanken über eine vielleicht vorhandene Möglichkeit, Molekulargewichte der Metalle nach zwei neuen Methoden zu bestimmen." The same subject was also attacked independently by G. Meyer (*Zeits. f. physikal. Chem.*, vii. 477), who constructed concentration cells of zinc, cadmium, lead, tin, copper, and sodium amalgams. Knowing the concentrations of the solutions of the metals in mercury (p and p_1) he was able (on the assumption, which has been confirmed by an independent investigation, that the metals dissolve in mercury in the atomic state) to calculate the electromotive force of the cells with the help of the above formula. Not only was the agreement between the found and calculated values of the electromotive force of the cells good, but the variation with temperature corresponded with the theory.

If the hydrogen occluded by palladium is simply dissolved or amalgamated with the palladium it ought to be possible to construct a hydrogen concentration cell, composed of palladium containing a small quantity of hydrogen, an electrolyte containing hydrogen ions (*i.e.*, an acid), palladium containing a large quantity of hydrogen, and if the calculated electromotive force of such a cell agreed with the experimental value, then we would have pretty conclusive evidence that the phenomenon of occlusion was simply one of solid solution.

Up to the present all attempts to work concentration cells with different alloys, other than liquid alloys or amalgams, have proved a total failure. The reason of this is not far to seek. It may be due to the non-homogeneity of the alloy from the very beginning; or, assuming that we could start with a perfectly homogeneous alloy, the moment it is dipped in the electrolyte there will be either a deposition on the surface of the electrode of the constituent metal which forms the electrolyte, or one of the metals composing the alloy will be dissolved out of the surface of the electrode, leaving the other exposed, so that we would practically get the electromotive force of the exposed metal. In the case of solid alloys diffusion is so slow as to be nearly negligible, whereas, in the case of liquid alloys or amalgams, it can take place with greater ease. It was hoped that in hydrogenised palladium diffusion throughout the mass would take place with as great ease as in liquid amalgams, and that the above source of difficulty would not operate adversely. It has been found, however, that diffusion, and consequently equilibrium, does not always take place so readily as was anticipated, although the difficulties introduced in this way are not insuperable.

Leaving out of account a few miscellaneous determinations of the electromotive force of charged palladium, the only literature bearing at all directly or indirectly on the subject is by C. G. Knott (*Proc. Roy. Soc. Edin.*, xii. 181) and by M. Thoma (*Centralblatt f. Elektrotechnik*, xi. 131), but the discussion of the results obtained by these observers is reserved for a later part of this paper.

Several different forms of cell were constructed, the electrodes consisting of palladium wires attached to gold wires, palladium foil both new and foil which had been thoroughly ignited in the blowpipe flame, palladium sponge shrunk on to platinum wires, palladium sponge and foil electro-deposited on gold foil, and palladium black existing as a layer on the surface of mercury electrodes. Of these forms of electrode one or two call for special mention. The palladium sponge shrunk on platinum (or gold wires) was prepared in the following way. The end of a hard glass tube about 8 mm. in diameter and 80 mm. long, was closed by means of a plaster of Paris plug. A wire was then pushed through the plug until it projected 10 or 12 mm. into the tube, and held in the axis of the tube until the plaster set. After drying, a quantity of palladium black was introduced and rammed tightly home. The open end was then attached to a Kipp's apparatus supplying pure hydrogen, and a current of the gas was passed until all the oxygen had been removed and the palladium black charged with hydrogen. On now igniting the whole tube the hydrogen was driven off, and the palladium black, which decreases many times in bulk on passing into sponge, shrunk itself on to the wire and formed a rod of sponge.

Although the electrodes, consisting of a layer of palladium black on the surface of mercury, which was connected by a platinum wire with the rest of the system, proved quite useless, they gave rise to other experiments which, although equally useless for the purpose, are worthy of being recorded. Such an electrode was charged electrolytically with hydrogen. A quantity of hydrogen which corresponded with what was required to remove all the oxygen from the palladium black was readily taken up. A hissing noise then occurred, and the palladium disappeared in the mercury. Apparently the function of the hydrogen was simply to clean the layer of oxide off the surface of the palladium black, which was then able to dissolve easily in the mercury. An attempt was therefore made to make a liquid palladium-hydrogen amalgam, but this was unsuccessful. Although electrolytic hydrogen was liberated for a considerable time on the surface of the amalgam, none was absorbed. In another experiment a quantity of palladium black was fully charged with hydrogen by direct exposure to the gas, and this, always kept in an atmosphere of hydrogen, was gradually tipped over into mercury contained in a bent branch of the tube. In order to prevent overheating, this mercury was kept cold in a bath of water, but the bulk of the occluded hydrogen was liberated immediately and the rest on standing over night. It thus seems impossible to prepare a palladium-hydrogen amalgam. Some of the solid palladium amalgam obtained by squeezing the excess of mercury through wash leather had the approximate composition $PdHg_5$.

A pair of the above electrodes of known weight which had been subjected to as nearly as possible the same treatment, was then placed in dilute sulphuric acid (*n*, and in some cases $0{\cdot}1n - H_2SO_4$ was used). The cell was then connected by means of a capillary syphon tube with a second cell, containing a platinum electrode. The function of the divided cell was simply to separate the electrode at which oxygen was evolved from the cell proper; and care was taken to prevent, as far as possible, the access of air to the hydrogenised palladium electrodes, or to the electrolyte containing them.

In series with the charging current was placed a hydrogen voltameter, so that the amount of hydrogen introduced into the palladium electrodes could be read off directly. Of course the charging had to take place so slowly that the whole of the hydrogen was absorbed by the palladium.

In order to determine the electromotive force of the cell, or the potential of either of the electrodes against a normal electrode—viz., zinc in zinc sulphate solution—a modification of Poggendorff's compensation method was adopted, with a capillary electrometer serving as a zero instrument. The capillary electrometer used was Ostwald's form of Lippmann's instrument, and was sensitive to two or three ten-thousandths of a volt. The working Leclanché cells were calibrated several times daily with a normal Carhart Clark cell.

In general, the potential difference between the two electrodes, immediately after the cell had been set up, was not zero. Before beginning an experiment, the electromotive force was brought as nearly as possible to zero either by re-igniting the electrodes, or by short-circuiting the cell in itself for a few days. In this way the electromotive force at the start could generally be reduced to a few thousandths of a volt.

A small measured quantity of hydrogen was then introduced into one of the electrodes. With the different forms of electrode this produced an electromotive force varying from 0·75 – 0·83 volt. On now adding approximately the same quantity of hydrogen (the weight of the electrodes being approximately equal) to the other electrode, the electromotive force of the cell diminished to nearly zero. Successive additional quantities of hydrogen were now introduced into this electrode. In most cases this produced only slight alterations in the electromotive force of the cell. In general, the alteration was so slight that it could safely be considered as experimental error or want of equilibrium. Where a marked difference occurred, this generally disappeared after the cell was allowed to stand for some time (in certain cases a few days were necessary). It would be very tedious to catalogue the scores of measurements made with the different cells and different electrodes. The following set has therefore been selected, and may be taken as typical of those obtained.

The electrodes in this case were cut from palladium foil which had been used in former experiments (*Phil. Trans.*, cxci. 150), and which was capable of occluding 846 volumes of hydrogen when exposed to the gas under the proper conditions (*loc. cit.*). The dimensions of the electrodes were about 30 mm. × 19 mm. × 0·025 mm., and they were known as β and ρ respectively. β weighed 0·2234 g = 0·0186 c.c., whilst ρ weighed 0·2453 g = 0·0204 c.c.

After ignition, the electrodes were placed in $n - H_2SO_4$, and short circuited over night. Next day, the electromotive force of cell was 0·0041 V. The changes which took place when the electrodes were charged with hydrogen are given below in tabular form,—the electromotive force, except when otherwise stated, being measured a minute or two after the charging current was interrupted.

Hydrogen contained in		E. M. F.	Potential against *Zn* in *Zn* SO_4.	
β	ρ	$\beta\rho$	β	ρ.
0·0 c.c.	0·0 c.c.	0·0041 Vβ –	1·5904 V.	1·5954 V.
0·72 ,,	0·0 ,,	0·8287 ,,	0·7686 ,,	1·5963 ,,
,, ,,	1·08 ,,	0·0029 ρ –	0·7677 ,,	0·7665 ,,
5·18 ,,	,, ,,	0·0053 β –	0·7684 ,,	0·7743 ,,
14·18 ,,	,, ,,	0·0469 ,,	...	...
After 6 minutes.		0·0352 ,,	...	...
,, 10 ,,		0·0293 ,,	...	...
,, 25 ,,		0·0205 ,,	...	...
,, 40 ,,		0·0176 ,,	...	...
,, 100 ,,		0·0147 ,,	...	...
,, 18 hours.		0·0071 ,,	0·7727 ,,	0·7798 ,,

When β was supersaturated with hydrogen with a current strength of 0·025 amp., the electromotive force of the cell $\beta\rho$ was 0·1001 V., but this gradually diminished during the course of time, and approximated to zero. With a higher current strength, viz., 0·3 amp., the electromotive force after some seconds was 0·1278 V., and this also gradually fell off. Finally, when both electrodes were supersaturated with hydrogen, and the excess of the gas allowed to come off, the electromotive force was 0·0029 V.

In considering the results contained in the above table, we ought to pay special attention to the cell containing 14·18 c.c. hydrogen in the electrode β and 1·08 c.c. in ρ, since in this case the gradual diminution of the electromotive force has been measured. In other cells the same effect was noted at other concentrations. Intermediate measurements were dispensed with in this instance, however, in order to study particularly a cell containing a considerable difference in concentration of the hydrogen in the two electrodes.

It is to be noted, then, that the electromotive force of the cell gradually diminished on standing and approximated to zero, or at least the initial electromotive force. The question therefore arises, whether we ought to consider the electromotive force of the cell immediately after the charging current was interrupted, the final value after standing for a sufficiently long time, or any intermediate value, as the true electromotive force. The choice of any intermediate value would be an arbitrary affair, and there is clearly nothing to recommend it. The first value, too, is very liable to be affected by an absence of equilibrium with respect to the equal distribution of the hydrogen throughout the electrode. The rate at which hydrogen would diffuse from one surface of the electrode to the interior, or to the other surface, would depend, other things being equal, on the concentration gradient of the hydrogen in the plate. For a plate approaching saturation, therefore, the time required before equilibrium sets in might be considerable. The final value of the electromotive force of the cell is thus more likely to be the correct electromotive force, and to be free from such incidental sources of error, to which we will return again presently, provided we have proof that the relative concentrations of hydrogen in the two electrodes have not undergone any alteration on standing. It is conceivable that equalisation of the hydrogen concentrations might take place, owing to bad insulation of the electrodes. In several cases the final concentrations were estimated electrolytically by determining the amount of electrolytic oxygen which was just necessary to remove the hydrogen, and it was found that they had practically remained unaltered. The hydrogen contained in ρ after standing eighteen hours was determined in this way. It was found that about 1·3 c.c. hydrogen was evolved in the voltameter before oxygen began to appear on the electrode ρ. The amount of hydrogen existing in ρ must therefore have been equal to or probably, since slight oxidation takes place, a little less than 1·3 c.c. We have therefore proof that the concentrations of hydrogen in the electrode β and ρ were practically unaltered on standing eighteen hours.

On the assumption that the hydrogen occluded by palladium is dissolved in the metal, and that such a cell is to be treated as a concentration cell, we can calculate its electromotive force from the above formula. Taking into account the slightly different weights of the electrodes β and ρ, the electromotive force of the cell having the ratio of concentration of the hydrogen in the two electrodes, 14·40 should be 0·0333 V. at 17° C. The actual electromotive force of the cell after standing eighteen hours was 0·0071 V. This is a maximum limit, however, and in all probability this electromotive force would have decreased considerably if the cell had been allowed to stand for a still longer period.

The conclusion to be drawn, then, is that the electromotive force of such a cell is either zero or else closely approximates to zero. In other words, the potential of a palladium electrode charged with hydrogen and immersed in dilute sulphuric acid is independent of the quantity of hydrogen occluded, at any rate, up to near the point of saturation of the palladium.

This result is in good agreement with that obtained by Thoma (*Centralblatt für Elektrotechnik*, xi. 131). The potential of a palladium wire against zinc in a saturated solution of zinc sulphate, was measured at intervals during charging, in some cases with the aid of a quadrant electrometer, in others with a galvanometer, and it was found that its potential remained constant at about 0·68 Daniell (0·75 V.) until minute bubbles of free hydrogen appeared on its surface. The potential then gradually diminished on further charging to 0·42 D, but on breaking the charging current the normal value was recovered. On removing the hydrogen by electrolytic oxygen the normal value was unaffected until all the hydrogen had been removed.

Thoma concludes by saying that, as long as all the hydrogen is absorbed by the palladium, its position in the electromotive series is independent of the amount of absorbed hydrogen, and that until the point of supersaturation is reached we are not dealing with a phenomenon of polarisation at all. Supersaturated palladium, on the other hand, gradually approaches the position of zinc the richer it is in hydrogen, but nevertheless always remains negatively electrified.

Knott, in a paper "On the Electrical Resistance of Hydrogenised Palladium" (*Proc. Roy. Soc. Edin.*, xii. 181), incidentally measured the electromotive force of a charged palladium wire against platinum in dilute sulphuric acid, and found that a slight charge of hydrogen makes the palladium strongly positive to the platinum, but this characteristic gradually diminishes till, finally, when the palladium is fully charged with hydrogen, the electromotive force is less than half its original amount.

The actual observed values are contained in the following table, the amount of hydrogen introduced in each case being estimated by the increase in weight of the palladium wire.

Mass.	E.M.F.
1·0000	−0·05
1·0014	+0·81
1·0017	+0·79
1·0023	+0·73
1·0045	+0·74
1·0068	+0·33

There can be little doubt that the first gradual diminution observed by Knott is due to partial polarisation of the palladium, and that, if sufficient time had been allowed for diffusion to take place, a constant value would have been obtained up to the point of supersaturation.

The gradual fall in the electromotive force of the cell we have examined (see table) is undoubtedly due to the same cause, and therefore we are justified in accepting the last number of the series as a *maximum* value, the true electromotive force of the cell, if it were possible to obtain it, probably approaching more closely to zero.

From the series of experiments which has just been described, and from others, the details of which have not been given, it appears, then, that the electromotive force of a cell, composed of palladium containing a small quantity of hydrogen, dilute sulphuric acid (*i.e.*, an electrolyte containing H ions), palladium containing a large quantity of hydrogen, is either zero or very nearly so. In any case, the electromotive force is out of all proportion to what we would expect if the cell behaved like a true concentration cell, and therefore it remains to be seen what interpretation can be put on the result. If it be granted that a concentration cell is not formed, this would seem to preclude the idea that the hydrogen is simply dissolved in the palladium, forming a solid solution. The formation of a definite chemical compound, on the other hand, would agree better with the facts, for the electromotive force would then be practically independent of the quantity of compound formed. The electromotive force of a metal dipped in an electrolyte, for example, is practically independent of the quantity of metal employed.

Whilst these electro-chemical experiments would seem to indicate the formation of a definite chemical compound, it still remains uncertain whether the compound so formed is Pd_8H_2, or whether it contains relatively more hydrogen than is represented by this formula. Experiments at low temperatures, such as those already suggested, might throw some light on this question.

Quite recently a new criterion for determining whether definite chemical compounds are contained in alloys, and if so what their composition is, has been brought forward by Liebenow (*Zeits. f. Elektrochemie*, iv. 201).

This method, which is based on measurements of the electrical resistance of the alloys or on their temperature coefficients, is unlikely to be of any assistance in the case of palladium hydrogen, for, in order to obtain satisfactory results, it would be necessary to prepare a large number of alloys, ranging from pure palladium, on the one hand, to pure hydrogen on the other; whereas, so far as we know at present, it is impossible to introduce as much as one per cent. of hydrogen into palladium.

In conclusion, I wish to acknowledge my indebtedness to Dr Ludwig Mond for valuable advice and assistance in conducting these experiments, and to the managers of the Royal Institution for placing at my disposal the resources of the Davy-Faraday Laboratory.

Some Contributions to the Spectroscopy of Hæmoglobin and its Derivatives. By David Fraser Harris, M.D., C.M., B.Sc. (Lond.), "Muirhead" Demonstrator of Physiology in the University of Glasgow.

(Read February 7, 1898.)

I.—*Details of the Preparation of HbO_2 and its Derivatives, for Demonstration to Large Classes.*

1. *Apparatus.*—The following are needed:—Electric or lime light; standard hæmatinometer (*i.e.*, of 1 cm. between the parallel sides) entirely of glass (for many pigments white glass flat-sided stoppered bottles do perfectly well); large direct-vision spectroscope (Messrs Hilger), or a Tollen's carbon-disulphide glass prism (base 5 inches, refracting angle 45°); a well-stretched dead-white screen to receive the spectrum 4 feet long at least, about 12 feet from the lantern.

Following Preyer (1), one can with such apparatus first of all demonstrate the different spectroscopic appearance according as blood is or is not diluted. Placing blood newly received from the slaughter-house (defibrinated blood) in the hæmatinometer, we show that it is opaque in a thick layer, no light passes to the prism at all, there is no spectrum, we can say that the percentage of HbO_2 is, at least, greater than 7·3. Dilute with pure cold water until a faint gleam of red light appears at the extreme left, the concentration of HbO_2 is now not less than 1 per cent.; continue the dilution until the first glimmer of extra-linear green appears, the percentage is now about ·9 per cent.; dilute until the first glimmer of intra-linear green appears, when the percentage is now ·7; add to this its own volume of water, making the percentage fall to ·35. From this point further dilution caused progressive faintness of the two bands, until the vanishing point of the fainter β band is just reached when the hæmoglobin is ·01 per cent.; continue the attenuation until the α band is alone visible, at which point the percentage can be stated to be certainly less than ·01 per cent.

Reduction of the two bands of HbO_2 to the one of HbO is a striking demonstration upon a large scale, but it is not easy to hit upon the precise strength of the HbO_2 solution that is most suitable.

It must be remembered that (1) before reduction is attempted the two bands of the oxidised pigment must be clearly seen by all to be two, *i.e.*, the intra-linear green must be well marked, and (2) the resulting single band must not be too faint when it does appear; for it must be borne in mind that it is always fainter than either of the bands it replaces.

To satisfy both these conditions, a percentage of ·6 will be found most serviceable. Add to a nearly full hæmatinometer of blood at this dilution a quarter of a test-tubeful of warm Am_2S_r in a few seconds the two bands will fade away into one.

II.—*Notes on Quantities of Materials used in preparing some of the HbO_2-derivatives for Large Scale Demonstration.*

1. *Neutral Met-Hæmoglobin.*—Of defibrinated sheep's blood diluted 1·5 times its volume with water, 140 c.c. are taken and warmed to 35° C.; to this add 15 c.c. of a 10 per cent. sol. of potassium ferricyanide—the most reliable substance for preparing this pigment in quantity; stir, filter. The band in red and that to the right of D are well seen; the others are not to be expected in a wall-spectrum.

2. *Alkaline Met-Hæmoglobin.*—To 140 c.c. of the pigment prepared as above, add 5 c.c. of NH_4HO; stir, filter; three of the four bands can be seen. It is misleading to describe either of these met-hæmoglobins as "chocolate coloured"; in solution, as above prepared, the neutral one is of a dark port-wine colour, the alkaline of a ruby glow in transmitted light. Blood, when treated with several other substances which form a certain amount of met-hæmoglobin (*e.g.*, $KClO_3$, K-permanganate, Amyl nitrite, Formyl aldehyde), does, no doubt, look "chocolate coloured," but that is largely due to the fact that the pigment is present in the midst of various proteid precipitates from the blood itself.

3. *Acid Hæmatin.*—This has to be very carefully prepared, else there will be precipitation and opacity. Dilute defibrinated

sheep's blood with its own volume of water, and to 140 c.c. of it, warmed to 35° C., add 5 c.c. of *glacial* acetic acid; stir and filter. This shows the characteristic band in the red. I find, with Stirling (2), that acid hæmatin is incapable of reduction with Am_2S.

4. *Alkali Hæmatin.*—To 140 c.c. of a ·7 per cent. of HbO_2 while at 35° C., add a stick 2·5 cm. long of solid KHO; stir, filter.

This is a pigment not well adapted for demonstration on a large scale, owing to the large amount of light cut out by its broad area of absorption in the orange, and to the very considerable dimming of all light to the right of F.

5. *Hæmochromogen.*—Of the above solution take 140 c.c. and dilute it to three times its volume, warm to 35° C., and add 15 c.c. of Am_2S, when the dark olive-brown solution almost immediately changes to a ruddy tint; it must not be overheated. Being *reduced* alkali hæmatin, a very striking experiment may be performed by oxygenating some of it and looking at its spectrum. The two characteristic bands in the green have disappeared and given place to that broad indefinite region of absorption characteristic of the unreduced alkali hæmatin. If, however, this area of dimming be now scrutinised, in a few seconds, towards its right border, the ghost of a dark band will be seen to form, and to become every moment more intense and definite, while a neighbouring band is found to be forming, though more faintly, to the left of it, and separated from it by an interval of clear green light that had previously been obscured.

6. *Acid-Hæmatoporphyrin.*—To 100 c.c. of concentrated H_2SO_4 add in the cold 5 c.c. of fresh undiluted defibrinated blood.

This can only be done in a glass vessel, and the liquids must be gently stirred to prevent any turbidity arising either from the $FeSO_4$ or from particles of carbon if some of the blood becomes charred. If specks of carbon do appear, the whole must be filtered through asbestos; and if too dark to yield the spectrum, must be diluted only by the addition of concentrated H_2SO_4; water must on no account be used. This iron-free pigment is very stable. It is somewhat interesting that treatment of blood with this strong acid develops the halitus sanguinis with considerable distinctness.

III.—*The Spectroscopy of Met-Hæmoglobin.*

This pigment appears under various chemical conditions; thus, exposure to the air of a thin layer of dilute blood, or merely corking dilute blood in a large bottle, is sufficient to form some. These solutions first become acid and show this pigment *before* there is any alkalinity with putrefaction and reduction of HbO_2 to HbO.

But, as is very well known, this pigment is formed when a nitrite, even amyl nitrite, or potass. permanganate, or $KClO_3$, or formyl aldehyde, or a dilute acid (3), acts upon the blood. The last mentioned is probably the method by which met-hæmoglobin appears in the urine, being produced in it by the weak acid, for it seems certain that one never finds acid or alkali hæmatin in urine, but only HbO_2, or HbO, or met-hæmoglobin. I kept HbO_2 in contact with both alkaline and acid urines for periods varying from thirty-six hours to two months, and only found alkaline hæmatin in alkaline urine after the latter period. I never found acid hæmatin in any urine whatever. Only strong acids can form it. In a clinical case of "blood in the urine" (carcinoma of kidney), I obtained a spectrum which could only be interpreted by recognising that we had both HbO_2 and acid met-hæmoglobin simultaneously present.

It was not acid-hæmatin, for—(1) Am_2S reduced the whole to HbO; and (2) dilution with half its volume of water caused the band in the red to disappear, which is precisely how that band of met-hæmoglobin behaves—it tends to disappear in dilute solutions with more celerity than any other band in the red. Thus there is no doubt this was *not* a case of met-hæmoglobinuria, but merely of hæmaturia in which some met-hæmoglobin was formed by the weak acid of the urine acting on the blood, either in the bladder or after being passed.

When met-hæmoglobin is reduced by Am_2S, it passes through the stage of HbO_2 before becoming HbO: this has given rise to the idea that it is "hyperoxygenated HbO_2"; the oxygen is, however, rather more firmly fixed in it than in HbO_2. When HbO, so produced, is re-oxygenated, only HbO_2 and not met-hæmoglobin is obtained; thus there is *no* pigment "*reduced* met-hæmoglobin" *per se*, in the same sense that there is a reduced alkali hæmatin.

per se, because met-hæmoglobin having been once reduced, cannot by oxidation be reconstituted. I fully corroborate those observers who lay stress only upon the band in the red in neutral and acid solutions—the only ones clinically met with and spectroscopically identical. The band in the red is alone to be relied on; and though it disappears upon dilution, it is stable compared with the two between D and E. In many urines, I believe, these two bands are really those of HbO_2, which pigment is present along with a certain amount of met-hæmoglobin; for after keeping such urines for a day or two, all the bands of met-hæmoglobin become more distinct, as though there had been a progressive conversion of HbO_2 into met-hæmoglobin. Such hints should be borne in mind before the serious pathological condition of met-hæmoglobinuria is diagnosed.

From the fact that both acid hæmatin and met-hæmoglobin possess a band in the red, these pigments may be spectroscopically confused. The following points distinguish them, apart, of course, from measuring the wave-lengths of their bands:—

1. In acid hæmatin the band in the red is nearer the C line.
2. It is usually the denser band of the two.
3. It is usually the broader band; always so in strong solutions.
4. In acid hæmatin there is almost always more yellow-green light in its spectrum.
5. The band in the red of met-hæmoglobin disappears on reduction, that of acid hæmatin does not.
6. Met-hæmoglobin "spontaneously" decomposes in time with reduction to HbO; acid hæmatin, being very stable, does not.

In connection with met-hæmoglobin, it is of interest to note that *it* is the pigment present in the material obtained from cases of hæmatemesis; of course, quite unaltered blood may be vomited, but if the material be kept for some time, a band in the red will develop.

I examined several cases of hæmatemesis: in one I was given a mass of soft, jelly-like consistence, of very dark red, *not* "coffee-ground" colour, unmixed with food. It was faintly acid, and when first examined showed the two bands of HbO_2, but on being put aside for 18 hours the band in the red appeared. Acid hæmatin need never be looked for in the material from hæmatemesis, since the acid of the gastric juice is so dilute. In confirmation of this, I mixed 12 c.c. of blood, diluted with half its volume of water, with 8 c.c. of ·2 per cent. of HCl.—result, met-hæmoglobin; whereas when, for glacial acetic acid, I substituted concentrated HCl. in the making of acid hæmatin, I obtained typical acid hæmatin. *Cf.* M'Munn (4) and Menzies (5) on this point. I agree with both.

IV.—*The Effect of a Rise of Temperature upon Hæmoglobin.*

The temperature of a water-bath was continuously raised from 10° C., and the solutions of HbO_2 taken from it were examined at intervals. Up to 40° C. there was no spectroscopic change whatever; between 40° C. and 55° C. a slight turbidity, but no change in the bands; between 60° C. and 65° C. increasing turbidity, but no change in the bands; at 67° C. the bands could just be seen amid general haziness, thus showing that the chemical integrity of HbO_2 is not compromised by a temperature as high as 67° C. On boiling, the solution became quite opaque, and the HbO_2 decomposed, hæmatin being precipitated.

V.—*On Sulph-Hæmoglobin.*

Reduced HbO is usually made by adding Am_2S, with gentle heat, to a solution of HbO_2; in a few seconds it gives the *one* broad band characteristic of the pigment.

If old filtered Am_2S be used, no other band appears; but if freshly made Am_2S be used, an additional band, one in the red at λ610, appears in the course of a week or so, and persists as long as the pigment is spectroscopically intact. There is no doubt this is the band of a pigment, due to the presence of the S, named by Lankester (6) "sulph-hæmoglobin." At first I thought it might be due to the presence of the NH_4, but found this view not tenable, for sulph-hæmoglobin can be prepared by passing pure H_2S through diluted blood, when a pigment spectroscopically identical with that obtained by Am_2S is formed. Thus, sulph-hæmoglobin is *not* a pigment *per se*; it is reduced hæmoglobin, so prepared chemically that a band in the red (presumably due to S) is present along with the well-known band of HbO. In other words, you cannot have *this* band in the red without also that of reduced HbO; the reverse is, of course, not only possible, but more usual.

VI.—*On the Spectra of those Pigments whose Absorption-Bands closely resemble those of HbO_2 or its Derivatives.*

1. *Carmine.*—Waller (7) says, "The discovery of a two-banded spectrum is not proof positive that HbO_2 is the colouring agent, for carmine gives a very similar spectrum." Dr Waller admits that they could only be confused upon hurried examination.

A solution of borax-carmine in standard hæmatinometer was diluted till of the same transparency as a ·3 per cent. solution of HbO_2; it showed the following differences from blood:—

(1) Each of the bands of carmine has its edges more hazy and ill-defined than the blood-bands.
(2) Carmine lets some blue light through; HbO_2 does not.
(3) More green light passes in the case of carmine.
(4) Concentrated solutions of carmine allow some violet light to pass; blood does not: carmine has a narrow zone of green at E.
(5) The interlinear green light is, in all strengths of carmine, deficient, while it is markedly present in the weaker solutions of HbO_2.
(6) The wave-lengths of the bands of carmine are—
The left-hand band from $\lambda 566$ to $\lambda 538$, centre at $\lambda 553$.
The right-hand band from $\lambda 522$ to $\lambda 500$, centre at $\lambda 511$.

2. *Magenta.*—This aniline dye, soluble in water, is in concentrated solutions very like HbO or HbO_2, of strength greater than 1 per cent. On dilution, the single broad band obscuring the orange and yellow regions gives way to two bands, remarkably similar to those of hæmochromogen. The right-hand band in the case of the magenta is, however, broader, fainter, and farther to the right in the green than is the corresponding band of hæmochromogen, while the left-hand band is rather broader than the left of the two of the blood-pigment.

3. *Roseine and Fuchsine* are like magenta; they remind one, at a certain degree of dilution, of hæmochromogen. The appearance, however, is hardly that of two bands; it would be more accurately described by saying that there is one very broad absorption area, whose darkest portion corresponds in position to the left-hand band

of hæmochromogen, the whole of the green region being invaded by a considerable amount of absorption. This appearance of bands emerging from a haze is in marked contrast to the very sharply defined bands of hæmochromogen.

4. *Red Writing-Ink* (not "copying-ink"). — An undiluted specimen cuts off all light from orange to violet, and thus resembles a solution of HbO_2, of strength greater than 1 per cent. Diluted with an equal volume of water, a little green to the left of the absorption-area is transmitted. A still more dilute solution resembles bilirubin, except that the left margin of the band is more hazy than is the case with the bile-pigment.

5. *Cochineal* (*Tinctura cocci*).—It differs from HbO_2 thus:—

(1) Both bands of coccus are broader than the corresponding blood-bands.

(2) They thus invade more of the green region, of which there is much less than in the case of HbO_2, for the bands of coccus are far more ill-defined on their edges.

(3) Compared with the blood-bands, there is not that relative difference of depth that exists between the latter.

(4) The bands of coccus, compared with each other, are, on a first glance, equally broad; if there is any difference of width between them (the haziness of their edges makes this very difficult of estimation), it is the left-hand one that is the broader—the opposite of which holds good in the case of HbO_2.

6. *Hœmatoxylin.*—This spectrum somewhat resembles that of HbO; it differs in that—

(1) In the case of hematoxylin, all the blue light is present; this is notably not so with HbO.

(2) The one broad band of hæmatoxylin extends much farther into the green region than does that of HbO, which does not cut out all the green rays.

(3) The hæmatoxylin band extends farther into the orange than is the case with HbO. Wave-lengths of hæmatoxylin band are from λ608 to λ474. To the naked eye hæmatoxylin is more purple than HbO.

7. *Saffranine.*—Soluble in water; in strong solution it very closely resembles HbO, except that the one broad band does not approach the D line so closely as in the case of the blood-pigment; no blue light is transmitted in either case. Upon diluting saffranine all the blue light passes, and the band locates itself much more to the right, so as to resemble the spectrum of urinary or biliary pigment.

8. *Eosine.*—In dilute solution this gives a band in the green at *b* very similar to the fourth and most distinct band of alkaline hæmatoporphyrin.

VII.—*The Spectroscopic Recognition of Blood-Stains.*

Quite fresh blood-stains, *e.g.*, before drying upon a handkerchief, yield with transmitted light the spectrum of HbO_2. Stains more than an hour old, and therefore quite dry, yield two bands; but if *brown*, give no bands, as hæmatin must be soluble and transparent to do this. Blood-stained filter-paper fourteen days old, when dry, gave the HbO_2 spectrum on the edge of the paper; after four months, and only after being wetted, that of acid met-hæmoglobin.

VIII.—*Circulating Blood.*

One must be prepared to find, even when the spectrum of circulating blood can be obtained, only *one* band, as indicating such a thickness of blood that the opacity allows of the appearance of only one band. If, for instance, the hand be held up before a powerful light, or to the sun at noon, one can see by the spectroscope, directed to the crimson glow between the fingers, a one-banded spectrum as of HbO_2 greater than 1 per cent.; similarly, light which has passed through the human pinna, or been reflected from the nail, or from the rabbit's fundus oculi, yields the same spectrum. If one uses the semitransparent ear of an albino rabbit, the blood sheet is just thin enough to yield the two-handed spectrum, which fades into the one-handed spectrum of the reduced pigment if one constricts the base of the ear for 30″. A two-banded spectrum can be seen if the frog's web (prepared as for microscopic examination) be interposed between the light and a spectroscope. If a ligature be now tightly tied round the thigh, one cannot get the two bands to give place to the one band, even after three hours—a most striking additional proof of the great relative difference in deoxidising energy possessed, on the one hand, by amphibian (cold-blooded) living tissue, and on the other, by mammalian (warm-blooded) tissue.

It demonstrates the very much more active deoxidative katabolism of the rabbit's tissues compared with the frog's. Since making these observations, I have seen Yeo's paper (8) upon the respiration of cardiac muscle, in which he finds, in the beating frog-heart, fresh mammalian blood perfused is reduced in from 30′–55′, showing, precisely as one would expect, the much more energetic katabolism in a frog's heart than in a frog's leg.

IX.—*The Guiac and Peroxide of Hydrogen reaction with Hæmoglobin Derivatives.*

The following pigments yield the blue colour of this test:—

Oxyhæmoglobin, even in very dilute solution, neutral and alkaline met-hæmoglobin, acid hæmatin, alkali hæmatin, and CO-HbO.

The following do *not* yield the blue colour:—

Reduced HbO, hæmochromogen, acid hæmatoporphyrin, the red ally of uro-hæmatoporphyrin. This latter list is interesting in that they are all more or less reduction-products of HbO_2.

Using this test to discover the minimal quantity of HbO_2 which it could detect, I find that it still yields a blue with so small an amount as 5 minims of blood, diffused through 750 c.c. of water; this represents ·04 per cent. of blood, which corresponds to ·0056 per cent. of HbO_2 in the water.

With the spectroscope I noticed that on diluting down one could detect ·625 c.c. in 1100 c.c. of water, and ·3125 c.c. in 1300 c.c. of water, although at this point the β band could only be viewed obliquely: at ·3125 c.c. in 1500 of water the β band had vanished, *i.e.*, the spectrum had just become atypical. This represents ·02 per cent. of blood, which is equivalent to ·0028 per cent. of HbO_2 in the urine; this figure is very nearly Preyer's for the minimum visibility of HbO_2, viz., ·003 per cent. Since the spectroscope can detect ·0028 per cent. of HbO_2 as compared with

·0056 per cent. by the guiac method, the instrumental is twice as delicate a method as the chemical.

The spectroscope could thus detect a loss of ·3 c.c. of blood by the urine per twenty-four hours.

X.—*Urobilin and Bilirubin.*

I believe the spectroscope may be of service, clinically, in cases of suspected choluria, especially in those in which Gmelin's reaction gives negative results. In urine, normal as to its depth of colour, its yellow pigment, urobilin, yields no absorption spectrum, nor, indeed, does an alcoholic solution of urobilin (prepared by M'Munn's method), if only no deeper than the tint of normal urine. With normal urine there is some haziness in the region of the F line, but it by no means amounts to a band. Urine evaporated to one-fifth its bulk, cooled and filtered from urates, *does* yield the broad, familiar band extending darkly over the F line and shading away into the green. Now, it is quite true that the absorption-spectrum of bile from the greenish-yellow pigment bilirubin is, at a certain degree of dilution, remarkably similar to that of urobilin, and under certain conditions would be practically indistinguishable from it; nevertheless, I believe that in so far as pathological urines are concerned, we *can* say whether the abnormal pigmentation is due to an excessive amount of urobilin or to the presence of bilirubin. For, first of all, from the fact that unless specially concentrated or chemically treated, normal urine gives no spectroscopic evidence of urobilin, if one sees in a urine a band at F, and no other bands, the presumption is that that band is due to bile-pigment. All doubt is, of course, dispelled if this urine also yields Gmelin's test, for urobilin does not give the play of colours. I am not forgetting that in choluria the urobilin may be itself increased, but as it is only in highly febrile urines that the urobilin band spontaneously appears (and when it does, is usually accompanied by bands more to the left), I am inclined to believe that the band at F in choluric urine is almost entirely due to bilirubin. If this increase of urobilin in choluria be anything like a common occurrence, it tends to confirm the contention emphasised in a former paper (9), that urinary pigment is not merely altered and reabsorbed intestinal biliary pigment, for in by far the greater number of cases of choluria there is *diminution* of bile entering the intestine, which should lead to less (and not more) urobilin, if the latter were derived from any pigment in the intestine. I do not here raise the question of the genetic relationship of biliary to urinary pigment, but shall be content to point out that even with the pocket spectroscope, the presence of bile-pigment in urine (even when *not* giving Gmelin's test) may be detected.

I examined a number of urines in cases of undoubted jaundice, with and without carcinoma of liver, none of which gave Gmelin's test; and by comparing them spectroscopically with urine concentrated to their tint, I was able to be certain that it was not urobilin that was the cause of their abnormal pigmentation.

As to the band at F in these cases, close scrutiny reveals some degree of difference; for, taking bilious and non-bilious urines of the same depth of colour to the eye, we notice—

1. The left margin of the broad band at F in the case of urobilin is much more hazy than in the case of bilirubin.

2. It does not extend so near to the red side as does the latter.

3. Very much more blue-violet light passes in the case of urobilin than in that of bilirubin; *i.e.*, while urobilin shows more blue, bilirubin shows more green.

4. The darkest zone of the bilirubin band is slightly nearer to the left than that of the urobilin band.

In other words, urine with a little bile-pigment resembles, both to the eye and with the spectroscope, non-bilious urine with an excessive amount of urobilin: given two urines, unsophisticated in any way, both with bands at F, the one in which the band is darker will probably contain bile-pigment, for the urinary pigment, in order to yield so dark a band, would either have to be present to a most unusual amount (such as is only reached by certain diseased urines), or have to be associated with a degree of concentration much higher than we are supposing to exist.

The spectroscope might be employed to distinguish the urine in carbolic-acid poisoning (carboluria) from that in choluria, for the former shows a distinct absorption in the orange region, coupled with a dimming of the entire green, features absent from the urine when containing either biliary or urinary pigment.

I had the opportunity of studying the urine in the rare disease, melanuria (melanotic sarcoma). Examined without dilution the brown-black fluid cut out all light except some red, and hence had no characteristic appearance; on dilution, it became so like urine with bile or carbolic acid, that the spectroscope cannot be said, in such a case, to be of much service.

Glacial acetic acid deepened the tint of the band in the dilute specimen; KHO had a similar effect.

XI.—*On Carbon-Dioxide-Hæmoglobin.*

I was led to look a little into this pigment from finding that defibrinated blood, which has been shaken up in a 1½ litre vessel full of pure CO_2 gas, no longer gives the two bands of HbO_2, and is of the true purple or 'venous' hue. Further, if CO_2 be passed through some defibrinated blood for some minutes, so as to be thoroughly mixed with it, fully reduced HbO can be seen, though not without trouble, on account of the difficulty of preventing re-oxygenation in our attempt to examine it. The following device succeeded. A stream of the gas was made to bubble up through the blood in a hæmatinometer, and the froth examined in a strong light. A very beautiful phenomenon was seen: an intact bubble gave a single-banded spectrum, but the instant it had burst it was oxidised, and yielded the two bands of HbO_2. The pellicle of the bubble was thin enough to allow sufficient light to pass to form a spectrum, while the bubble bursting upon the glass gave for a moment a smear of oxidised pigment before it drained itself down into the fluid below. The rapidity of the oxygenation was very notable.

Quite otherwise is the behaviour of a dilute *solution* of HbO_2. I diluted defibrinated sheep's blood till it gave the spectrum of HbO_2 of 1 per cent., and for 2¼ hours passed through it a vigorous stream of CO_2 gas. The dilution had destroyed the red corpuscles by their having imbibed water and burst, so that under the microscope not a red disc was to be seen. At the end of 2¼ hours the HbO_2 was quite unreduced and had nothing of a venous hue. Thus, in order that CO_2 may reduce HbO_2, the physiological integrity of HbO_2 and the morphological condition of the red corpuscle must both be uncompromised. Does CO_2 'unite' with the red discs? Presumably it does, since it does not form any union with pure HbO_2 in solution when the discs have been destroyed. But if it does *not* unite with HbO_2, and yet *does* unite with the physiologically intact disc, it can only unite with some constituent of the corpuscle other than the pigmentary. To this extent I would seem to agree with Bohr (10); though I could not say the union was with the globin any more than with the hæmatin, seeing that CO_2 effects no change upon hæmoglobin in solution. In other words, CO_2 effects changes of reduction only upon *blood.* What is noteworthy is the extremely unstable or loose character of this compound of CO_2 with the red disc, which can in an instant be reoxidised, *i.e.*, O can displace the CO_2 very readily. The artificial HbO, made by reducing HbO_2 with Am_2S, is, in comparison, quite *stable,* for if reoxidised, it shows the two bands, and then they fade away to the one again.

It is, in short, easy to preserve the Am_2S-HbO intact for months, whereas the CO_2-HbO is a most evanescent production. I am very much inclined to regard the union of CO_2 with the discs as not a chemical one, but much more of the nature of a physical, interstitial, or intermolecular one in the meshes of the protoplasm of the intact red corpuscle,—HbO_2 having no chemical affinity for CO_2, while it has a very strong one for O. This state of matters has a most direct and important bearing upon the needs of the body as regards these gases, seeing that the red corpuscle of arterial blood must be able to carry its burden of respiratory oxygen to the remotest recesses of the tissues, and not till then give it up to the lymph, whereas the red corpuscle of venous blood must be ready to part with its dissociable CO_2 as soon as it has reached the pulmonary capillaries.

XII.—*On Carbon-Monoxide-Hæmoglobin.*

It is well known that chemists regard CO-HbO as "a very stable pigment," and one that "resists putrefaction for a long time." I tried to answer the question 'how long?' and in doing so found a difference in stability between what one might call a large and a small quantity thus:—a large quantity (250 c.c.) was bottled upon 30th January 1895, and showed the two bands upon 23rd of February, and up to 15th of March, whereas a small quantity put up in a test-tube 30th January had by 18th of February become HbO, with putrid smell. In between six and eight weeks the CO-HbO in large amount becomes changed first to reduced HbO and then (after five months) to alkali-hæmatin. As to its power of resisting reduction by Am_2S, the point of importance seems to be whether the CO-HbO is or is not freshly made. Some fresh CO-HbO with $\frac{1}{8}$ of its volume of fresh Am_2S was put up on 18th February, and after three months was still CO-HbO, unreduced, whereas some ten days' old pigment with $\frac{1}{8}$ of its volume of Am_2S, became in twenty-eight days reduced to hæmochromogen. The statement that CO-HbO is a very stable pigment is fully justified.

XIII.—*On the Permanence of HbO_2 and its Derivatives, and upon their Resistance to Putrefaction.*

In February 1895 I examined a number of test-tube preparations of dilute (·25 p.c.) solutions of dog's HbO_2 which had been hermetically sealed up by Professor M'Kendrick in 1888. Into them had been put bits of living tissue to see how these would affect the pigment; the results on that point are too discordant to warrant generalisation, but the condition of the tubes after seven years is interesting. The HbO_2 had originally, of course, not been heated, but the air above it must have been pretty well sterilised by the heat necessary to fuse the glass of the tube.

Tube 1,	in 1888	HbO_2 + portion of eye,	result in 1895 HbO,	result in 1897 HbO.
,, 2,	,,	HbO_2 + cerebral cortex,	,, HbO,	,, HbO.
,, 3,	,,	HbO_2 + piece of skin,	,, HbO_2,	,, HbO_2.
,, 4,	,,	HbO_2 + nothing,	,, HbO,	,, HbO.
,, 5,	,,	HbO_2 + nothing,	,, HbO,	,, HbO.
,, 7,	,,	HbO_2 + skin,	,, HbO_2	,, HbO_2.

Of these seven, four had become reduced, and of these four two had no tissue in them, while two tubes, which after nine years were still HbO_2, had skin in them; thus I do not think the results as to tissue action are of much value, but the significant fact remains that a dilute solution of mammalian HbO_2, practically sterilised, can remain for nine years chemically in *statu quo.* Further, those that were HbO in 1895 were HbO in 1897; thus the reduced pigment in dilute antiseptic solution is as stable a form as the oxidised.

There are differences in the behaviour with regard to putrefaction between defibrinated blood and dilute aqueous solutions of HbO_2. Thus—

Sheep's or ox's blood in a corked bottle, even when not quite full, loses in about twenty-four hours its crimson hue, and becomes claret coloured, while in a few hours more the spectroscope shows it to be HbO. This putrefaction-HbO is very easily re-oxidised into HbO_2, *i.e.*, the pigment itself has *not* become decomposed, its chemical integrity has not been compromised, although it has been reduced in and by decomposing blood. The pigment cannot be said to be decomposed so long as the HbO can become HbO_2 again. After 5 months, corked-up blood is alkaline, the pigment hæmochromogen, so that it must have passed through the stage of alkali-hæmatin.

The pigment in decomposing blood probably oscillates between these two states according to the amount of available oxygen present, for decomposing sheep's blood 3 years old mixed with H_2O in a hæmatinometer (for it is too opaque when undiluted), showed the band of alkali-hæmatin; probably the hæmochromogen had been oxidised in the process.

A *dilute* solution (·3 p.c.) of HbO_2 in a test-tube loses its brightness on the fourth or fifth day, becoming slightly turbid, of a pale rusty-brown colour, slightly acid in reaction, and yields the spectrum of met-hæmoglobin; at the end of 3 weeks it had become HbO, which, however, was easily oxidised to HbO_2 on being shaken with air. Thus, while a dilute solution of the pigment of blood passes through an intermediate stage of met-hæmoglobin, the pigment in blood itself does not, but is reduced as its first change—a result evidently due to the development in decomposing blood of some reducing materials which are absent from weak, watery solutions of the pigment alone.

I found that thymol, as Yeo (8) had found phenol, delayed the appearance of these reducing decomposition-products in blood.

XIV.—*On certain other Pigments under Observation for Three Years.*

1. *Reduced Hæmoglobin*, made by Am_2S, well corked, and not shaken with air, was still HbO at the end of five months. At the end of six months it was hæmochromogen, at the end of two years

it was alkali-hæmatin, presumably a re-oxidation after the reduction due to putrefaction was ended.

2. *Neutral Met-Hæmoglobin* (made with K-ferricyanide) was unchanged for seventeen days; at the end of four weeks it was HbO, which could be oxidised to HbO_2; at the end of five months it was HbO; at the end of two years it was no longer HbO, there was a precipitate, and the supernatant liquid was yellowish. Another specimen at the end of two years was still HbO, but after three years had become hæmochromogen.

3. *Alkaline Met-Hæmoglobin.* — After five months this was alkali-hæmatin; at the end of three years it was a yellowish liquid, with a shading at F.

4. *Acid Hæmatin.*—At the end of three months it was intact; at the end of two years it showed its characteristic band in the red, although the solution had become greenish-yellow; at the end of three years the band in the red was still quite distinct.

5. *Alkali Hæmatin* at the end of five months was in no way altered; at the end of two years its characteristic absorptions over D and in the violet were present, though the colour of the solution had become greenish.

6. *Acid Hæmatoporphyrin.*—At the end of three years quite unchanged.

7. *Hæmochromogen* (made by reducing the KHO alkali-hæmatin with Am_2S) had a rather curious history.

By the seventeenth day two specimens of it—one with and one without thymol—had become of the most beautiful clear *green* colour, the reaction was alkaline, and there was no precipitate. It still showed the two characteristic bands in the green, but had in addition a broad area of absorption in the red (to which it owed its green colour).

The pure solution at the end of two years was yellowish, with the faintest possible band at F. The solution with thymol had become alkali-hæmatin at the end of two years, and was still the same at the end of three; it had lost a great deal of its red colour.

8. *A Solution of CO-HbO* made on 31st January 1895, and reduced with Am_2S on 8th of February, was in two years hæmochromogen.

A solution made on 1st of February, with thymol added, was a yellowish fluid, with some red precipitate, at the end of two years. A solution of CO-HbO, to which Am_2S had been at once added, still showed after three years the two bands between D and E.

A solution of CO-HbO to which nothing had been added, was at the end of two years a pale yellowish fluid.

Some deductions can be drawn from these observations, extending over three years, upon the permanence of HbO_2 and its derivatives. They may be arranged in order of stability thus:—

Very stable.—Acid Hæmatoporphyrin and its red ally in urine (a-Hæmatoporphyrin) (9); HbO_2 when in dilute antiseptic solution or when dried, Acid Hæmatin; then Alkali Hæmatin (whether chemically pure or made by putrefaction); Carbon-monoxide-Hæmoglobin; Hæmochromogen (of putrefaction).

Moderately stable.—HbO; Alkaline Met-Hæmoglobin; Neutral Met-Hæmoglobin.

Unstable.—Chemically Pure Hæmochromogen.

Very unstable.—HbO_2 in presence of putrefaction.

In connection with these pigments under observation for a very long time, it is noteworthy that in several instances, after reduction had done its work, there was formed a precipitate of something reddish, while something yellowish and transparent went into solution. This yellowish liquid was noticed in dilute solutions of HbO_2, HbO, CO-HbO, Hæmochromogen, Acid Hæmatin, Neutral Met-Hæmoglobin. This yellow pigment was *not* the yellow pigment of serum, because it was observed in dilute artificially-produced solutions of HbO_2 or its allies.

I venture to think we have here a test-tube version of what occurs to blood-pigment in the body, chiefly in the liver, viz., first a reduction of the 'effete' hæmoglobin, followed by a decomposition of it into two portions, an iron-containing compound that can be demonstrated in the liver-cells, and iron-free substances—the parents of the biliary and urinary pigments, greenish or yellowish in colour.

In a former paper (9) I tried to construct, based upon the experimental work of others, a genetic tree of these pigments, but at that time had unfortunately not noticed a paper by Dr Eichholz (Cambridge) (11). Two of his results confirm the views arrived at in that paper. He says, "A body very closely resembling urobilin may by prolonged and complete *reduction* be produced from hæmatin in acid solution;" and "*incomplete* reduction furnishes a body also resembling urobilin, but which immediately reverts to the state of uro-hæmatoporphyrin." This latter is an orange pigment, and would "resemble urobilin." M'Munn thinks that its red ally and it itself are progressive stages in reduction from hæmatin. Eichholz did not evidently stop at any stage till he reached one resembling urobilin, but it is perfectly possible that, even artificially, one passes through the stage of α-hæmatoporphyrin.* In the body, if this pigmentary katabolism to urobilin be disturbed or incomplete, the red or orange (α- or β-) hæmatoporphyrin may appear in the urine; sulphonal seeming to have a powerful influence in this direction in the case of the red or α pigment.

XV.—*The Spectroscopy of Freshly-drawn Blood.*

Under this head I made a large number of observations from time to time, but owing to the very great mechanical difficulties of examining the blood soon enough, I do not feel justified in reporting on any except the following:—

Blood from the internal jugular of a dog, chloroformed until respiration had ceased, gave the one-banded HbO very rapidly changing to HbO_2 upon oxidation.

A large dog, chloroformed till respiration ceased, yielded from int. jugular HbO, from left ventricle HbO_2.

A cat, suddenly killed by a blow on the head, gave with blood of right ventricle HbO, with blood of left ventricle HbO_2.

In a month-old rabbit, chloroformed till respiration ceased, blood of right ventricle was HbO with one band rapidly becoming two on oxidation; from internal jugular HbO also.

So far as I am entitled to form an opinion, I confirm M'Munn in saying, "the blood in the right side of the heart and in the veins is reduced as soon as the animal has ceased to breathe, but the HbO_2 of the blood in the left side of the heart and aorta does not become reduced for some time after death" (12); how long after death it would be difficult to say, but I find one experiment upon the point.

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* I suggest this name instead of Meio-de-oxy-hæmatoporphyrin, which denotes a particular view.

A cat was killed by a blow on the head, and the body lay for eighteen hours, when blood from the left ventricle gave the single band of HbO very easily oxidised; blood from the right auricle gave HbO not so easily oxidised.

On the Crystallisation of HbO_2 *of Rodents' Blood.*

For the last three years I have kept crystals of HbO_2 under observation. Some were from the rat and were mounted both in water and in balsam (by Stein's method, defibrinated blood mixed with balsam). It is interesting that HbO_2 will not crystallise in plasma; the blood needs to be defibrinated, and the red corpuscles must give up their HbO_2 before crystallisation can be depended on. The relation of this fact to the needs of the circulation is too obvious to require mention. Whilst watching the familiar crystallisation in water of the HbO_2 from the rat, I noticed that a considerable number of the spicules, whether singly or in stars, were *colourless*; in every other respect they resembled by transmitted light their bright crimson neighbours.

There were more colourless crystals in the water preparation than in the balsam—a fact evidently related to the higher solubility of HbO_2 in water. It was also noticed that the water round about these colourless groups was stained deep red. What were these colourless crystals? It has been suggested they were crystallised proteid-globin; but this involves the breaking up of HbO_2, and the pigmentary portion—the hæmatin—is not soluble in water. If they were still HbO_2, colourless HbO_2 is a substance unknown to physiological chemistry. At the same time, there is no other term by which they can be alluded to than "colourless hæmoglobin," or the chromogen of HbO_2. More recently, I have observed HbO_2 crystals in balsam from the guinea-pig become within three weeks 'bleached' to this colourless condition; it certainly looks like a reduction of the HbO_2 to colourless chromogen, probably effected by changes in the defibrinated blood surrounding the crystals.

On the other hand, why should not all similarly formed crystals of HbO_2 become similarly altered?—*the majority do not.* I have preparations in balsam three years old showing crimson crystals both from rat and guinea-pig. Such preparations still give the two bands of HbO_2 in the microspectroscope. (I may here mention that hæmin crystals do not do so.)

It seems evident that there are two fundamental forms of HbO_2 crystals in the rat—(1) the elongated rhomboidal prism; (2) the thin flat plate: if the former tapers at both ends, we have the acicular; if only at one end, the clavate. The plate may be a *regular* hexagon, as Halliburton (13) found, so that this form is not absolutely confined to squirrels' blood, or it may be an irregular hexagon, a pentagon, or other polygon (size of rat's regular hexagons is from ·002 to ·003 mm.).

I noticed the formation, as it were, of certain of the flat plate crystals (regular hexagons included), by the lateral coalescence of radially disposed rows of granules, or of acicular or clavate crystals disposed in one plane; while, after three weeks, I saw certain apparently homogeneous crystal-plates breaking up into rows of granules, radially disposed, or into radiating acicular or clavate crystals. Some plates broke up into radiating feathery forms.* In the cases in which the thin flat plate was formed at first *as such*, *i.e.*, quite homogeneously, it did not break up radially, but cracked into a number of polygonal areas, like a mosaic. Some of these thin flat regular hexagons are after three years quite intact. Further, I corroborate Halliburton that the regular hexagons are formed near the periphery of the preparation, *i.e.*, where drying went on most vigorously—the rapidity of loss of moisture being evidently a factor in determining the size and details of form of a crystal.

Guinea-pigs' blood mixed with balsam sometimes yields some very large tetrahedra (·05 mm. side) of a bright crimson colour. Occasionally one finds aborted forms—thick triangular plates with upward sloping edges (·04 mm. side)—evidently the foundation upon which the pyramid was to have been built. Some of those thick plates were yellow, not red; and after three weeks became eroded on the edges, which were previously particularly sharply cut.

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* This breaking up of crystals into simpler elements I have also seen in very old cholesterine crystals.

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January 1898.

Note on the Effect of Mixed Diet as Regards Salivary Digestion. By W. G. Aitchison Robertson, M.D., D.Sc., F.R.C.P.E.

(Read July 4, 1898.)

The following research was made with the object of determining whether the digestion of starch is affected favourably or unfavourably by its admixture with other articles of food.

As regards the quantity of saliva secreted per twenty-four hours, this is stated to vary from 200 to 1500 grammes. Investigating this in my own case I found that the average amount was 400 grammes.

Method of Investigation.—Ten grammes of each of the various foods as prepared for the table were accurately weighed out, placed in a large tube, and heated to 38° C. Two cubic centimetres of mixed human saliva were then added. A series of these tubes were prepared, and the whole was placed on a water-bath kept at a uniform temperature of 38° C. At intervals a tube was removed, and the condition of its contents noted.

Porridge forms a fairly easily digested article of food. The greater the dilution of the porridge, however, as with water or milk, the greater is its ease of digestion; in fact, well-made water gruel is one of the most easily digested of foods.

The reason why potatoes are so frequently badly borne by the stomach is probably due to the manner in which they are prepared. If, after being boiled, they are finely powdered, so that the saliva can gain ready access to the starch granules, they are easily and rapidly digested. On the other hand, when potatoes are sent to the table whole, the probability is that they will be imperfectly masticated, and swallowed in fragments of more or less large size. In such a case the saliva can only act on the outer surfaces of the fragments, and the result is a very imperfect digestion. Were potatoes prepared properly, I feel sure that they might be prescribed as frequently as other farinaceous foods for invalids or dyspeptics.

As regards the digestion of bread, the experiments showed that it was much less acted upon by saliva when eaten alone than when taken along with some indifferent fluid, as water. The greater moistening of the bread which results produces a marked rapidity in the action of ptyalin.

Newly-baked bread is not so rapidly acted upon by saliva as stale bread, but the ultimate degree of starch conversion is greater in the former than in the latter. So far as these experiments show, stale bread is not more easily digested than newly-baked bread.

Bread in a light and spongy condition (as is met with in well-aerated varieties, *e.g.*, Vienna bread) is more rapidly acted upon by saliva than when less spongy. Such bread, however, does not ultimately undergo any more complete digestion than does ordinary bread.

The addition of butter to bread has little or no effect on amylopsis by ptyalin. Cheese, on the contrary, has a stimulating effect on the salivary secretion; and, apart from the consideration of its digestibility, it has a helpful action in promoting starch proteolysis.

The addition of milk to bread causes a remarkable enfeeblement of the salivary ferment, while broth exerts a slight restraining influence on it.

When bread is chewed along with an infusion of tea, only about one-half the amount of sugar is formed as compared to the same proceeding when bread and water are alone employed. Coffee has no great effect in hindering or slowing starch proteolysis, and the same is true of cocoa.

Beer seems to exert a stimulating action on ptyalin, as even from the earliest period the amount of sugar formed is in excess of that produced in the presence of water alone.

Alcohol, even in dilute solution, retards salivary digestion of starch, but the action is much less marked than in the case of infusions of tea. Whisky permanently lessens the activity of ptyalin.

Wines have a very marked inhibitory influence on the digestion of starch by saliva, and this is almost wholly due to their acidity. Even after three hours' digestion in the presence of sherry, port, or claret starch undergoes hardly any conversion.

The more dense, the less broken down, or the firmer the jelly in which the starchy food is when undergoing salivary digestion, the less rapid and extensive is the proteolysis.

Amylaceous substances are more easily acted on by saliva when thoroughly moist than when more or less dry.

Comparison between Different Starchy Foods masticated, then kept at 30° C. for thirty minutes.

Food.	Unchang'd Starch.	Soluble Starch.	Erythro-dextrin.	Achroo-dextrin.	Percent. sugar formed.
1. White bread alone	much	much	much	much	39·28
2. Bread water	little	,,	,,	,,	32·47
3. Newly-baked bread water	much	,,	,,	present	34·13
4. Bread crust water	,,	,,	fair amt.	,,	33·0
5. Vienna bread (stale) water	,,	,,	much	much	35·14
6. Vienna bread (new) water	,,	,,	,,	,,	38·96
7. Bread butter	,,	,,	,,	,,	31·27
8. ,, cheese	,,	,,	,,	,,	37·16
9. ,, milk	,,	,,	,,	some	28·56
10. ,, broth	,,	,,	,,	,,	33·57
11. ,, tea	,,	,,	,,	,,	22·99
12. ,, coffee	,,	,,	,,	,,	29·30
13. ,, cocoa	,,	,,	present	present	39·83
14. ,, beer	,,	,,	much	much	30·86
15. ,, dilute spirit	,,	,,	,,	present	35·05
16. ,, strong spirit	,,	,,	,,	,,	31·46
17. ,, sherry	nearly all	—	—	—	6·76
18. ,, claret	,,	sm. amt	—	—	8·23
19. ,, port wine	much	much	fair amt.	fair amt.	13·29
20. Potato, powdered	little	—	—	much	20·28
21. ,, in fragments	—	—	—	—	21·28
22. Porridge, alone	nearly all	much	much	trace	25·72
23. ,, water	some	,,	sm. amt.	—	29·9
24. ,, milk	much	,,	fair amt.	—	23·88
25. Rice, boiled	,,	,,	,,	little	12·36
26. Corn flour, boiled	trace	,,	much	,,	38·13

On Torsional Oscillations of Wires. By Dr W. Peddie.

(Read June 20, 1898.)

(*Abstract.*)

This paper is in continuation of two others, on the same subject, previously communicated to the Society. It consists of two parts, the first experimental, the second theoretical.

The wire which was experimented upon was the one which was used in the previous experiments, and the apparatus was that described in the latter of the communications above referred to.

In the first paper (*Phil. Mag.*, 1894) it was shown that the equation

$$y^n(x+a)=b,$$

where n, a, and b, are constants in any one experiment, represents to a very high degree of accuracy the relation between y, the range of oscillation, and x, the number of oscillations. In the second paper (*Trans. R. S. E.*, 1896), the result of attempts to determine the nature of the dependence of n upon (1) magnitude of initial oscillation, (2) amount of "fatigue" of the wire, were described. It was found to be impossible, under the subsisting conditions as to fatigue, to separate the effects of fatigue and initial range.

The first series of experiments described in the present paper was conducted under the condition of excessive fatigue of the wire. The effect of magnitude of initial range was found to be practically eliminated. Thus it was possible to express all the results of that series by a general formula, of the above type, in which n and b are absolutely constant, while a is given in terms of the initial range.

In the second paper above alluded to, it was shown that, throughout the series of experiments therein described, the product nb might be regarded as constant. And it was pointed out that this was, so far, an accident depending on the size of the scale-unit. The importance of the result lay in the fact, that a scale-unit which made the product constant could be found; and that, if the product were constant, a critical angle of oscillation, at which the

value of the loss of energy per oscillation is independent of n, must exist.

In the first series described in the present paper, the (constant) product of n and b, has a value about one-half of the former, the diminution being due to a great change in the value of b, caused by the excessive fatigue employed. Excessive fatigue makes the value of n tend to unity.

A recalculation of the constants in the formulæ in Table I. of the first paper is made by the method now used; and it is shown that, while nb is not constant, log.nb is a linear function of n—the two quantities increasing simultaneously. This implies the existence of a critical angle greater than that subsequently obtained, so that the critical angle is diminished by fatigue.

It is further shown that log.b may be regarded, in each series of experiments, as a linear function of n. With the observed values of n, it is impossible to tell which of log.b or log.nb is more accurately a linear function of n—though both cannot be truly such a function simultaneously. The chief experimental result obtained is independent of this question. It is found that *in every series of experiments, if conditions under which* n *had the value unity are attainable, the value of* b *is absolutely constant.* This seems to indicate a quantity which is dependent only on the nature of the substance of which the wire is composed.

A modification of the apparatus was employed to compare the times of out and in motions during the oscillations. It was found that the time of inward motion exceeds that of outward motion through the same range.

It is also shown that the magnitude of the total period of oscillation has no observable effect on the values of the constants in the equation given at the commencement of this abstract.

In the first paper on this subject communicated to the Society, the equation just alluded to was deduced from the assumption that the defect of the potential energy from the value which it would have in accordance with Hooke's Law is proportional to a power of the angle of distortion. In the theoretical part of the present paper the assumption is made that molecular groups on the average obey Hooke's Law in their distortions, and that definite groups have a definite limit of distortion beyond which they break down and instantaneously re-form. The loss of energy in oscillations of the system is due to the rupture of such groups.

If the assumption that the breaking limit of distortion is evenly distributed among groups be made, the theory indicates that the relation $y(x+a)=b$ holds between range of oscillation and number of oscillations. This is the relation which was found to hold under great fatigue. Thus the effect of fatigue may be to produce this even distribution. In this case Hooke's Law is not obeyed—a term involving the square of the distortion appearing; so that, in an outward oscillation, the distortion is an elliptic function of the time. The theoretical law is shown to accord very closely with the results of observation.

In the second half of the inward motion, according to the theory, the inward acceleration is less than that at the same stage in the outward motion by a constant amount. It is always less at any stage. Thus we deduce the experimentally found relation between the times of out and in motions over a given range.

Further, if the wire, after its first positive distortion, be stopped just short of the zero and be again distorted in the positive direction, the theory shows that, while the same law connecting distortion with distorting force holds, the co-efficient of the term which involves the square of the distortion is greatly reduced. Thus Hooke's Law is much more nearly obeyed in the second distortion. This is in complete agreement with Wiedemann's observations (*Phil. Mag.*, Jan. 1880). On the other hand, if the zero be passed, the wire is reduced to its original condition.

If the assumption that the number of groups which have a definite breaking distortion is constant for all distortions be replaced by the assumption that it is proportional to a power of the distortion, the term, in the expression for the distorting force, which represents the deviation from Hooke's Law involves a power of the distortion different from the second. Thus the assumption, made in the first paper on the subject, that the defect of the potential energy, from the value that it would have in accordance with Hooke's Law is proportional to a power of the distortion, is deduced as a consequence of a certain, uneven, distribution of number of groups in respect to the extreme limit of sustainable distortion.

The general relation between torsion and set is also discussed.

It is not to be supposed that good agreement of the results of theory with the results of experiment necessarily constitutes a proof of the particular theory. The aim in the paper is rather to show how completely the general action of imperfectly elastic materials may be accounted for upon simple and reasonable views regarding molecular statistics.

The Strains produced in Iron, Steel, Nickel, and Cobalt Tubes in the Magnetic Field. Part II. By Professor C. G. Knott, D.Sc., F.R.S.E.

(*Abstract.*)

(Read June 6, 1898.)

The most important part of the paper has to do with the behaviour of certain iron and nickel tubes, considerably shorter and narrower than those discussed in Part I.* Two iron bars, A and B (A being twice the length of B), and a nickel bar, B, were bored out by successive stages, so as to give three series of tubes of increasing bore. Each tube was subjected to four distinct experiments. These were—

1. Measurements in various magnetic fields of the corresponding changes of volume of bore.
2. Measurements in the same fields of changes of length.
3. Measurements in the same fields of changes of volume of the material of the tube.
4. Measurements in the same fields of *apparent* external changes of volume, the tube being plugged and treated as a bar.

From these measurements the principal coefficients of strain at the inner and outer walls were determined. An element originally spherical became changed into an ellipsoid, whose principal axes were parallel to the axis of the tube, parallel to the radius of the tube passing through the element, and perpendicular to these two directions. The corresponding coefficients of strain are distinguished as the longitudinal elongation λ, the radial elongation ν, and the tangential elongation, μ.

In nickel λ is always negative, and μ and ν generally positive. In the narrowest bored tubes μ is occasionally negative. In the B-tubes λ reaches a value of nearly -25×10^{-6} in field 500 (C.G.S. units); μ almost touches the value $+7 \times 10^{-6}$; and because of the very small and practically negligible value of the cubical dilatation, μ has the value $+18 \times 10^{-6}$.

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* See *Trans. R.S.E.*, vol. xxxviii. pp. 527–555 (1896).

With nickel tubes formed by coiling thin plates, λ reaches the value -30×10^{-6}, while μ is found to be $+18 \times 10^{-6}$, ν being consequently $+12 \times 10^{-6}$. This excess in value of μ over ν in very thin-walled tubes, and the excess in value of ν over μ in tubes of thicker walls, are points of some interest.

In iron and steel, λ is positive in low and moderate fields, and negative in high fields. The ratios μ and ν tend to have opposite signs from λ; but the tendency is considerably modified in the tubes of narrower bore. As the bore is increased and the thickness of wall diminished, the well-marked maximum in λ occurs in a lower field. The cubical dilatation is always positive. The following values for iron tube B V will show the nature of the changes which occur in these ratios as the field is gradually increased.

ELONGATIONS × 10^6.

Field	At Inner Surface.			At Outer Surface.		
	λ	μ	ν	λ'	μ'	ν'
190	+2·1	−1·5	− ·4	+2·1	−1·25	− ·65
310	0	− ·4	+ ·6	0	− ·1	+ ·3
500	−2·2	+ ·8	+1·6	−2·2	+1·0	+1·3

Very similar values are found with the other tubes discussed.

Among other general conclusions arrived at the following may be stated:—

The apparently capricious character of the results obtained in Part I. finds a ready explanation. The bore dilatation $(\lambda + 2\mu)$ appears as a *differential* effect, being the sum of two quantities of, in *general*, different signs; and of these, sometimes the one and sometimes the other has the greater value.

Except in the case of very thin walls, a circular element in any transverse plane, perpendicular to the axis of the cylinder, becomes, when the cylinder or tube is magnetized parallel to its axis, an ellipse, with its major axis pointing towards the axis of the tube. When the walls are thin, the ellipse into which a small circular element is changed has its minor axis situated radially. The ellipse increases in excentricity as the distance from the axis diminishes. In nickel this ellipse is greater in area than the original circle; in iron it is, in a general way, less or greater according as λ is positive or negative.

Cobalt behaves very like nickel, but the elongations are about one-third the value. The cubical dilatation is always negative, attaining a value of $-\cdot 18 \times 10^{-6}$ in field 500.

The paper ends with a discussion of the work done by the magnetic forces in straining the metals against the intermolecular forces on which the elasticity of the substance depends.

On Consonant-Sounds. By R. J. Lloyd, D. Lit., M.A., Hon. Reader in Phonetics at the University College, Liverpool. (With a Plate.)

(Read May 2, 1898.)

The prime object of the following paper is to assist in deciphering the irregular traces which represent the consonants in a phonographic record, by investigating *a priori*, from the causes which create the consonant, the elements which probably lie entangled in the tracing to be interpreted. Accurately speaking, the difference between vowel and consonant is not one of nature, but of function. To define either vowel or consonant, it is necessary first to define a syllable. All human speech proceeds in rapid alternations of louder and softer, more sonorous and less sonorous. These alternations vary considerably in energy; any one of them may be twice as long, or twice as loud, or twice as sudden in its rise or in its fall as its next neighbour. They seem, in fact, to tend both in duration and in form and in energy rather to a successive change than to any regularity; but each of them is a syllable. A syllable, then, is a wave of sonority, one climax of sound, with its accompanying rise and fall. Accurately speaking, this climax is a subjective one. It usually has its objective counterpart in a climax of amplitude, which again has its counterpart in a maximum of depth in the phonographic furrow; but this is not always the case. Noise does not produce the cumulative subjective effects of tone; it is therefore less loud than tone, when proceeding from vibrations of equal amplitude. Every speech-sound is a mixture of noise and tone; in vowels the proportion of noise is very small, though never quite negligible; but in consonants it ranges from a very small to a very large proportion. Hence the irregularity of consonantal phonograms reaches its height in those consonants which contain the largest amount of noise, that is, of more or less irregular vibration. Still there is a difference between noise and noise, though it is less definite and less assignable than the difference between tone and tone. Noises differ in duration, in suddenness of rise and fall, in the degree of their irregularity of vibration, and in the average period of those vibrations. This average period may also increase or decrease during the utterance of a given sound, and may do so more or less rapidly. These distinctions are all apprehensible by the ear, and they are vital to the recognition of consonants. The function of a vowel is to constitute the strong sound which forms the climax of a syllable. The function of a consonant is to connect those stronger sounds together, so as to form waves of sonority, *i.e.*, syllables. Every syllable must have a vowel, but it need not have a consonant; a vowel by itself is a wave of sonority, because it always has a rise and a fall of intensity, however brief and feeble.

Consonants serve, however, both to make the minima of sonority more marked, and at the same time to enable us to multiply enormously the number of significant syllables at our command, by changing the consonants.

The essence of a consonant, then, is simply to be less sonorous than the adjacent vowel; and the essence of a vowel is to be more sonorous than the adjacent consonants. Some speech-sounds are so sonorous as to be practically always used as vowels; others are so wanting in sonority as to be practically always used as consonants. But there are some of a middle kind, which can be used either way. When this is to be done, their relative sonority is in practice heightened or reduced by an addition or subtraction of stress, *i.e.*, of lung-pressure. In this way it is easy to pronounce *aia* or *aua* so as to make both *i* and *u* function distinctly as consonants. (I use the letters here and elsewhere in their Latin values, except in spelling English words.) But the *s* used in *hissing*, the *sh* used in *hushing*, the *l* in *bottle*, the *n* in *eaten* (eat'n) each function distinctly as vowels, because they constitute, or have the power to constitute, each by itself, a syllable.

But some syllables begin or end with more than one consonant. When syllables are built in this way, inherent differences of sonority again obtrude themselves. The syllable is essentially a wave of sonority, one rise and one fall. Therefore consonants cannot be built into it at random, but only in the order of a rising and falling sonority. A syllable can no more begin with *lt* or *nt* than it can end with *tl* or *tn*. This is simply because *l* and *n* are always more sonorous than *t*. But in the contrary order there is no difficulty: compare *bolt* with *bottle*, *wound* with *wooden*, and the like. But when the inherent difference of sonority is small, the required rise or fall may be created by addition or subtraction of stress; compare *wist* and *wits*, *tsar* and *star*.

The essential weakness of consonant sounds is generally based upon a relative obstruction of exit. M. Marichelle in his excellent little work, *La Parole d'après le Tracé du Phonographe* (Paris, 1897), makes this the universal test of the difference between vowel and consonant. But there is practically no *h* in French, otherwise M. Marichelle would have seen that this criterion is faulty. The exit of *h* is generally just as wide as that of the following vowel (*v. infra*). Phoneticians, however, use this fact of constricted exit to classify consonants in two different ways, first, as to the mode, and secondly, as to the place or places of the constriction. They then divide them a third time, according to the manner in which the larynx is concerned in their respective production. Under the first head it will be simplest to regard at first only three chief categories—fricative, plosive, and nasal. A fricative sound such as *z* or *h* is distinguished by the frictional rush of breath through a more or less obstructed opening; a plosive, such as *p*, *b*, *t*, or *d*, is marked by a sharp percussion, the result of the sudden creation or sudden removal of a complete stoppage; a nasal also implies a complete stoppage in the mouth, but with free exit through the nose.

Under the second head, the places of constriction normal to English and Scotch pronunciation are fully enumerated below. Under the third head there are three well-marked categories of consonants—toned, whispered, and spirate. In a toned consonant the larynx is vibrating—producing tone; in a whispered consonant it is in a hissing position—producing whisper; in a spirate consonant it is wide open—giving nothing but a full supply of breath. Of the examples already given, *s*, *f*, *h*, *p*, and *t* are spirate; and *v*, *z*, *b*, and *d* are ambiguous, because they may be either toned or whispered. But the toned and whispered classes are precisely of equal extent; it will suffice, therefore, simply to remember that every toned symbol has always a whispered value, acoustically very different from the toned value, and highly important to this investigation. It is impossible to talk accurately about speech-sounds without using an accurate set of symbols. In the following table the symbols *p, b, m, f, v, s, z, t, d, n, k, w,* and *h*, are used in their ordinary values. For the rest the following are keywords: *θ*, *thin*; *ð*, *then*; *ʃ*, *she*; ʒ, *azure*; j, *ye*; g, *go*; *η*, *sing*; hw, *what*; χ, Sc. *loch*; x, Sc. *licht*. They are taken chiefly from the alphabet used by the *Association Internationale Phonétique*. The inverted *ɹ* stands for untrilled *r*.

Locality of Constriction.	Fricative.		Plosive.		Nasal.	
	Spirate.	Toned or *w*.	Spirate.	Toned or *w*.	Spirate.	Toned or *w*.
Lip to lip, ...			*p*	*b*		*m*
Lower lip to upper teeth, . . .	*f*	*v*				
Point and blade of tongue to upper teeth, .	*θ*	*ð*				
Fore-blade to fore-gums, and teeth to teeth, ...	*s*	*z*				
Whole blade to after-gums, and teeth to teeth, ...	*ʃ*	ʒ				
Point of tongue to gums (alveolars), .		*ɹ*	*t*	*d*		*n*
Front of *dorsum* of tongue to hard palate, .	*x*	*j*				
Back ,, ,, soft ,, .	χ		*k*	*g*		*η*
The same, and lip to lip, } simultaneously, .	*hw*	*w*				
In various places, ...	*h*					

Only two English consonants fail to fall into the above scheme of classification, namely, *l* and trilled *r*. They both belong to the sixth line of the table, but they find no place in the columns, because they are not strictly either fricative, plosive, or nasal.

I. THE SPIRATE FRICATIVES.

f.

In so complex a matter it is important to begin from the simplest examples. The simplest class will be found to be the spirate fricatives; and one of the simplest of this class is *f*. The conditions of the production of *f* are (1) closure of *nares*, (2) larynx freely open and breathing, (3) a non-frictional passage between tongue and palate, (4) a frictional constriction between lower teeth and upper lip. The words frictional and non-frictional are (and will be) used to indicate that the issuing breath does or does not produce audible friction. The above conditions are exhaustive; and they are essential. If the nose is open, the breath flows out that way, either silently or in a nasal aspirate; if the larynx is open, but not breathing, there is silence; if it is breathing, but not freely open, there is either tone or whisper, and the *f* is transformed into a toned or whispered *v*; if the tongue is moved up, so as to make the inner passage at all frictional, the *f* is changed into *θ*; and if the outer constriction is relaxed till it ceases to be markedly frictional, the *f* melts away into a kind of *h*. It ought to be possible, starting from these completely determined conditions of *f*, to discover something about the resulting sound.

In *f* there is but one source of sound; it is the frictional noise of the breath struggling through the narrow interstices between the teeth and the lip. But this noise awakens resonance in the cavity behind; and though this resonance has not the perfect regularity of a musical tone it is regular enough to convey a less definite sensation of its own frequency, *i.e.*, of its pitch. In this respect, *f* serves as a preliminary type of all fricative consonants. Every fricative consonant contains frictional noise, and resonance consequent upon it. The simplicity of *f* is that it has friction in one place only, resonance apparently in one cavity only, and no imperative alteration either in the friction or in the resonance, *i.e.*, neither of them need change in character from beginning to end of the consonant. These fricative resonances have some important features in common. One and all, they are prompted by irregular noises, in cavities of somewhat irregular shape. Such cavities never have an exact proper tone, like a pipe, or a string, or a tuning-fork; they respond to any stimulus which is sufficiently near to their proper tone. Hence the character which these resonances present to the ear. They are weak through mutual interference; they are indefinite, so that it is impossible to fix their position in the octave by ear to an exact semitone; and to what octave they belong it is always utterly impossible to say. Nevertheless, when they are forced by any change of articulation to change their pitch, the change in the whole body of resonance is felt very accurately. When the resonance of *f* is carefully listened to, it is found to be far from constant in pitch. The resonance of *f* in *fee* has, in my own case, the pitch *a*, octave uncertain; that of *f* in *fall* is about *f*—five semitones lower. But it is best to study the consonant *f* apart from any vowel.

It is a further good reason for beginning with *f* and those fricatives which most resemble it, that they can be produced in isolation, and prolonged *ad libitum* without at all losing their characteristic effects on the ear. It is a long time before the phonetic observer acquires the conscious separate command of each of his organs—velum, nares, larynx and tongue. After the consciousness of their separate action comes the consciousness of their several positions, at any given time. This consciousness grows in accuracy by actual inspection and handling, and by nothing else. But it is at first quite unreliable, and must be trusted no further than it can be tested by the mirror, the probe or the finger, or by diagrams set out from the testimony of these. When these cautions are observed, the following observations may be carried out concerning *f*.

When freed from connection with any vowel, the resonance of *f* can be carried a long way both up and down in pitch, without at all spoiling the *f* itself. Its whole range in the individual speaker is about two octaves—an octave each way from the *f* in *fall*. Add to this range another octave for the difference between infant and adult resonances, and it becomes clear that the essential quality of *f* is but vaguely linked with the actual pitch of its resonance. The quest made in that direction is therefore a vain one: we must look elsewhere for the acoustic essence of *f*, or of any other fricative. But it is not vain to ask whence that resonance comes. It seems clearly to arise in that free passage between the tongue and palate which is an essential part of the articulation of *f*. This is evidenced by the fact that in the upper octave of its range this passage lengthens or shortens itself in proportion to the fall or rise of the pitch of the resonance. It is impossible to lengthen the passage any more than this; and the further fall is secured by bulging the passage, till at last it becomes a considerable cavity. The upper octave, with its simple lengthening and shortening of tube, affords some criterion of the real pitch of the resonance. In my case, the tube at its longest is about 100 mm., at its shortest about 50 mm.

Its effective length will always be rather longer, because it has some width, and does not open at both ends abruptly into free air. The highest resonance in my case is therefore doubtless, f^4, 2816 v.d.; and the lowest, f^2, 704 v.d. The former is the pitch of a narrow pipe 60 mm. long.

The whole sound of *f*, in isolation, appears to consist of this single resonance and the original friction at the teeth. Three other sounds in our table seem to enjoy an equal simplicity of composition. Not till these are being treated can we profitably ask wherein *f* differs acoustically from every other consonant.

θ.

This is the *th* in *thin*. Its relationship to *f* is shown by the fact, already stated, that if, during the articulation of an ordinary *f*, the point and blade of the tongue are moved slightly upwards, so as to create an aerial friction between the tongue and the upper teeth, the *f* is then transformed into *θ*. The resonance rises at the same time about a semitone. This appears to be due only to the slight fore-shortening of the tube, which now ends inside rather than outside of the upper teeth. This slight difference persists in connection with given vowels, *e.g.*, my *θ* in *theme* gives b^3, 1980 v.d.; in *thaw* it gives $f^3\sharp$, 1491 v.d. With these compare those of *f* in *fee* and *fall*. But when *θ* is tried in isolation it is found to have a rather less range of possible resonance than *f*. The reason for this, however, is not acoustic, but physiological: the tongue-tip is no longer free; it must remain always in close proximity to the upper teeth. This stiffer position prevents the tongue from acting so freely as before: it especially prevents that sinking and withdrawal of the tongue-tip, by which the bulging of the passage can be carried so far in *f*. Thus the range of the *θ* resonance is shortened considerably at the lower end. In my case it reaches only from a^2, 880 v.d. to f^4, 2816 v.d.

There is no evidence that *θ* contains more than two audible elements, namely, this resonance and the original friction: and there is no essential difference in pitch between this resonance and that of *f*. Such differences as do exist seem due to physiological accident only. Where, then, is the acoustic difference between *θ* and *f*? There is an undoubted difference in the original friction. In *f* the length of the frictional passage is at most the thickness of the teeth, about 3 mm. But in *θ* it is from 4 to 6 times this length: the resistance to the breath is much greater: and these conditions are essential; for if the application of the tongue is relaxed in such a way that friction only takes place against the tip, and not also against the blade of the tongue, the sound becomes much more like *f* than *θ*. Whether this difference in the exciting noise leads also to any marked difference in the resonance excited, it would be premature to say. The ear testifies that in passing from *f* to *θ*, the slight rise in pitch is accompanied by a great increase in shrillness of quality. This may be due only to the new friction. But the narrowing of the tube has made it more capable of producing an overtone. Some trace of the first overtone might reasonably be looked for.

Here the matter must rest until the phonographic traces of *f* and *θ* have been sufficiently magnified and accurately compared. The successes of Dr Boeke and M. Marichelle in obtaining magnified photographs of phonographic tracings justify great hopes of successful comparison. But such a comparison will be very difficult to follow out, unless care is taken that the resonance is at the same pitch in both examples. This might be secured pretty nearly by recording them in connection with an identical vowel. But there are pitfalls in all pronunciations of connected sounds, which will be noted by and by; if they could be recorded at the same pitch in isolation, it would be much better. The frictional noises will doubtless come out in the phonogram as distortions of the resonance. These ought to be much longer and deeper in *θ* than in *f*; and if there is an overtone, it ought to show itself in a tendency of the resonantal vibrations to flatten or notch themselves in the middle.

x and χ.

These are the sounds of *ch* in the Scotch words *licht* and *loch*. They are heard also in German words like *ich* and *doch*. They are very much alike in sound, and are best examined together; for they differ little in analysis, except in the actual pitch of resonance. But German phoneticians agree in representing them by two different signs, and there is in practice a certain physiological gap between them, which will be explained later. As in *f* and *θ*, the conditions of their production are four. Two of these are the same as in those cases, namely closed nares, and a larynx fully open and breathing; and the other two are identical in kind, namely, a frictional constriction and a resonant passage. But there is this broad distinction, that the frictional constriction is now situated, not at the outer, but at the inner end of the resonant passage. This has an important influence on the acoustic result. The frictional noises of *f* and *θ* work backwards upon the resonant passage; but they also work forwards to the listening ear, and make their own direct, unmodified impression upon it. In *x* and χ this is not so. The frictional noises can only reach the ear through the resonant passage, and the result is that they are all more or less damped, except those to which the resonance of the passage just happens to respond; and these merge themselves into the rest of the resonance. The longer the passage the greater is the damping of the noises, and the nearer does the resonance approach to the regularity of musical tone. Hence the resonance of χ is notably less rough in quality, and more definite in pitch, than that of *x*. But this does not appear to arise from any considerable difference in the original noise. The χ constriction is formed against the soft palate, and the *x* constriction against the hard palate; but the degree of obstruction and the length of the frictional passage seem to be fairly constant, and to resemble those of *θ* rather than those of *f*. The *dorsum* of the tongue is incapable of framing a short frictional orifice against the palate, like that framed by *f* against the teeth. When the resonances are listened to in isolation there is found to be no necessary gap between *x* and χ: they extend in me without a break from $f^4\sharp$, 2982 V.D. down to c^2 528 V.D.; but this lower limit can only be reached by vigorous rounding and protrusion of the lips. The resonant passage is found at the same time to undergo a progressive change of shape, very similar to that described for *f* and *θ*; excepting always that its wider aperture is no longer found behind, but before. For this reason it is much easier of inspection, and its changes can be noted by the most unassisted observer. Its range of transformation is greater than that of *θ*, because the tongue-tip is again free, and the bulging of the passage can be carried out to its fullest extent. Measurements in the upper octave of resonance are much the same as before. When the resonance is f^4, the length of passage is about 50 mm.; an octave lower, it is about 100 mm.: the lower octave and a half are obtained by bulging, as in f. In the upper octave the difference between an f passage, a θ passage and a x passage is very simple and obvious: the θ passage sharply converges; the χ passage sharply diverges, and the f passage does neither. After the changes of this upper octave have been noted in the x and χ passage by the eye, those of the θ passage are readily imagined, for the same octave, by conceiving a reversal of the passage, end for end. The cause which creates in practice a certain gap between x and χ is the organic facility of creating a frictional passage, either (1) just behind the alveolars, or (2) between the post-dorsum and the velum. The so-called "front" vowels, such as *i* and *e*, connect themselves most easily with the former; but the "back" vowels, *a*, *o*, *u*, with the latter. Thus it is the vowel in *licht* and *loch* which determines the consonant, making the one x and the other χ.

A question here suggests itself to the investigator, which must be considered before going further. The resonances of f and θ came from behind the constriction. How do we know that no resonance comes from behind the constriction of x and χ? To this question it is impossible to answer quite positively. The ear does not recognise any such secondary element. But a weak secondary element might conceivably be there, and be essential to the x and χ, though not separately audible. In whispered fricatives there is such a secondary element, which at first escapes separate recognition, but may easily be felt by comparing f with whispered *v*, or θ with whispered ð, or any similar pair. The secondary resonance in x or χ, if it exists, must be weaker still, for it eludes the ear under all circumstances.

Moreover in x and χ, the state of things inside the constriction is not what it is in f and θ. Inside the constriction of f and θ it is essential that a resonant tube shall have been framed; inside that of x or χ the formation of such a tube seems to be sedulously avoided. The observer, feeling inside the x constriction, is surprised to find how abruptly it terminates inwardly: the tongue slopes down as nearly perpendicularly as it can. In the higher pitches of x, as the passage outside the constriction grows shorter and shorter, this abrupt decline of the tongue inside the constriction is maintained by a strong pull of the muscles running from the inner surface of the lower jaw to the tongue-bone and root of the tongue. The growing contraction of these muscles is to be easily felt outside, when the resonance of *x* is raised step by step to its highest. It seems to be of the essence, therefore, of *x* and χ, that their constrictions shall open as abruptly as may be into the pharyngeal cavity; and thus the formation of a resonant tube inside the constriction, resembling that of *f* or *θ*, is avoided, apparently with intention and effort. Something of the same kind happens with *f* and *θ*. Their resonant passages cannot open so abruptly into the pharynx as that of *x*, because that is not their constricted end: but the same sharp declivity may be felt at the inner end of either passage; and the same co-operation of the muscles under the jaw may be felt externally, when the pitch of either resonance is carried towards its upward limit. If any of these consonants, *f*, *θ*, *x*, χ, does possess an inner, weaker resonance, it is certainly of low pitch; for the larynx is wide open, the body of air to be agitated extends right down to the lungs, and its aperture is relatively small. It remains to be seen whether any such elements can be detected in phonograms. As for the principal resonance of *x* and χ, it does not differ essentially from that of *f* or *θ*; the difference must again be sought, not in the resonance itself, but in the nature of its distortions. These will probably be less random and less violent than those of either *f* or *θ*, especially in χ.

h.

The production of the consonant *h* reposes upon four primary conditions, two of them again identical with, and the other two similar in kind to, those for *f*, *θ*, *x*, and χ; but it differs from any of these in its less degree of constriction, and its greater force of breath. These two conditions hang necessarily together; for the *h* passage would, as a rule, on account of its wideness, let the breath pass without any audible friction, if it were not expelled with more than ordinary force. Hence comes the name *aspirate*, which is used to distinguish *h* and all other feebly constricted spirates from those of closer constriction. Aspirate sounds other than *h* are rare in English, but will be found to demand attention whenever the acoustic nature of *l*, *m*, *n*, *η*, *r*, *ɹ*, *p*, *t*, or *k*, is studied. In *h*, as in each consonant treated hitherto, our first aim must be to define the two variable conditions—those which serve to differentiate it from every fricative previously described. These are (1) the friction, and (2) the resonance. It is the distinguishing feature of the *h* friction that it does not take place at any definite point: in fact it is convenient to regard as the purest or most typical *h* that kind of aspirate which creates friction at all points of the voice-passage, from the pharynx to the external air. This implies that in such an *h* the degree of obstruction is fairly uniform; and this in turn implies that the resonant passage is of fairly uniform calibre throughout. This position of the organs does not materially differ from that which they naturally assume in sighing; and thus the pitch of a sigh differs little from that of this typical *h*. In me it seems to be about e^3 1320 V.D.

Leaving *h* for a moment, it may be remarked that not only *h*, but every fricative, tends to what may be called a neutral type, when pronounced in isolation. Underlying all our articulations, there is the principle of economy of adjustment. When we start from a state of rest and end in a state of rest, the articulation departs usually from the position of rest, the ordinary silent position, so far as is necessary, and no further. Hence *f*, θ, and χ, no less than *h*, have what may be called a neutral pitch in every individual. In myself I find that this neutral pitch is, for *f*, about f^3, 1408 V.D.; for χ, $f^3\sharp$, 1491 V.D.; and for θ, g^3 1584 V.D. This steady rise from *h* to θ corresponds well with the changes in the oral passage. In *h* the passage embraces the whole distance from the pharynx to the external air, and it is wide all through; in *f* it is slightly foreshortened, and much narrowed at one end: in χ it recovers a little length forward, but loses considerably in length behind; and in θ foreshortening again occurs in stronger form than in *f*; and the advance of the tongue-tip draws the whole tongue forward, and causes the pharyngeal cavity to extend itself part way up the back of the tongue.

But in *h* this neutral or isolated value has some special features. It is true that the articulation and pitch of *h*, even more than that of *f*, θ, *x*, or χ, is pulled hither and thither, in actual speech, by the adjacent phones. But, through all these changes, *f* is still *f*, θ is θ, and nothing more: *x* and χ are still either *x* or χ: but *h* is no longer quite neutral *h* to the ear, when it is drawn much away from this unbiassed articulation. And the reason is clear. This neutral *h* keeps the upper and lower sides of the voice-passage as far as possible, parallel and equidistant. We cannot depart very far from that arrangement in any direction, without making the passage distinctly narrower in some places than in others; and if narrower, then more obstructive and more frictional. Whenever the friction of *h* is thus localised, it at once departs more or less from the simple neutral sound of *h*. Yet this is the kind of *h* which usually occurs in actual speech. Whether the departure from type can be recognised by the hearer or not, depends on the extent of the departure, and the acuteness of the listening ear. One kind of departure is patent to everybody: it is when the constriction is so local and so tight as to transform the *h* into a *x* or χ. Precise and forcible speakers may often be heard to pronounce *he who* as *xī* χ*ū*. But A. J. Ellis, the translator of Helmholtz, noticed many years ago that a mere ordinary *h* often carries the impress of the following vowel. In other words, if the pronunciation of a word beginning with *h* is arrested before the vowel is actually begun, the ear can often recognise, from the *h* alone, what vowel was going to follow.

This phenomenon is only one instance of a habit of language so wide-spread that it may be called a law. It is simply that the organs are always endeavouring during the articulation of any phone, so far as the conditions of that articulation will allow, to put themselves into the readiest position for commencing the articulation of the next phone. Conceive in your mind the intention of pronouncing the word *blue*: you will find your tongue in the *l* position before you have even begun to sound the *b*. In the phenomenon called *umlaut*, so important to all the Germanic languages, this influence extends even further back; for it modifies the preceding vowel. The reason why our *h* conveys, as a rule, not only the sound of *h*, but an inference of the following vowel, is simply that the organs have already, during the *h*, assumed something like the prospective vowel-position; so much so, indeed, that the oral or forward resonance of the vowel is produced and heard, with the distinctive breathy timbre of the *h*. Thus a quick ear infers the vowel, though it does not really hear it; for the appropriate hinder (pharyngeal) resonance of the vowel is wanting, being impossible with an open larynx. Even the *h* which is here called neutral carries with it the same inference in actual speech, for it arises before the "neutral" vowel ʌ (see *Proceedings* of this Society, vol. xxii. p. 99).

Whether there is any hinder resonance in *h* is another question. The only thing certain is that, if present, it is not the same as the hinder resonance of the following vowel. The openness of the constriction favours the exit of such a resonance: but the ear does not testify to the presence of any second resonance, though on whispering *hi*, *ha*, *ho*, etc., it does recognise easily that, in passing from *h* to the vowel, a new (hinder) resonance has been added. We arrive therefore at the same conclusion for *h* as for *f*, *θ*, *x*, and χ, that, if it has a second resonance, it is deep and very feeble.

It follows from what has just been said that, in connected speech, the resonance of *h* will be simply the oral resonance of the following vowel. Certain qualifications of this statement must be made later; it is absolutely true only of that part of *h* which immediately adjoins the vowel. But this shows that the resonance of *h* has at any rate the same range as that of the oral resonance in vowels, which I have tabulated from phonographic data (*Journal of Anat. and Phys.*, vol. xxxi. p. 251) as extending, in adult male organs from f^4 2816 V.D. down to d^1 287 V.D. This result is roughly corroborated by the observation that every fricative yet studied can be turned into an *h* by sufficiently relaxing its constriction, and reinforcing the breath. This observation is best carried out with *x* and χ, because their range is the widest ($f^4\sharp$, 2982 V.D. down to c^2 528 V.D.). When their articulations are relaxed into the *h* position a fall of resonance can always be heard; because the passage is always widened and lengthened. But the fall is unequal, because the widening and lengthening are unequal; they are greater in χ than in *x*. Hence the lower limit of the *h* resonance drops further below that of the χ resonance than its higher limit drops below that of the *x* resonance.

Still, in the greater part of its range, the resonance of *h* covers again the same ground which we have seen to be covered by *f*, *θ*, *x* and χ: and again the characteristic of the consonant is found, not in the pitch of the resonance, but in the nature of its modifications and distortions. Wherein, then, do these differ? The ear affirms at once that there is a total loss of those high, shrill, hissing vibrations which are produced by the forced passage of air through narrow apertures. When we are able to compare sufficiently magnified phonograms of *x* or χ and *h* (if possible, at identical pitches of resonance), we shall probably find the distortions of *h* to be relatively long and gentle. Overtones, also, are apparently not to be expected. The fact that the friction does not originate in one place, but traverses greater and less portions of the voice-passage may have some characteristic effect on the phonogram, but of what kind it would be rash to prophesy. The phonogram of this weak sound *h* will probably be very difficult to obtain in a very clear form. It will be easier to recognise that frictional disturbance is weaker in the resonance of *h* than in the other fricatives here studied than to find out the nature of the disturbance itself.

s and *ʃ*.

These are the sounds of *s* in *see* or *so*, and of *sh* in *she* or *show*. It is convenient, at this stage, to treat them together, because they have a curious parallelism, resembling that which has been seen to exist in *f* and *θ*. But they are just as distinct from each other as *f* and *θ*, and far more distinct than *x* and χ, which are really complementary rather than parallel, and together constitute only one series of similar sounds. In the table of consonants already given, it is seen at a glance that the articulation, both of *s* and *ʃ*, is somewhat more complex than that of *f*, *θ*, *x* or χ. Each of them has two constrictions; it has not only a close approach of a certain part of the tongue to a certain part of the hard palate and gums, but it has also an approach of the lower to the upper teeth. When this second constriction is not properly carried out, both *s* and *ʃ* become much feebler, and lose nearly all their distinctive timbre. The organic effect of these two constrictions is to create between them in both consonants a fore-cavity of small size and extremely irregular shape. The shape of its aperture is more irregular still, being framed between the opposed edges of the teeth. The office of this cavity does not appear until the rest of the articulation is explored.

But even when the inner constriction, and what lies behind it, is examined, the essential difference between *s* and *ʃ* escapes observation at first. It is easy to discover that in both of these consonants (as in *f* and *θ*, but not as in *x* and *χ*) there is a resonant passage behind the constriction. This passage is less accessible to observation than any such passage that has been yet examined. But it is evidently subject to exactly the same kind of modifications which have been seen in all the other cases. The resonance can be made to move, by modifications of the passage, through two octaves of pitch. And that the modifications are of the same kind as before, is evidenced externally, at the upper end of the scale by the same tension in the muscles beneath the jaw, and at the lower end of the scale by the same vigorous rounding and protrusion of the lips. Internally also the lengthening and shortening of passage which creates the upper octave of resonance can be directly observed; and though the bulging of passage which creates the lower octave is less accessible to direct observation, the same muscular feelings are experienced as in former cases. Therefore when the resonance is found to extend through two octaves, from *g* to *g*, we conclude as before that the real range is from g^4 3168 V.D. down to g^2 792 V.D. We expect a slight rise above previous resonances, because the passage has again been foreshortened by a few millimetres, through the retreat of its orifice from the teeth to the alveolar ridge, and the result stated is consistent with that expectation.

Up to this point there is nothing to indicate any difference between *s* and *ʃ*. There are accidental differences to be noted presently; but just as in *f* and *θ*, so here no essential difference is to be found between those two consonants, except in the nature of the frictional exit from the resonant passage. In *ʃ* this orifice is framed on a much larger scale than in *s*. It is not only wider, but it extends much further back. The *s* orifice is as short a one as the tongue can possibly frame. It is also framed against the ridge of the alveolars; so that the inner passage widens immediately into non-frictional dimensions. But if this *s* is changed into *ʃ*, the tongue at once arches itself upwards, so that the whole fore-tongue is brought close to the upper slope of the alveolars, and thus a frictional channel far longer than that of *s*—longer even than that of *θ*—is created. The noises created in such a passage are naturally deeper and stronger than those of *s*. It is doubtless in compensation for the greater resistance of so long a channel that it is at the same time made less narrow than that of *s*.

During these observations the office of the fore-cavity begins to reveal itself. It appears to act chiefly as a resonator. This fact does not come to light in ordinary circumstances. In fact most investigators have thought that the chief phenomenon involved was the impact of the out-going air upon the teeth; for within ordinary limits there is no sign that this resonator is adjusted in any way to reinforce the sounds poured into it from the inner passage. But that is apparently because a cavity so irregular in shape, and of such irregular orifice, has a certain considerable range of resonance, within which it will continue to respond fairly well, without readjustment. But when either *s* or *ʃ* is carried through the whole range of its resonance, there is a very palpable instinctive readjustment of this fore-cavity, and especially of its orifice. When the resonance, either of *s* or *ʃ*, is carried to the highest point, there is an evident effort to increase and disencumber the orifice, by widening and spreading the lips. But when, on the other hand, it is carried to the lowest point, there is strong effort to diminish the orifice and to increase its resistance, by pursing and protruding the lips. The aim of these adjustments seems manifestly to be a kind of tuning, whereby the proper pitch of the cavity is roughly adjusted, so as to reinforce the sounds which it is receiving from the inner passage.

It may be remarked in passing that the German (*sch*) is frequently articulated with rounding and protrusion of lips. It is therefore probable that it has often a lower resonance than the normal English *ʃ*, about to be described. The neutral *ʃ* and the neutral *s* have greater importance than any of the sounds hitherto called neutral; because neither *s* nor *ʃ* allows itself to be so readily drawn away from its neutral articulation (and for the individual, neutral resonance) as any consonant yet treated. Already, in carrying either *s* or *ʃ* through the whole range of its resonance, the articulations at both ends of the scale, but especially at the lower end, have been found more laborious; whilst the resulting sounds have been less powerful. Why should these two consonants tend more strongly to a fixed type of articulation than any previously described?

It is simply that there are now two resonances instead of one, and that these two resonances have to be adjusted so as to reinforce each other. It is no longer permissible to arrange the available air-space into any shape which will produce a resonance; it must be arranged into two portions whose resonances shall be nearly identical. This is a problem which, were it not for the above-named devices of spreading and pursing the lips, and for the considerable range which the resonance of the fore-cavity possesses, could only be satisfied by one specific type of articulation. Such an articulation would be as fixed as that of a vowel, and for the same reason—that the resonances are not severally free, but must be more or less accurately adjusted to each other. In English, in fact, this seems to be almost the case with *s* and *ʃ*; for all English speakers have a strong aversion to employ either rounding or protrusion of lips in ordinary speech; and, though the range of the resonance of the fore-cavity is considerable, there is always a natural preference for the middle of that range, because it gives the strongest resonance and the most forcible consonant. Hence the resonances both of *s* and *ʃ* appear to be far less mobile, in English at least, than the resonances previously studied. They are drawn aside like the others, no doubt, by adjacent phones, but how far and how quickly can only be ascertained from a study of connected phonograms.

When it is endeavoured to determine for *s* and *ʃ* respectively this neutral type, a curious difference reveals itself—the accidental difference already referred to; the neutral pitch of *ʃ* is some 4 or 5 semitones lower than that of *s*. This is heard at once when *s* and *ʃ* are articulated in immediate succession, in either order; and a careful comparison of the articulations discloses the reason. It is evident that the upward arching of the fore-tongue, which takes place in passing from *s* to *ʃ* (see *ante*) disposes the organs to the formation of a longer palatal passage; whilst in *s* the necessity of avoiding friction in that passage causes the tongue to hold off somewhat from the palate, and disposes it to trend more rapidly downwards into the open pharynx. Thus it happens that the most convenient *ʃ* passage is about $\frac{1}{4}$ longer than the most convenient *s* passage, and its pitch about 4 semitones deeper. An additional convenience is that the same movement tunes the fore-cavity also; for it draws the tongue-tip 2 or 3 mm. backwards, it enlarges the space under the tongue, and thus creates the required fall of pitch in the fore-cavity without any other movement whatever.

My neutral *s* has a pitch of $e^4\flat$ 2500 v.d. to e^4 2640 v.d., and my neutral ʃ, one of $b^3\flat$ 1980 to c^4 2112 v.d. The remarkable thing about *s* is that its neutral pitch is nowhere near the middle of its possible range. The other consonants, it is true, have their neutral pitch somewhat above the middle of their range, but the neutral pitch of *s* is within 3 or 4 semitones of its highest extreme. The old tinfoil phonograph failed more conspicuously to reproduce the keen sounds of *i* and *s* than any other phones. It is, therefore, quite according to expectation when we find *s* to possess ordinarily the highest resonance of all consonants, and *i* of all vowels, and when we also find that their height is not dissimilar (*i* has one resonance about f^4 2816 v.d., see table *infra*).

But it must be always remembered that this high pitch is not of the essence of the sensation called *s*: it is merely an accident, and not even a universal or necessary accident, of human physiology. Not only *s* and ʃ, but every consonant yet treated, can be produced by human organs at identical pitches of resonance. Wherein then does their difference from the rest, and from each other, reside? It is easiest to answer this question by remembering that *s* differs from ʃ, and ʃ differs from *θ*, chiefly in the possession of a resonant fore-cavity. It is the reinforcement derived from this cavity which gives to both of them their great superiority in sonority over all other spirate fricatives. This fact has its counterpart in the relative depth and distinctness of their phonograms. But the difference in kind between a phonogram of ʃ and a phonogram of *θ* must be sought, as before, in the distortions of the resonance. So also, in a less accurate sense, with *s* and ʃ: for there may in this case be some slight difference in the original friction made against a hard body like the teeth and a soft body like the alveolars, respectively. But whilst in *f* and *θ* the friction was exterior, and its noises came to the ear in their crude original state, the frictions of *s* and ʃ are interior, and their noises can only reach the ear through the resonant cavity, which must modify them immensely. An influence of this kind has been already noted in the case of *x* and χ: but the influence of the fore-cavity of *s* and *ʃ* is probably much stronger than that of the divergent tube of *x* and χ. The effect of that influence will be to damp all the frictional noises which are either graver or more acute than the range of the resonance of the cavity; whilst all noises within that range will be more or less exaggerated.

In the phonogram of *ʃ*, therefore, we ought perhaps to find the resonance subject to very strong distortions, but none of them either much longer or much shorter than the waves of the resonance itself. The same remark does not quite apply to *s*; the original frictional noises are so much acuter and weaker as not to afford the groundwork for similar results. The frictional noises are perhaps renewed at the teeth. In any case the ear seems to testify that they survive in an acuter form than those of *ʃ*. Here, as elsewhere, the appearance of good phonograms must be awaited.

Effects of Combination: Glides.

It is necessary to say something here about the effects of combination with other phones upon the acoustic composition of the class of consonants just treated. For though the spirate fricatives, and, indeed, all fricative consonants, can be produced and studied in an isolated form, they are never found isolated in actual speech. The very name 'consonant' indicates that in ordinary language they are never sounded alone. There are some consonants which it is difficult, or even impossible, to sound alone. Such sounds are essentially connective and transitional. Unlike the fricatives, they never consist, and never can consist, of a succession of similar sounds: it is of their essence to change continuously; in other words, they are not continuant, but gliding. But there are conditions of combination in actual speech, as will perhaps have been already gathered, which cause the fricatives themselves to glide through very considerable changes, even in the duration of a single utterance of any one of them. We cannot consider their possible combinations at this point exhaustively, because all other consonants remain yet to be explored; but we can consider their combinations with vowels, which are not only in themselves the most important combinations, but which also serve, for the reason above indicated, to facilitate a subsequent study of those consonants which are always and essentially of a gliding nature.

Limited as our problem here is, three classes of cases at once offer themselves for solution: (1) that of a spirate fricative beginning from silence and ending in a vowel, (2) that of such a fricative leading from a vowel to silence, (3) that of it joining two vowels. The last case varies greatly. If the two vowels connected are identical, this case is simpler than the other two: if not, it is more complex. Let us start from the simplest case, remembering always that the principle which rules the process of articulation is economy of adjustment—so long as that adjustment suffices to attain the sound desired.

Before attacking this problem it is necessary to lay aside certain prepossessions, derived from orthography, which, from an acoustic point of view, are more or less misleading. We are apt to think that the combinations *asa* or *oso* contain but three sounds each, *a-s·a* and *o·s-o*; and the ear seems to confirm this impression. But a little consideration shows that there are really five sounds in each case: there are not only two vowels and an intervening consonant, but also two brief, yet inevitable, gliding sounds, the one connecting the first vowel to the consonant, and the other connecting the consonant to the second vowel. When the pace of articulation is sufficiently relaxed, the existence of these glides becomes quite evident to the ear; and the reason of their existence becomes evident too. The posture of the organs in the *a* or *o* articulations is very different from their posture in *s*. In passing from *a* or *o* to *s*, two principal movements have to be carried out—a complete opening of the larynx, and a raising of the tongue-tip some 12 or 20 mm. These movements are, of course, attempted simultaneously. It is the duration of the longer one, therefore, which determines the duration of the glide. In this case the tongue-adjustment probably takes the longer time of the two. There is generally an instinctive effort to make glides as short as possible. They are by-products, necessary evils, which the speaker produces, one may say, against his will, and stifles as far as he is able. As a rule, he succeeds in making them so brief that they are not separately audible; but they still exist in a sub-sensible form, and constitute no negligible fraction of the duration of the phonogram. Consider that the whole duration of the *s* may be only $\frac{1}{10}$ sec., and compare that with any possible rapidity of the transitional motions which create the glide: the glide must be, in this case at least, half as long as the consonant. How is it, then, that the glide is not separately heard? This arises chiefly from the persistence of auditory sensations. The sensation of the vowel does not subjectively cease at the point where the vowel vibrations terminate in the phonogram. It persists during the whole duration of the glide, and practically blots it out by its more powerful timbre. It follows, however, from the principle that the duration of the glide is the duration of the longest movement involved, that glides are not normally of at all equal length. The combination *bl* in *blue*, already instanced, is practically glideless, because the *l* articulation can be, and is, framed inside the *b* articulation, and simultaneously with it. In combinations of *h* + vowel, there is a real glide, because the larynx must have time to shut before the *h* can be transformed into a toned vowel; and there is inevitably a certain portion of this time during which the *h* has been spoiled by the narrowing of the glottis, though the chords are not yet vibrating, and the toned vowel is, therefore, not yet begun. But this period must be exceedingly short; and in speaking of such combinations in a former passage we have neglected it (see also *infra*). But sometimes the transitional movement is much more cumbrous than these, and takes much longer time. At times, indeed, it is impossible, by any attainable rapidity, to prevent the glide being audible to a quick ear. Take the English word, *eel.* During the *ee* the tongue is presented convex to the hard palate, with a passage of some 50 sq. mm. section between them. For *l* this must be entirely changed. The tongue-tip must be withdrawn to the alveolars; the passage must be shut up, and new passages opened at each side; the convex curvature of the tongue-blade must be exchanged for one slightly concave. Hence the sound of the *ee* and the sound of the *l* are never contiguous, and the dull intervening sound is always long enough to be separately apprehended by a quick ear. In French *fil* this is not quite the case. The French *l* is "dorsal," not "coronal," *i.e.*, it is articulated with the blade, not the tip, of the tongue against the alveolars, and does not require so great a change of curvature.

But something depends here on the speaker. Some speakers exert themselves much more to minimise their glides than others. Southern English pronunciation is conspicuously less alert than Northern English in this respect; and French is more alert than either. It is through this fondness for gliding articulations in Southern English that nearly all its vowels have developed a "tail" of obscurer sound which almost makes them into diphthongs.

Besides these preliminary remarks on glides in general, it is necessary also to make some general remarks about vowels. I have elsewhere (*Phonetische Studien,* 1890–2) examined the articulations of all the cardinal vowels in detail, and have shown that they appear to be in every case designed to produce at least two resonances, the one proceeding from the oral, and the other from the pharyngeal, part of the articulation. For our present problem it suffices to know very little about the latter, but the details of the former are important. The following list of oral (adult male) resonances is mainly compiled from my table of such resonances, all calculated from phonographic data of sung vowels, in *Journal of Anat. and Phys.*, vol. xxxi. p. 251. But these agreed well with those which I had previously derived (*loc. cit.*) from direct observation. I have, therefore, added, from direct observation, two resonances of English vowels (marked *) which are wanting in the other list.

Kind of Vowel.	Approximate English Key-word.	Oral Resonance.
Close *i*	mar*i*ne	f^4 v.d. 2816
* Open *i*	p*i*t	$d^4\sharp$ 2500
Close *e*	r*e*in	c^4 2112
Open *e*	th*e*re	$f^3\sharp$ 1508
Front *a*	m*a*n	f^3 1431
Back *a*	f*a*ther	c^3 1082
* Open *o*	l*aw*	$g^2\sharp$ 834
Close *o*	note	$d^2 - a^1$ 623–444
Open *u*	p*u*t	c^2 528
Close *u*	br*u*te	$c^1 - d^1$ 314–287

What strikes the eye at once in this list is that the range possible to the oral resonance of the vowels covers largely the same ground as the range possible to the several resonances of each of the spirate fricatives themselves. And the reason is not far to seek.

In *Phonetische Studien*, vol. iii. p. 264, I showed that the mechanism which produces the declining scale of resonance exhibited in this table is precisely that which has been seen to vary the resonance of each of these spirate fricative consonants. The oral articulation of close *i* is a pipe-like passage along the palate, about 50 mm. long. This passage increases up to open *e*, when it is about 100 mm. long. The further fall of resonance is produced step by step, by bulging the passage, and contracting gradually its two apertures—the labial, by which it opens into the outer air, and the velar, by which it communicates with the pharynx. The diagrams in *Journal of Anat. and Phys.*, vol. xxxi., show the same thing.

The passage in which these modifications are made is almost identically the same passage in which, by similar modifications, the resonance of each spirate fricative has been conducted through a series of similar changes (see *ante*). *There is, therefore, as a rule, for every spirate fricative, a possible articulation approximating more or less closely, in resonance, form, and position, to the oral articulation of any vowel which may happen to be adjacent to it.*

Recurring now to combinations of the type found to be simplest (*asa*, *oho*, *ifi*, etc.), it is evident, from the principle of economy, that the articulation of a consonant so situated will depart as little as possible from that of the vowel by which it is flanked on both sides. But when it is asked how much this least possible amounts to, the answer varies somewhat with the vowel, and still more with the consonant. There is one region of the articulation, however, where the same things happen always in combinations of this class; this is the pharyngeal region. Four things always happen there, the closing of the larynx to form the first vowel, the opening of it to form the consonant, the closing of it again for the second vowel, and the opening of it at the finish. It is the second and third of these four which concern us here, because they help to create the on-glide and the off-glide, respectively, of the fricative consonant. But there is in most cases another element in both of these glides, resulting from the changes taking place simultaneously in the oral part of the articulation; and where this stronger element is present, it will perhaps be hopeless to look for any distinguishable traces of the brief and feeble glides developed in the pharynx.

But in the combinations of *h*, *ihi*, *ehe*, *aha*, *oho*, *uhu*, these glides may possibly be decipherable, because they are the only necessary glides involved. The oral articulation of *h* need not change in the least from that of the vowel by which it is flanked. The only necessary change is the opening and shutting of the larynx. The opening of the larynx puts an end to two things,—the tonic vibrations of the chords, and the resonance of the pharyngeal cavity. But the latter, at least, will not perish quite instantaneously; it has a period of 280 to 800 V.D., and doubtless has time to undergo a rapid fall of force and change of period, even during the swift opening of the chords.

Next in simplicity is the case of *f* and its compounds, *ifi*, *efe*, *afa*, *ofo*, *ufu*. One effect of the remoteness and disconnection of the lips from the other vocal organs is that the articulation of *f* and all other labials interferes relatively little with that of other kinds of sound with which they are associated. The only movement which takes place in the cases here to be considered is a slight raising of the lower jaw and lower lip, enough to bring the latter into contact with the tips of the upper teeth. This raising is greatest for *afa* and *ofo*, because the jaw is lowest in *a* and *o*; and it is least for *ifi* and *ufu*. The reduction of the labial orifice will tend in every case to produce some drop in the oral resonance; but the greatest reductions (in *a* and *o*) will also have great compensations in the reduction of the bulk of the oral cavity. In no case, probably, will the resonance of the *f* be carried very far away from the oral resonance of the adjacent vowel, except perhaps in *ufu*; for *u* has often an oral resonance lower than that of any possible *f*. But note what is said below about *uθu*, etc.

Very similar results hold good for *θ*, when placed between identical vowels. We note again, however, in reference to *uθu*, that whilst the resonance of *θ* could not be carried below a^2 880 V.D., the oral resonances actually recorded of *u* are only 314–287 V.D.

The gulf here apparent is not really so wide as it looks. Relatively to most of the resonances here studied, the resonances of vowels are much less mobile; but they are not all equally wanting in mobility. Mobility increases in the same order that oral resonance decreases—*i*, *e*, *a*, *o*, *u*.* All the resonances of *u* yet calculated are for isolated *u*. There can be no doubt that between consonants *u* will show much higher oral resonances than these. It would not be hard to phonograph the syllable *θūθ*, and show at once how far the *θ* is able to pull the *u* resonance upwards, and how far the *u* is able to pull the *θ* resonance downwards. The constitution of the glides in such a phonogram would also be very interesting. So also, in their own degree, would be phonograms of *θōθ* and *θāθ*; but the mobility of *i* and *e* is relatively slight.

The combinations of *x* and χ—*ixi*, *exe*, *aχa*, *oχo*, *uχu*, show a further increase in the intractability of the consonant, especially in *x*. The resonance of χ in *aχa*, *oχo*, *uχu*, can be heard to fall step by step, though hardly *pari passu*, with the fall of the oral resonance of the vowel, but in *x*, whether the adjoining vowel be close *i* or open *i*, close *e* or open *e*, or front *a*, the convenience of articulating against the alveolar ridge is such that the *x* resonance is not drawn very far away in any case from the "neutral" value, due to that position, and estimated below for myself at 2500 V.D. When the oral articulation and resonance of the vowel are far away from these, *e.g.*, in the case of open *e*, or front *a*, adjusting movements are needed, of some duration, and there will doubtless be two glides, of rising and falling pitch respectively, corresponding to them in the phonogram.

The like combinations of *s* and *ʃ* appear to exhibit a still further decrease in the power of the adjacent vowel to make the resonance of the consonant conform to the oral resonance of the vowel. In the combinations *isi*, *ese*, *iʃi*, *eʃe*, this intractability of the consonant is not much observed, because all the resonances lie near together, in the four-accented octave. But this is by no means the case in the other forms, *asa*, *oso*, *usu*, *aʃa*, *oʃo*, *uʃu*. In these there is a wide gap between the neutral resonance of the consonant, and the oral resonance of the vowel; and the consonant refuses to give way very far. Under these circumstances there is a palpable on-glide and off-glide added to the consonant, as described for *x*—longest, of course, for *u*, and shortening for *o* and *a* successively.

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* On mobility of *u*, see Helmholtz, *Sens. Tone*^{3}, p. 110; of *u* and *o*, see tables above quoted: of *a*, see *Proceedings* of this Society, vol. xxii. pp. 110–113.

The above results can be conveniently represented in their entirety as a kind of struggle between the vowel striving to impose something like its own oral resonance on the consonant, and the consonant striving to realise its own untrammelled articulation and "neutral" resonance, and even to bring the oral resonance of the vowel, as far as it will come, towards the same standard. The order of steadfastness in the vowels is *i*, *e*, *a*, *o*, *u*—the last the least steadfast. The relative steadfastness of these consonants, the spirate fricatives, may be exhibited as under. The figures refer to my own voice.

Spirate Fricative.	Range of Resonance.	Neutral Type.	Influence of Neutral Type.
h	V.D. 287–2816	1320	Imperceptible.
f	704–2816	1408	Very slight.
θ	880–2816	1584	Slight.
χ	528–	1491	Moderate.
x	–2982	2500	Strong.
ʃ	792–3168	2046	Very strong.
s	792–3168	2570	Very strong.

The higher the position of a consonant in the above list is, the more closely does the oral resonance of the vowel succeed in assimilating the resonance of the consonant to itself; and the lower, the less closely. But in the latter case the transition is not abrupt, but gliding,—much more so than appears to the ear.

The other possible combinations of spirate fricative and vowel need not detain us long. They are all to a certain extent more complex cases, and had better wait until we have actual phonograms to work from. It seems likely that a spirate fricative, beginning from silence, will begin from the neutral type, where that type is strong; but less near to it, where that type is weak. But as it approaches the vowel the same mutual influences will pass between them as in the case where the vowel flanked the consonant on both sides: and if there is any great difference between its initial and final resonance, there will be a gradual change in the resonantal vibrations, right through the consonant. It is no longer a continuant consonant, consisting of similar parts repeated many times, but a gliding consonant, consisting of parts gradually and steadily changing. But it is not a *glide* in the special sense here attached to that word—the sense of a purely connective sound; every part of it belongs to the consonant, because every part of it contains that special or specially situated friction and that special distortion of the resonance which make the consonant. Such a consonant, however, may also need a true glide to connect it with the following vowel, but such a glide never contains more than a brief fading-off or growing-up of the characteristic friction of the consonant.

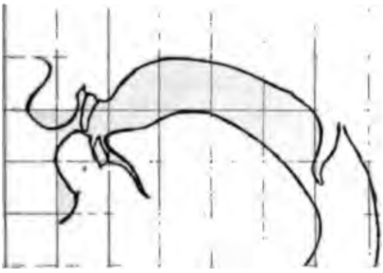
The case of the spirate fricative leading from vowel to silence is the reverse of all this, and may be left for the reader to work out. The case of a spirate fricative between dissimilar vowels is the case most frequently occurring in actual speech; and in most cases it will naturally produce both a gliding consonant and a pair of connective glides.

Looking back on the rude sketch of connected speech which we are now able to put together, from the limited material of vowels and spirate fricative consonants, we are struck by the way in which all its elements tend to link and interpenetrate each other.

Sometimes the resonance of the consonant becomes absolutely identical with one resonance of the adjacent vowel; sometimes, again, two such resonances draw nearer without becoming identical; but even then the gap is bridged by a ladder of gradual changes. There is plenty of room for such ladders; for the absolutely inaudible glide, not $\frac{1}{20}$ sec. long, has room for scores of successive resonantal waves, each differing imperceptibly from its neighbour.

Bearing these facts in mind, it is possible to understand the view which M. Marichelle has taken of consonants (*op. cit.*, p. 128), "Les périodes de la consonne, soumises à l'action continuellement modificatrice de la fermeture ou de l'ouverture progressives, ne sont que les formes mêmes de la voyelle, plus ou moins altérées." At the end of a paper treating of the spirate fricative consonants

SPIRATE FRICATIVES.



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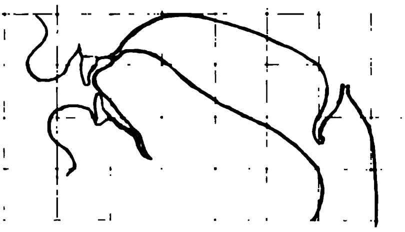

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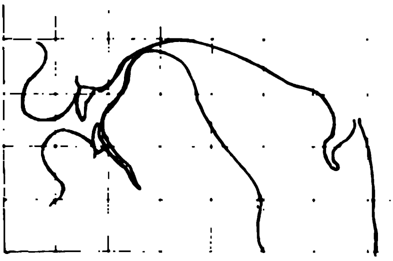

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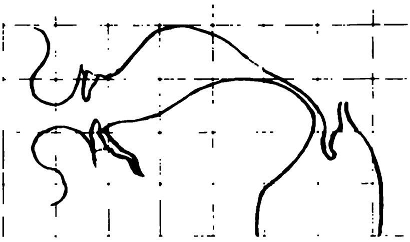

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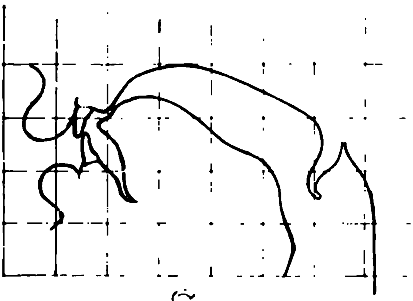

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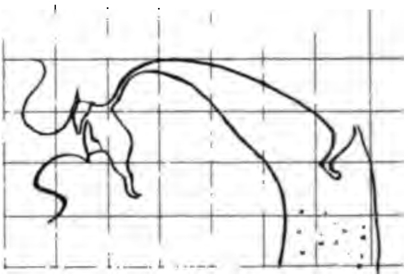

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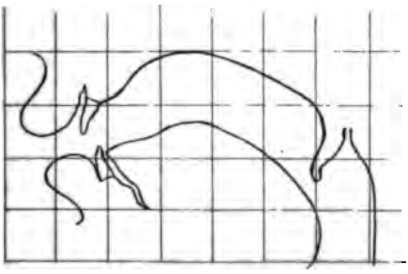

H

NEUTRAL POSITIONS

(ENGLISH & SCOTCH).

in their character of independent, self-sustaining sounds, it is impossible to endorse this statement in its literal meaning. But to anyone starting from the phonographic end of the evidence, it may well seem that the vowel so interpenetrates the consonant that the latter is only a modification of it. But we have seen, I think, that the consonant also interpenetrates the vowel, and has also its own independent acoustic foundation.

It is unfortunate that M. Marichelle's beautiful enlargements do not comprise one single specimen of a spirate fricative, either singly or in combination, though for *ʃ*, at any rate, he has evidently (note, p. 87) some legible phonograms. Only four of his numerous diagrams contain any consonants at all: they are phonograms of the combinations *aja* (Fr. *aya*), *aka*, *ba*, and *wa* (Fr. *oi*). But *j*, *k*, *b*, and *w* are all too complex to be taken as initial studies of consonant-sounds: *j* is fricative, but toned: *k* is not toned, but it is plosive, and therefore essentially gliding: *b* and *w* are both toned and gliding. None of these could be profitably studied till the simpler case of the spirate fricatives had been dealt with. I am sorry that they can be only dealt with here so imperfectly. The need of precise phonographic evidence is manifest in every detail: and even in principle there are probably lurking errors which will only be corrected by objective facts. Even there, however, the refutation of a wrong hypothesis may at once reveal the alternative and true view of the case. The notes on which this article is based were made about four years ago, but they seemed to be, in parts, so perilously deductive—it seemed, in short, so rash a thing to construct phonograms *a priori*, when the real thing might be published any day and confute them—that they were laid aside, to await objective confirmation. Direct objective confirmation has not yet come; but the fine phonograms just mentioned have a certain relationship, though disguised and broken, to those of the sounds studied here; and they have prompted me at length to publish these notes, in the hope chiefly that they may direct the attention of those engaged on phonograms to these simplest consonants, hitherto universally neglected, though by far the most likely subjects for successful analysis. I hope now, after this necessary preface, to be able shortly to take up the more complex sounds which remain.

Note on Crystalline Hydrates of Sodium Thiosulphate. By W. W. Taylor, M.A., B.Sc. *Communicated by* Professor CRUM BROWN.

(Read June 20, 1898.)

During the winter session I had occasion to prepare several supersaturated solutions for lecture demonstration. Among them was a solution made by fusing crystals of sodium thiosulphate. Next day this solution had deposited a considerable quantity of crystals, but had not solidified. The crystals were separated from the liquid, washed, dried, and analysed. Their composition agreed approximately with the formula $Na_2S_2O_3, 2H_2O$. Other crystals having this composition were then obtained by driving off part of the water from fused crystals of the pentahydrate, and allowing the solution to cool.

They were analysed by conversion into sodium sulphate.

I.—1·2186 gm. substance gave 0·9027 gm. $Na_2SO_4 = 24{\cdot}02$ per cent. Na.

II.—1·3977 gm. substance gave 1·0320 gm. $Na_2SO_4 = 23{\cdot}95$ per cent. Na.

	Calculated.	Found.	
		I.	II.
$Na_2S_2O_3, 2H_2O$.	23·74 % Na	24·02 %	23·95 %

In order to find out the temperature condition for the formation of this hydrate, a solubility curve was constructed from the data given by Mulder * and by Kremers.† There appeared to be a change in the curve between 50° C. and 60° C., but the data were insufficient to fix the temperature with accuracy.

I therefore made a series of solubility determinations, extending from 0° C. to 100° C. The form of apparatus used was that described by van't Hoff.‡ The stirrer was driven at about 3000 revolutions a minute by means of an electric motor; and the temperatures were constant to 0·1° C. Below 50° C. the solutions were in contact with $Na_2S_2O_3, 5H_2O$; above 50° C. in contact with $Na_2S_2O_3, 2H_2O$; and at 50° C. solutions were prepared in contact with each of them, and found to have the same concentration.

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* Dammer, *Handbuch der Anorganischen Chemie*, ii. b., 163.

† *Jahresbericht*, 1856, 275.

‡ *Vorlesungen über bildung u. Spaltung* von Doppelsalzen, p. 54.

The analysis of the solutions was carried out as follows:—A weighed quantity of the solution was made up to a known volume, and portions of 10 c.c. were titrated with starch and standard iodine. The mean of two or more titrations was used in the calculation.

The following is the solubility table, expressed in grams of anhydrous sodium thiosulphate in 100 grams of water.

Temperature.	gm. $Na_2S_2O_3$ in 100 gm. H_2O.
0·2°	52·67
12·2°	62·64
19·9°	70·10
30°	84·68
40°	102·6
45°	119·7
*50°	170·9
50°	169·7
52·5°	178·5
55·6°	190·1
60°	206·7
72°	237·8
80·5°	248·8
90·5°	254·2
100°	266·0

If these values are represented graphically, the curve is seen to consist of two parts, which intersect at 50° C. The transition from $Na_2S_2O_3, 5H_2O$ to $Na_2S_2O_3, 2H_2O$ takes place at this temperature. It is also the melting-point of the former, the saturated solution at this temperature having the same composition as the solid. (See Diagram on next page.)

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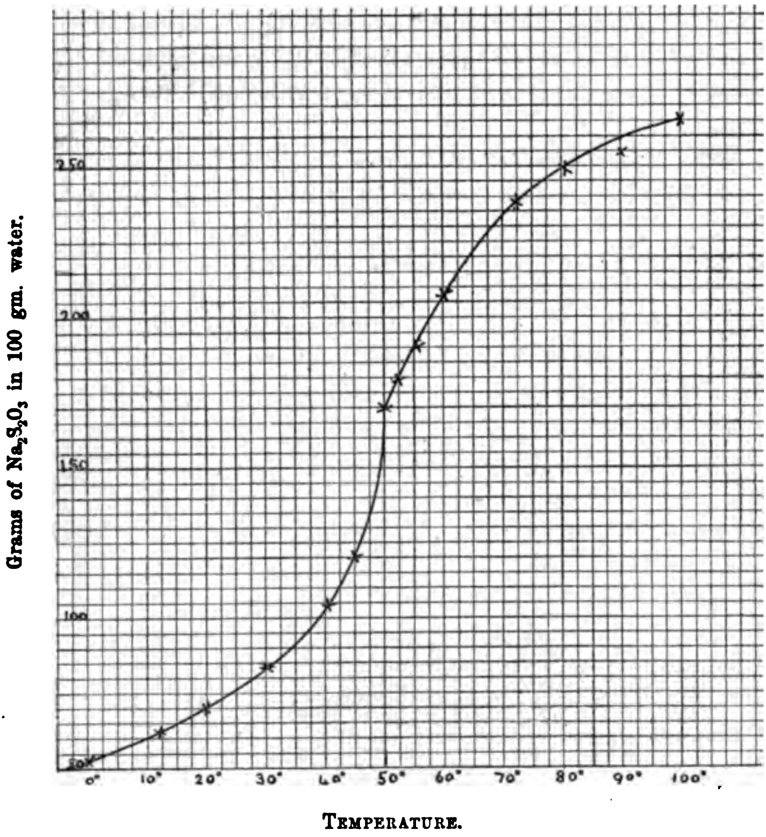
* This solution was in contact with $Na_2S_2O_3, 5H_2O$; and the temperature was raised from 45°. The next solution was in contact with $Na_2S_2O_3, 2H_2O$; and the temperature was lowered from 55°.

Large quantities of pure dihydrate were prepared in the following way:—Pure recrystallised sodium thiosulphate was dried over sulphuric acid in a vacuum desiccator. The anhydrous salt so obtained was dissolved in fused pentahydrated salt, and the solution kept at 53° C. Crystallisation was induced by adding a few fragments of the dihydrate to the solution. The crystals were removed from the liquid, placed in an oven at 53° C. on filter-paper, and covered over with glass to prevent loss of water. They were quite clear and transparent. The analysis, carried out as in the solubility determinations, agreed closely with the formula.



I.—2·3651 gm. substance gave 1·9163 gm. $Na_2S_2O_3 = 81·03$ per cent.

II.—(A different sample) 0·7340 gm. substance gave 0·5970 gm. $Na_2S_2O_3$ = 81·35 per cent.

Calculated for $Na_2S_2O_3$, $2H_2O$.		Found.	
		I.	II.
$Na_2S_2O_3$,	81·46	81·03	81·35

Note.—After the greater part of this work had been completed, I became aware of a paper on supersaturation by Dr Nicol, which recently appeared in these *Proceedings.*[*] In his paper Dr Nicol mentions that supersaturated solutions of sodium thiosulphate deposit crystals, the formula of which is probably $Na_2S_2O_3$, $2H_2O$.

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[*] *Proc. Roy. Soc. Edin.*, xxi. (1897), p. 473.

The Freezing-Point of Aqueous Solutions of Sodium Mellitate. (*Preliminary Note.*) By W. W. Taylor, M.A., B.Sc. (*Communicated by* Professor CRUM BROWN.)

(Read July 18, 1898.)

As is well known, the molecular depression of the freezing-point of aqueous solutions of electrolytes is greater than 1·87, van't Hoff's constant, with which the molecular depressions of non-electrolytes have been found to agree extremely well. This is in accordance with the dissociation theory; and for binary electrolytes the molecular depression may approximate to 1·87 × 2, for ternary electrolytes to 1·87 × 3, etc., but cannot exceed these values.

Substances such as potassium ferrocyanide, which, according to the dissociation theory, might be expected to give values approximating to 1·87 × 4 or 1·87 × 5, have not been found to give molecular depressions greater than 1·87 × 3. This has been held to be an objection to the dissociation theory.

At Professor Crum Brown's suggestion, I have determined the freezing-point of aqueous solutions of sodium mellitate in order to ascertain whether its molecular depression approximates to the theoretical maximum, 1·87 × 7.

Sodium mellitate was prepared as follows:—A slight excess of mellitic acid was added to a boiling dilute solution of sodium carbonate; after expulsion of the carbonic anhydride, the solution was exactly neutralised with pure dilute solution of sodium hydrate, using phenolphthalein as indicator. After concentration over sulphuric acid, crystals separated out. They were washed and twice recrystallised from water.

Analysis of the air-dried crystals gave numbers corresponding to the formula $Na_6C_{12}O_{12}, 17H_2O$.

I.—0·6137 gm. substance heated to 135° C. lost 0·2350 gm. = 38·30 per cent. H_2O. 0·6137 gm. substance gave 0·3332 gm. Na_2SO_4 = 17·61 per cent. Na.

II.—0·5723 gm. substance gave 0·3142 gm. $Na_2SO_4 = 17·80$ per cent. Na.

	Calculated for $Na_6C_{12}O_{12}, 17H_2O$.	Found.	
		I.	II.
Na	17·74	17·61	17·80
H_2O	39·24	38·30	

The formula given in Beilstein's *Handbuch der Organischen Chemie*, on the authority of Erdmann and Marchand,* is $Na_6C_{12}O_{12}$, $18H_2O$. On reference to their paper, I found that the loss on heating to 160° C. is stated to be 38·88 per cent., which closely agrees with the formula given above. As at that time the formula assigned to mellitic acid was $C_4H_2O_4$, the 38·88 per cent. loss was nearer $6H_2O$ than to $5H_2O$.

Erdmann and Marchand gave no sodium estimation.

The salt dissolves in water without any indication of hydrolysis; the most dilute solutions that were used were neutral to phenolphthalein.

The arrangement of the apparatus was a modification of that used by Abegg.†

The glass cylinder of 200 c.c. capacity was supported by cork wedges in a metal air-chamber, the depth of which was much greater than that of the glass.

The metal cylinder was fixed in the wooden lid of a large porcelain cylinder, which, in turn, was surrounded by a large wooden box packed with waste. The freezing mixture of pounded ice, salt, and water was placed in the porcelain cylinder, and the temperature remained constant to within 0·1° C. during a series of determinations lasting several hours. The stirrer consisted of a platinum disc provided with two stout platinum wires fused into glass tubes, which moved vertically in brass slides. The length of stroke of the stirrer was easily adjusted to the depth of liquid by means of a movable eccentric on the driving pulley. The stirrer was worked by an electric motor, and the rate was kept constant at 32 strokes a minute throughout all the experiments.

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* *Liebig's Annalen*, 68 (1848), 327.

† *Zeit. für phys. Chem.* xx. (1896), p. 207.

The thermometer is an ordinary Beckmann thermometer, graduated into hundredths of a degree C. The temperatures were easily read to 0·001° C. by means of a telescope.

The method of procedure was that described by Abegg (*loc. cit.*).

The following table contains the data of the experiments performed. In the table—

a = grams of solution.
b = grams of $Na_6C_{12}O_{12}$ (anhydrous).
n = number of gram-molecules in 1 litre.
Δ = the observed depression of the freezing-point.
Δ/n = the molecular depression.
α = the molecular depression divided by 1·87 (van't Hoff's constant).

SERIES I.—*Temperature of bath* – 1·30° *to* – 1·25° *C.*

a	b	n	Δ	Δ/n	α
71·48	0·06121	0·00181	0·020	11·07	5·92
73·48	0·12243	0·00352	0·034	9·67	5·17
78·47	0·27455	0·00738	0·064	8·67	4·64
83·45	0·42666	0·01079	0·091	8·44	4·51
88·44	0·57878	0·01381	0·112	8·11	4·34
98·49	0·88531	0·01896	0·144	7·59	4·06

SERIES II.—*Temperature of bath* – 1·00° *C.*

a	b	n	Δ	Δ/n	α
51·46	0·09581	0·00393	0·037	9·42	5·04
53·47	0·19162	0·00756	0·062	8·20	4·39
63·45	0·6678	0·02221	0·165	7·48	3·97
69·05	0·9059	0·02770	0·195	7·04	3·77

The two series of determinations were made with two independent solutions of sodium mellitate.

It is my intention to continue these experiments on a larger scale during the winter, and to extend them to salts of pyridinpentacarboxylic acid, and of other polybasic acids.

On the Electrolysis of Ethyl Potassium Diethoxysuccinate. By Prof. Crum Brown and Dr H. W. Bolam.

(Read July 18, 1898.)

The method of electrolytic synthesis of dibasic acids described by Crum Brown and Walker has hitherto been found to be applicable to dibasic acids of the formula C_nH_{2n} $(COOH)_2$, whether normal or with side chains, and to camphoric acid, which is no doubt a saturated cyclic compound. In all other cases which have been tried the anion is oxidised and broken up.

One of us long ago found that this was the case with tartaric acid, and in 1894 v. Miller and Hofer showed that when the salts of monobasic hydroxy-acids are electrolysed the anion is oxidised and broken up. They showed that this is the case also with methoxyacetic acid, so that it would appear that the replacement of H by OR′, where R′ is an alkyl, leads to the same destruction of the anion as does the replacement of H by OH. We examined ethoxysuccinic acid, and found that here also no ethereal layer is formed on the electrolysis of the ethyl potassium salt.

Very different was the case with diethoxysuccinic acid.

Professor Purdie has shown that it is possible to add the elements of alcohol to fumaric and maleic ethers by means of sodium and anhydrous alcohol (*Trans. Chem. Soc.*, 1885, vol. 47, 856, and 1891, vol. 59, 468). He gave us permission to apply this method to acetylendicarboxylic ether, where, by the addition of two molecules of alcohol, diethoxysuccinic ether should be formed.

Acetylendicarboxylic acid was prepared according to Baeyer's directions by the action of alcoholic potash on dibromsuccinic acid. On treating this acid with alcohol and sulphuric acid the acetylendicarboxylic ester was obtained in small quantity, the yield being from 25 to 30%.

The acetylendicarboxylic ester was then mixed with twice its weight of alcohol, and then a small quantity of sodium ethylate dissolved in alcohol was added. The solution usually remained alkaline for 36 hours, and then a further small quantity of sodium was added. After allowing to stand at the ordinary laboratory temperature for 10 days the solution was still alkaline. The quantity of sodium added varied somewhat, but did not exceed one-fifth of an atom of sodium to one molecule of acetylendicarboxylic ester.

On pouring into water an oil separated, which was shaken out with ether, the ethereal solution dried over ignited sodium sulphate and the ether distilled off. On fractionating under reduced pressure the whole of the product came over within two or three degrees, and the boiling point was fully 25° higher than that of the acetylendicarboxylic ester, namely, 145–146° at 17 mm. pressure.

Analysis gave the following results:—

0·1801g substance yielded 0·1352gH_2O and 0·36gCO_2.

		Calculated for Diethoxysuccinic Ester.
C . . .	54·52%	54·96%
H . . .	8·34%	8·34%

The same diethoxysuccinic ether was obtained by the action of sodium ethylate in alcoholic solution in the cold on dibromsuccinic ether, prepared from fumaric acid by the action of bromine and subsequent esterification. Rather more than the calculated quantity of sodium was added, and the solution was kept alkaline for a fortnight. The oil thus got boiled at 144–145° at 16 mm. pressure.

On hydrolysis with alcoholic potash a white pearly potassium salt was formed, insoluble in alcohol, but easily soluble in water. From the aqueous solution the insoluble lead and calcium salts were prepared by double decomposition. Analysis of these salts gave numbers agreeing fairly well with the theoretical numbers for salts of diethoxysuccinic acid.

The diethoxysuccinic ether prepared in these ways is the unsymmetrical body, the acetal derivative of oxal-acetic ether. Michael and Bucher have already shown that this is the case (Ber. 1895, 2511).

If diethoxysuccinic ether is allowed to stand with concentrated hydrochloric acid for 24 hours and the solution then made alkaline with caustic potash and boiled—on neutralising with acetic acid and adding calcium chloride a precipitate of calcium oxalate is produced, the oxal-acetic acid first formed giving oxalic acid and acetic acid.

The half saponification of the diethoxysuccinic ether was effected in the usual way by means of alcoholic potash in the cold. The ethyl potassium salt is easily soluble in alcohol and in water.

Upon electrolysis of the concentrated aqueous solution, the number of volts being 12 and of ampères 3 to 4, an oil was formed, which was soluble to a slight extent in water. This oil was removed with ether, and the ethereal solution dried over ignited sodium sulphate. On distilling off the ether a faintly yellow-coloured oil was left, having a strong odour. From this oil, on standing in the desiccator, large rhombic plates crystallised out, these were drained off, freed from oil by pressing in filter paper, and dried on porous tile.

Analysis gave numbers which agreed fairly closely with the theoretical numbers for tetraethoxy-adipic ether, namely:—

0·137g substance yielded 0·1178gH_2O and 0·2851gCO_2.

		Theory.
C . . .	56·76%	57·14%
H . . .	9·55%	9·00%

A molecular weight determination by Beckmann's boiling point method, using ether as solvent, gave the molecular weight 326 instead of 378.

This crystalline electrolytic product is very stable, being unacted on by 10% alcoholic potash in the heat and by 20% alcoholic potash in the cold. On boiling with aqueous baryta it was possible to detect the formation of alcohol by means of the iodoform and the benzoyl chloride tests, but the ester appears to suffer decomposition by this treatment.

On treating with concentrated hydrochloric acid no ketipic acid is formed. It therefore appears probable that the electrolysis product is the diacetal derivative of $\alpha\alpha'$ diketoadipic ether,

$$EtO\cdot CO\cdot C(OEt)_2\cdot CH_2\cdot CH_2\cdot C(OEt)_2\cdot CO\cdot OEt.$$

We are engaged in the investigation of this point as well as in experiments on the non-crystalline product of electrolysis.

Note on the Passage of Water and other substances through Indiarubber Films. By R. A. Lundie, M.A., M.B., B.S.C., F.R.C.S.Ed. (With a Plate.)

(Read December 20, 1897.)

Indiarubber has come to be so nearly synonymous in popular language with *waterproof* that it seems almost as much of an impertinence to inquire whether it really is so, as it would be to ask whether lead is heavy or iron strong. The question, however, came before me in a practical shape; and I have made out some facts, in attempting to answer it, which may be of interest.

A water-bed is simply a hollow indiarubber mattress, capable of being filled with water and secured by metal screws. Such beds are largely used in the treatment of paralysed and feeble patients; and it is well known to nurses and others who have much to do with them that they need every now and then to be filled up in order to keep them comfortable. I first became acquainted with this fact in the course of medical work at the Longmore Hospital, where there are always many of them in use. Every three months or so a considerable quantity of water needs to be added to each of them, because it becomes so slack that the patient sinks through the cushion of fluid and rests on the solid mattress below, at the points where pressure is greatest. It seemed to me that this circumstance was worth inquiring into. Is it simply due to stretching of the material, or is there an actual transudation through the indiarubber? As I could get no answer to the question from people or from books, I determined to interrogate Nature for myself.

By the kind permission of Professor Tait, I was enabled to perform some experiments on the subject in the University Physical Laboratory.

I took some of the thin indiarubber balloons used as playthings, filled them with water, secured the necks by a double tie with strong thread, and weighed them from time to time. Different sizes and different makes were used, but all gave very similar results.

It was soon evident that the balloons all lost weight at a rate which was fairly constant for each under the same conditions. Some were exposed to the air of the room, some placed in a chamber kept saturated with water-vapour, and some in a chamber kept dry by sulphuric acid. The loss fell in the moist chamber to $\frac{1}{15}$ or $\frac{1}{20}$ that in ordinary air, and was more than doubled in the dry chamber (see Pl. figs. I. and II., and Tables A., B., F.).

The thickness of the rubber films, in their distended condition, varied from 0·1 to 0·025 mm. The extremes of the observations made are as follows: the numbers represent for twenty-four hours the decrease in radius of the balloon in micromillimetres ($\mu = \text{metre} \times 10^{-6}$); or, which comes to the same thing, the amount passing through per square metre reckoned in cubic centimetres:

In the air of the room . . .	11 and 32
In the dry chamber . . .	34 and 74
In the moist chamber . . .	·8 and 2

These are the *extremes* recorded. For each balloon the divergence is less. The loss was not proportional to the thinness of the rubber; though the thicker balloons, on the whole, lost less than those of thinner make.

One of the balloons showed a small leak when it had been filled; it occurred to Dr Knott to close it with a piece of gummed paper. Next day it was found to have been leaking a little; but the leakage ceased, and this one behaved afterwards in the air of the room just like the others.

It seemed clear, then, that the loss observed was closely analogous to evaporation, for it depended on the pressure of water-vapour in the air surrounding the balloon, not upon the hydrostatic pressure within it, which was practically the same under all the conditions.

These experiments were all performed between the 19th July and the 6th October. This month (December) I find that similar balloons of thickness 0·025 mm. lose weight much more slowly—at a rate representing 4 to 6 cub. cm. per square metre per day, or only about $\frac{1}{5}$ of that which obtained with corresponding balloons during the autumn. This difference no doubt depends on the lower temperature and consequent lower vapour-pressure of water, under ordinary conditions, at this time of year.

During this month I have also been making similar experiments with indiarubber of greater thickness. With pieces of indiarubber tubing, though I have found a gradual loss of weight in most cases, I have not had sufficiently satisfactory results to enable me to speak of definite numbers; but with football bladders, about 0·5 mm. thick in their distended condition, I find a loss of weight corresponding to an escape of from 0·75 to 3 cubic cm. per sq. metre per day. I am sorry I did not try these thicker balloons in summer, when higher temperatures and larger differences in weight make the results more satisfactory. But, taking and averaging the results that are comparable, it appears that rubber twenty times thicker than the toy balloons allows almost $\frac{1}{3}$ as much water to pass through it; the loss is thus very far from inversely proportional to the thickness of the rubber. This suggests that the resistance to the passage of water through indiarubber is probably to a large extent a *surface* phenomenon, whether the obstacle be at the inner or the outer surface, or at both; and that its diffusion through the substance of the rubber is comparatively easy.

I have estimated, from the rough data given me by the hospital nurses, that in order to explain the slackening of the water-beds actually observed by leakage of water alone, it would be necessary to assume a rate of loss about as great as was found to occur from the thin balloons in ordinary air during summer; and seeing that these beds are made of rubber at least 1 mm. thick, it seems hardly likely that the transudation can be as rapid as this. The loss must, however, be rather greater in a water-bed in actual use than in a laboratory experiment such as I have made at the same air-temperature; for the temperature, and therefore vapour-pressure, of the water inside is raised a little by the body-heat of the patient lying upon it.

Some escape of water through the indiarubber there certainly must be; and it gives, at all events, a partial explanation of the usual behaviour of water-beds.

I thought it would be interesting to find out at the same time whether other liquids could also pass through the indiarubber films. I chose alcohol, as it has no sensible action on indiarubber.

On filling similar balloons with absolute alcohol and methylated spirit, I found that the loss was enormously greater than in the case of water. Here, however, it did not remain constant, as with the water balloons, but showed a steady and somewhat rapid diminution at each weighing. It was influenced in the same direction, but to a very much less extent, by exposure to dry and to moist air; that is to say, it was increased a little in dry air, and diminished in moist air (see Pl. fig. III., and Tables K. and M.). The clue to this is probably given by an interesting and unexpected result. When a water and spirit balloon were placed together in the moist chamber, the water balloon not only did not lose weight, but actually gained slightly; and on one occasion, when one of the spirit balloons happened to burst in that chamber, the gain of the water balloon beside it was very considerable (fig. I. B., and Table B.). Similarly, no doubt, when a balloon containing alcohol is placed in an atmosphere containing water-vapour, water-vapour passes through the film in consequence of the lower pressure of water-vapour within it. In a moist atmosphere this effect must be increased, and in a dry one abolished or reversed.

Taking the first observations in the case of each balloon, the figures, reckoned in the same way as for water, are as follows:—

	Extreme rates of loss.
For alcohol, ...	369μ and 516μ
For methylated spirit, . . .	339μ and 497μ

—that is to say, from 15 to 30 times as great as the loss of water under similar conditions.

It was noticed in the course of the experiments that while the outside of the balloons was free from pigment at the beginning of the observation, stains of the same colour as that with which the balloons were dyed frequently appeared on the paper on which they were standing; and sometimes there was sufficient colouring-matter on them to soil the fingers. I therefore determined to try whether any diffusion of salts in solution in water took place. I filled bladders with strong solutions of magnesium sulphate and ammonium chloride. The former speedily began to leak, apparently from some chemical action of the sulphate on the indiarubber; but the latter showed no leak, and in ordinary air lost weight just like the water-bladders. They were then immersed in jars of water, and there gained weight slightly but steadily; while the water in the jar showed a slight but decided and constantly increasing reaction to silver nitrate solution, showing the presence of a chloride in the water surrounding the bladder.

These balloons began to leak by minute apertures three or four weeks after they had been filled. But in spite of this, they continued to gain weight when immersed in water, though more slowly than before.

I am indebted to Professor Crum Brown for giving me references on the subject, especially to papers published in 1866 by Payen[*] on the question of the permeability of indiarubber, in which he describes experiments of the same kind with bladders filled with water. His papers deal mainly with the microscopic structure of indiarubber, which he found to be penetrated by fine pores; and he maintains the theory that these explain the well-known results of Graham with regard to the diffusion of gases through indiarubber, as against Graham's own theory that the gases pass through the membrane in a state of solution.

Payen's balloons were, in their distended condition, ·5 – 1 mm. in thickness, and the results varied with the state of the rubber.

Pure (unvulcanised) rubber allowed 23 c.cm., vulcanised rubber only 4 c.cm., to pass per square metre in twenty-four hours.

The temperature was 15° C., not very different from the temperature at which my experiments were made in summer. The rubber with which I have worked has all been more or less vulcanised.

With regard to the explanation of the phenomena observed, I would only remark :—

(1) That though Payen finds pores in the indiarubber, he fails to show that they are continuous throughout it.

(2) That the fact that carbonic acid penetrates a rubber film far faster than other lighter gases, even than hydrogen, seems to require some other explanation than is given by assuming the rubber to behave merely as a porous septum.

To summarise my results, then, I think I have shown—

(1) That indiarubber, at least in layers up to 0·5 mm. thick, is steadily, though slowly, penetrated by water.

[†] *Compt. rend.*, 63, 533 ; *Jour. Pharm.*, [4], 4, 357 ; *Zeitschrift für Anal. Chem.*, 6, 109 ; *Chem. Centralblatt*, 1867, 93.

(2) That the rate of penetration depends upon difference of pressure of water-vapour on the two sides of the indiarubber, not upon hydrostatic pressure, and increases considerably with increase of temperature.

(3) That indiarubber is similarly, but far more rapidly, penetrated by alcohol.

(4) That it also permits diffusion of some substances dissolved in water.

These facts may need to be taken into account in dealing with indiarubber, both in medical and scientific work.

I am indebted to Professor Tait for his kind permission to work in his laboratory; to Professor Crum Brown for references to the literature of the subject; and to Dr Knott for assistance with the experiments and calculations.

Summer Experiments.

(Only a few of the most characteristic results are given here.)

	Thickness of Rubber.	Date.	Weight (grammes).	Loss of weight per day (grammes).	Decrease of radius per day (micro-millimetres μ = metre $\times 10^{-6}$).	
A. Filled with *water*.	70 μ.	July 19	587·7	...	...	
		,, 23	584·2	0·88	26	In air of room.
		,, 27	581·3	0·70	21	
		Aug. 4	576·1	0·64	19	
		,, 7	573·9	0·75	22	
		,, 10	571·2	0·88	26	
		,, 14	568·1	0·79	24	
		,, 28	558·4	0·69	21	
		,, 31	558·3	0·033	00·83	Water chamber.
		Sept. 14	550·1	0·58	17	Air of room.
		Oct. 6	539·6	0·47	15	
B. Filled with *water*.	95 μ.	July 19	571·6			
		,, 23	569·9	0·43	13	Air of room.
		,, 27	568·2	0·41	12	
		Aug. 4	564·6	0·45	13	
		,, 7	564·6	...	...	Water chamber.
		,, 10	564·3	0·05	2	
		,, 14	565·3	−0·25	−8	Do. *with alcohol balloon.*
		,, 16	564·2	0·55	17	Air of room.
		,, 21	570·2	−1·20	−36	Water chamber *with three alcohol balloons*, one of which burst in interval.
		,, 24	568·4	0·61	18	Air of room.
		,, 28	568·3	0·025	00·76	Water chamber.
		,, 31	566·5	0·58	17	Air of room.
		Sept. 14	561·4	0·36	11	
F. Filled with *water*.	55 μ.	July 19	119·3			
		,, 23	118·1	0·29	25	Air of room.
		,, 27	115·3	0·71	61	Dry chamber.
		Aug. 4	113	0·28	24	Air of room.
		,, 10	108·2	0·81	72	Dry chamber.
K. Filled with *absolute alcohol.*	40 μ.	Aug. 4	159·2			
		,, 7	141·1	6·05	516	Air of room.
		,, 10	126·2	4·96	459	
		,, 12	118·8	3·71	362	
		,, 14	112·6	3·07	312	Water chamber.
		,, 16	106·6	3·02	317	Air of room.
	50 μ.	,, 21	97·6	1·79	197	Water chamber.
		,, 28	79·1	2·65	322	Dry chamber.
M. Filled with *methylated spirit.*	50 μ.					
		Aug. 4	135·1			
		,, 7	118·5	5·52	486	Air of room.
		,, 10	105·5	4·34	417	
		,, 12	98·8	3·32	313	Water chamber.
		,, 14	91·5	3·67	391	Air of room.
		,, 16	85·2	3·12	348	Dry chamber.
		,, 21	75·2	2·00	237	Water chamber.
	70 μ.	,, 28	57·9	2·47	333	Air of room.
		,, 31	51·5	2·10	322	

LUNDIE ON THE PASSAGE OF WATER AND OTHER SUBSTANCES THROUGH INDIA-RUBBER FILMS.

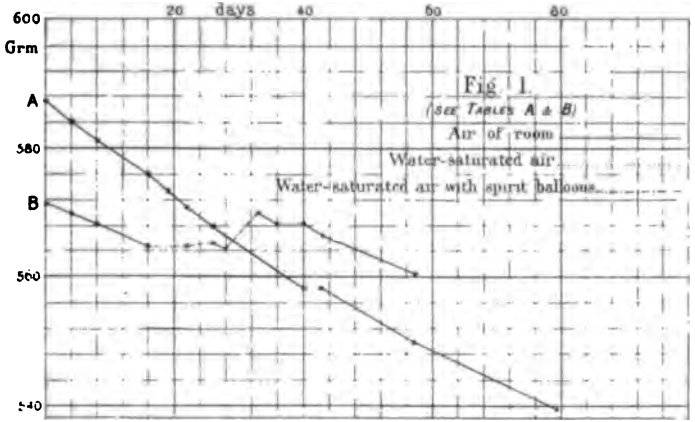
Fig. I.
(SEE TABLES A & B)

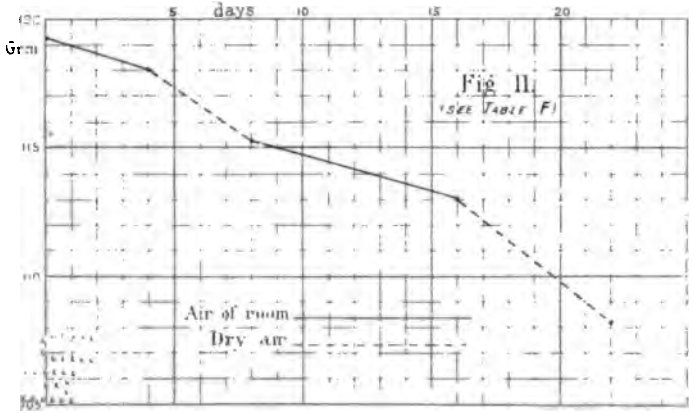
Fig. II.
(SEE TABLE F)

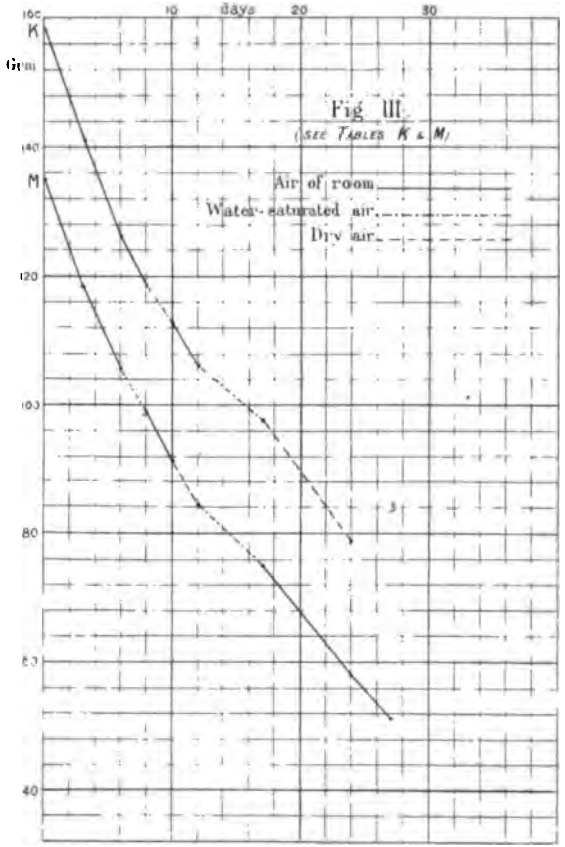
Fig. III.
(SEE TABLES K & M)

MU

Note on Finding the Logarithmic Sines and Tangents of Small Arcs. By J. Burgess, C.I.E., LL.D.

(Read July 18, 1898.)

In geodetical and astronomical computations it frequently happens that we have to use the logarithmic sines or tangents of small arcs, or to find the small angles corresponding to their artificial sines or tangents; and in all trigonometrical tables directions are given to guide the learner in these operations. It might seem, then, superfluous to refer to such a matter. A variety of methods in the solution of a simple problem has, however, sometimes advantages, and the method I have been in the habit of using, though obvious enough, is not usually given.

In Vega's great *Thesaurus Logarithmorum Completus* (1794), based on A. Vlack's *Tables*, the rules for the functions of small arcs make use of second differences; * and the 7th and later editions of Hutton's *Tables* (1830, 10th ed., 1846) follow the same method. Thus—

Q being the required (or given) logarithmic function, A the next less tabular value, and Δ^1, Δ^2, the first and second differences,—

Then $Q = A + x\Delta^1 + \frac{x.x-1}{2}\Delta^2 +$ etc., or approximately, $Q = A + x\Delta^1 - \frac{1}{2}x(1-x)\Delta^2$.

That is,

$$\log.\sin.(a+x) = \log.\sin.a + [x\Delta^1 - \tfrac{1}{2}x(1-x)\Delta^2].$$
$$\log.\cos.(a+x) = \log.\cos.a - [x\Delta^1 + \tfrac{1}{2}x(1-x)\Delta^2].$$
$$\log.\tan.(a+x) = \log.\tan.a + [x\Delta^1 \mp \tfrac{1}{2}x(1-x)\Delta^2].$$
$$\log.\cot.(a+x) = \log.\cot.a - [x\Delta^1 \mp \tfrac{1}{2}x(1-x)\Delta^2].$$

In the case of the tangent and cotangent the upper sign in the last term is used when $a + x < 45°$, and the lower when $a + x > 45°$.

For the angle:—we have respectively for the excess of the angle above the tabular value for a,—

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* The rule given in Shortrede's *Logarithmic Tables* (1858) applies only when the fraction of a second is ½.

For sine, $$x = \frac{Q - \log.\sin.a}{\Delta^1 - \frac{1}{2}(1-x)\Delta^2};$$

For cosine, $$x = \frac{Q - \log.\cos.a}{\Delta^1 + \frac{1}{2}(1-x)\Delta^2};$$

For tangent, $$x = \frac{Q - \log.\tan.a}{\Delta^1 \mp \frac{1}{2}(1-x)\Delta^2};$$

For cotangent, $$x = \frac{Q - \log.\cot.a}{\Delta^1 \mp \frac{1}{2}(1-x)\Delta^2}.$$

This requires first the division of the difference $Q - A$ by Δ^1 to obtain an approximate value of x, and then a second division with the approximate value of $\frac{1}{2}(1-x)$ thus found, in order to obtain a closer approximation to x. And for arcs very near 0° or 90° when the functions vary rapidly and the use of third differences would be required, it is recommended to find the corresponding natural sine, tangent, etc., for which only first differences are necessary, and then to find the logarithm of this, in order to obtain the correct values. These processes are cumbersome.

In Hutton's *Tables* (10th ed., p. xxxvii.) Maskelyne's rules, given in his introduction to Taylor's *Logarithms* (1792), are stated as "often useful," but no examples of their use are given. These rules are quite empirical, but for small arcs, under 2°, they are very convenient and accurate, and have often been published, as in Galbraith's *Mathematical Tables* (1827), Shortrede's *Logarithmic Tables*, etc. They are expressed by the formulæ—

$$\log.\sin.a'' = \log.\sin.1'' + \log.a'' - \tfrac{1}{3}(\log.\sec.a'' - 10)$$
$$\log.\tan.a'' = \log.\tan.1'' + \log.a'' + \tfrac{2}{3}(\log.\sec.a'' - 10).$$

When $a = 5°$, or $a'' = 1800''$, the error in the 7th place of decimals is $-5{,}611$ or $0''\cdot023$;

„ $a = 10°$, the error in the 6th and 7th places of decimals is $-90{,}428$, or $0''\cdot757$ in sine and $0''\cdot711$ in the tangent;

„ $a = 10°\ 34'$, the error in the 5th, 6th, and 7th places of decimals is $-112{,}861$, or $1''$ in sine and $0''\cdot966$ in tangent;

„ $a = 15°$, the error in the 5th, 6th, and 7th places of decimals is $-463{,}445$, or $5''\cdot9$ in sine and $5''\cdot5$ in tangent, etc.;

the divergence rapidly increasing above 10°.

The method I have found most useful in all cases is nearly as simple as this, and does not require the use of the secant. It is represented by

$$\log.\sin.(a+x)=\log.\sin.a+\log.(a+x)-\log.a;$$
$$\log.\tan.(a+x)=\log.\tan.a+\log.(a+x)-\log.a.$$

Where a and $a+x$ may be stated in degrees, minutes or seconds, and decimals of the same denomination: Thus, taking Vega's example, to find the arc of which 5·6271691 is the log. sine, we have—

$\log.\sin.(a+x)$ . . .		5·6271691
$\log.\sin.8''=a$.	ar. co.	4·4113351 – 10
$\log.8''$		0·9030900
$a+x''=8''{\cdot}7416$, log. . .		0·9415942

Conversely to find the log. tan. of 2° 42′ 56″·44 (= 9776″·44)—

log. tan. 2° 42′ 56″ . .		8·6760614
log. 9776″·44 . . .		3·9901807
log. 9776″ .	ar. co.	6·0098388 – 10
log. tan. 2° 42′ 56″·44 .		8·6760809

But, while the usual tables give the sines and tangents for each second up to about 2°, few of them give the values for seconds beyond this, and some not even for the first 2°. We may, however, apply this method with a correction that will enable us to find the values for any arc in the quadrant.

For the sine we readily find the difference between 5° + 60″ computed by the above method and the true value for log.sin. 5° 1′ to be – ·0000036,828. Similarly the errors of the method are—

at 5°	– 36,828	at 30°	– 224,692
10°	– 73,708	35°	– 263,940
15°	– 110,815	40°	– 304,073
20°	– 148,264	45°	– 345,255
25°	– 186,179	etc.	

To above 20°, these differences are represented, within less than half a unit, in the seventh decimal place, by $7,39 \times a^\circ$, and for x'' the correction will be $-0,1232ax''$.

Let, as before $a + x'' = 2^\circ\ 42'\ 56''\cdot 44$, to find the sine.

Here $a^\circ = 2\cdot 7$ and $x'' = 56\cdot 44$

$-0,1232 \times 2\cdot 7 = -0,33$ and $-0,33 \times 56\cdot 44 = -18,4 =$ correction.

Then, log. sin. $2^\circ\ 42'$		$8\cdot 6730804$
$a = 2^\circ\ 42' = 9720''$ . .	ar. co. log.	$6\cdot 0123337 - 10$
$a + x'' = 9776\cdot 44$ . .	. log.	$3\cdot 9901807$
	correction	-18
log. sin. $2^\circ\ 42'\ 56''\cdot 44$ . .		$8\cdot 6755930$

This value is correct to the last figure of the decimal.

For all angles to about 50°, but especially for those above 20°, the correction is expressed very nearly by—

$$-x''\{0,1228a + (,0000031\dot{5}a - ,00003)a^2\}.$$

Thus, at 40°, it will be—

$$-x''(4,913 + ,000096\dot{2} \times 1600) = -x'' \times 5,067.$$

For the tangent, the differences or errors for 1′ are as follow :—

at 5°	+ 73,883	at 30°	+ 504,922
10°	+ 149,237	35°	+ 620,914
15°	+ 227,885	40°	+ 756,284
20°	+ 311,752	45°	+ 918,424
25°	+ 403,138		

To about 8° the correction would be + 14,8 per degree for 60″, or

$$+\frac{14,8a^\circ}{60} \times x'', \text{ or } +0,24\dot{6}a \times x''.$$

Thus, for log. tan. 2° 42′ 56″·44, we have—

$$0,24\dot{6} \times 2^\circ\cdot 7 \times 56''\cdot 44 = +37+$$

And, log. tan. $2^\circ\ 42'$		$8\cdot 6735628$
$a = 9720''$ . .	ar. co. log.	$6\cdot 0123337 - 10$
$a + x'' = 9776''\cdot 44$ . . .	log.	$3\cdot 9901807$
	correction	$+37$
log. tan. $2^\circ\ 42'\ 56''\cdot 44$ . .		$8\cdot 6760809$

But for larger arcs, the above differences do not yield a simple empirical formula,* and as the log. cosine is easily obtainable by the usual process, it is better, in cases where extreme accuracy is required, to employ the formula: $\log.\tan.\, a = \log.\sin.\, a - \log.\cos.\, a$.

Or, we may find the difference to be added to log. tan. a for an increment of x'' by the formula—

$$\Delta \log.\tan.\, a = \log.\left(1 + \frac{\sin.\, x}{\sin.\, a \cos.(a+x)}\right).$$

Thus to find the log. tan. 80° 2′ 37″ or log. cot. 9° 57′ 23″. The log. sin. 9° 57′ 23″ = log. cos. 80° 2′ 37″ is found as before.†

log. sin. 37″ . . .	6·2537766
log. sin. 80° 2′ . . ar. co.	0·0066041
log. cos. 80° 2′ 37″ . ar. co.	0·7622087
·00105339 . . . log.	7·0225894
1·00105339 . . log.	0·0004572
log. tan. 80° 2′ . . .	10·7551611
log. tan. 80° 2′ 37″ . .	10·7556183

But, it is easier to use the sine and cosine, thus—

log. sin. 80° 2′ 37″ .	9·9934096
log. cos. . . . ar. co.	0·7622087
log. tan.	10·7556183

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[*] Approximately the correction may be represented by $+\frac{x''}{60}(14{,}685a + {,}016015a^2 + {,}00008447a^3 + {,}0000797a^4 - {,}0000011566a^5 + {,}000000010737a^6)$. The *comma* is used to indicate the separation of the 7th and 8th places of decimals.

[†] 9° 57′ is nearly 10°; and $-{,}123 \times 10 \times 23'' = -28{,}3 =$ corr.

log. sin. 9° 57′ (35820″) .	9·2375153
log. 35843″	4·5544044
log. 35820 . . . ar. co.	5·4458744 − 10
corr.	− 28
log. cos. 80° 2′ 37″ . .	9·2377913

On the Theory of Archimeric Segmentation and its bearing upon the Phyletic Classification of the *Cœlomata*. By A. T. Masterman, D.Sc.

(Read June 6, 1898.)

In a recent paper embodying the results of my work upon *Actinotrocha*,* I was led to suggest a theory of the segmentation of cœlomate animals, the essential features of which were as follows:—

1. There can be demonstrated, in the morphology of the *Cœlomata*, two distinct types of segmentation—(*a*) a primitive or archimeric type, having in its constitution certain evidences of a radial origin, and (*b*) a secondary or metameric segmentation, superposed upon the former and bearing evidence of a bilateral origin.

2. A certain number of the *Cœlomata* retain, under varied disguise, the archimeric segmentation throughout life, together with a number of other primitive characters in common. These groups, being genetically allied, may be conveniently classified in one division, for which the name *Archi-cœlomata* is proposed.

3. In the truly segmented animals, such as the *Annelida*, *Arthropoda*, and *Eu-chorda*, the bilateral or metameric segmentation (of the posterior archimeric segment) tends to completely replace the archimeric. In at least two of these groups there has been an independent evolution of metameric segmentation.

In attempting to bring forward facts in support of this theory, one may approach the subject from three stand-points.

(*A.*) *Firstly*, an attempt may be made, by general reasoning, to construct a hypothetical ancestor of the *Cœlomata* as a group. In this way may be obtained a schematic outline of an organism possessing each system of organs in its most archaic condition.

(*B.*) *Secondly*, taking this as a central type of the organisation of the *Archi-cœlomata*, several of the lower divisions of the cœlomate animals may be shown to possess a greater or less number of these features, such that they will naturally fall together under the one major division, conveniently termed the *Archi-cœlomata.* These features may be morphological or ontogenetic.

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* *Quart. Journ. Micros. Science*, Aug. 1897.

(*C.*) *Thirdly,* evidence may be derived from morphology and from ontogeny of the metamerically segmented groups that a secondary segmentation has been phyletically superposed upon the primary. Thus, these animals may be shown to pass through ontogenetic stages which closely resemble the archi-cœlomate type (and, in consequence, some of the *Archi-cœlomata*), and to possess in their morphology more or less vestigial traces of the archimeric segments.

(*A.*) In the classification of the *Triploblastica* the only sound basis of phyletic value upon which to rest is the condition of the mesoderm or third layer, and bound up with this is the cœlome and its segmentation. It is impossible to enter here into the whole discussion of the theories with respect to the primary origin of the cœlome, but I would adhere to that originally propounded by Mr Sedgwick* as the most in harmony with the facts of morphology, ontogeny, and physiology.

Its essential feature is "that the somites of segmented animals are derived from gut pouches, which are homologous with the pouches of *Cœlenterata.*"

Not only are there a great number of morphological and ontological facts which point to the truth of this hypothesis, but it is confirmed by physiological considerations. Thus, the pouches of the *Actinozoa* present special differentiations of the primitive endoderm, the walls of which perform the functions of the muscular and reproductive systems, and the cavities of which contain a nutritive circulatory fluid. In the lowest *Cœlomata* the cœlome performs precisely the same series of functions, and only differs essentially from the 'pouch' of the *Cœlenterata* by its loss of organic continuity with the endodermic gut-wall, whilst in the history of the cœlome in the higher *Cœlomata* each function becomes differentiated and confined to a special area of the organ, the morphological expression of which is the eventual division of the primitive cœlome into discontinuous elements, each with its own distinct function.

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* *Quart. Journ. Micros. Science,* 1886.

It is necessary to emphasise these points with regard to the theory here adopted, because there are other theories of the cœlome which, by an appeal to a judiciously-selected series of morphological facts, have been made to bear a superficial semblance to probability, although violating the elementary principles of physiological differentiation.

I allude in particular to the suggestion that the cœlome is to be derived from the gonads of the *Pseudocœla*. If once it be granted that a mass of reproductive cells have, in the course of phyletic history, lost their sexual specialisation and given rise, in the same individual, to muscles, connective and other mesodermic tissues of the differentiated cœlome, there is surely no impediment to assuming that the *Cœlenterata* have been derived from the *Cœlomata*, or any one phylum from any other that may suit the individual fancy.

Intimately bound up with Mr Sedgwick's theory of the cœlome is that of the blastopore. To quote the author's words—"it necessarily follows (from the consideration of the *Peripatus* embryo) that the mouth and anus of the *Triploblastica* are derived from the gastrœa mouth, *i.e.*, cœlenterate mouth." * I need not quote the arguments which were adduced for this theory by its author; but, since it was suggested, further research has only afforded additional evidence in its favour. †

This mode of derivation of the triploblastic mouth and anus assumes the existence of a radial cœlenterate ancestor of the *Cœlomata*. In other words, an organism in which the parts were arranged in radial symmetry, about an axis passing through the mouth to the aboral pole. We, therefore, go no further than the theory to suppose that the gut-pouches were also radial, which implies that their number must have been at least three. Their possible number appears to lie between three, four, and five. The higher numbers, which are multiples of either of these, must have been derived from a prior stage with the lowest factor, whereas a radial symmetry of seven, eleven, or thirteen units has no precedent in animal morphology. The same remark applies to the simplest factor, *i.e.*, three. We are, therefore, left with the tetraradiate and pentaradiate forms of symmetry for consideration.

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* A. Sedgwick, *loc. cit.*, p. 67.

† *Cf.* development of *Serpula*, *Peripatus*, and *Mollusca*.

All the lowest forms (*Cœlenterata*) present a tetramerous symmetry, both pelagic and sedentary, whilst the *Echinodermata* are the great pentamerous group. Besides being of higher organisation than the *Cœlenterata*, these forms give evidence of owing their pentamery to a sedentary past and a peculiar hypertrophy of one side of the bilateral ancestor.

As the hypothetical form we are discussing must have been a Cœlenterate or little above it, we are apparently justified in assuming that it was tetramerous.

As regards the habitat of this organism, its radial symmetry indicates either a pelagic or sedentary existence, and there are numerous reasons for holding that it was the former. Brooks * has insisted upon the importance of the pelagic habitat as the dwelling-place of the primitive types, and on this assumption the pelagic stages of the littoral fauna acquire a phyletic significance.

Some time ago † several reasons were adduced for regarding the pelagic ontogeny of certain of the fishes to be primitive and of important phyletic significance, and the demersal type of ontogeny to be secondarily acquired. The same considerations apply to a large extent to the invertebrate forms with pelagic larvæ. Amongst these, perhaps, the most important is that the typical pelagic ontogeny is without yolk, a secondary means of nourishment which cannot have its phyletic equivalent. As an important factor in the recapitulation of phyletic history an egg must, by the nature of the case, have no yolk, and must, from its earliest existence, obtain its own nutriment, so that, as these conditions are most nearly approached in a typical pelagic ontogeny, we are justified in regarding the pelagic habitat as primitive.

Lastly, we are attempting to follow the evolution of the highest and most progressive types of living beings, and it is very questionable how far it is possible for organisms which have to any considerable extent adapted themselves to a sedentary existence, to again become free, and thereafter attain the highest position in the

* W. K. Brooks, *The Genus Salpa*, Baltimore.

† A. T. M., *Natural Science*, March 1897.

animal scale. Examples taken from the *Actinozoa*, *Echinodermata*, *Polyzoa*, *Lamellibranchiata*, and *Tunicata*, of species which have again acquired a locomotor habit tend to show that the loss of organs undergone during the sedentary stages is irremediable and limits the further advance in differentiation. In other words, as has been recently pointed out,* we have no proof that organs once lost have ever been re-evolved.

It therefore follows that although there are numerous instances of sedentary types again assuming a pelagic or littoral existence, yet the ancestral history of such forms as the *Arthropoda* and *Chordata* probably has been passed in the pelagic or littoral regions. In the case of this earliest cœlomate ancestor, its radial symmetry leaves us no choice between these two; a pelagic environment is the only possible alternative.

We are therefore led to regard the remote ancestor of the *Cœlomata* as having been a radially symmetrical tetramerous cœlenterate with a ventral axial mouth (with functions of mouth and anus) and four gut-pouches separating off from the alimentary canal to form four cœlomic pouches.† All four of these 'cœlomic' pouches will form reproductive and muscular elements, and will contain a nutritive circulatory fluid. Belonging to the pelagic plankton, it must have had an existence somewhat similar to that of many pelagic medusæ of the present day.‡ (Stage I.)

The next differentiation is that of the mouth (blastopore) into mouth and anus. The first step in this is the approximation of the lips of the stomodæum in two opposite radii in correlation to the hypertrophy of the ingestive and egestive functions, respectively, in the two radii at right angles to them. If we assume the former presence of an ectodermal involution (cœlenterate stomodæum), as in the *Actiniaria*, then the fusion of the walls in two radii would leave the ectoderm as a pair of ectodermal funnels, the metazoan stomodæum and proctodæum, connected with the functions of ingestion and egestion respectively. In ontogeny, the stomodæum and proctodæum would naturally arise as separate invaginations.

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* E. W. Macbride, *Natural Science*, January 1897.

† *Cf.* the suggestive remarks of Korschelt and Heider (pp. 344–345, English translation).

‡ By this it is not implied that this organism was morphologically comparable to the medusæ of the present day. To this view (Kleinenberg, Balfour) it has been objected by Korschelt and Heider (pp. 342–343, English translation) that the medusa presents a higher type of locomotion, and we may add of ingestion. Another indicated difficulty is the absence, in medusæ, of an apical nervous system. It has been assumed below that the apical ganglion was present in the pelagic cœlomate ancestor, and that in such a form as the trochosphore it has become secondarily shifted to the new apical pole at the apex of the pre-oral lobe. For figure of this Stage I., see *Quart. Journ. Micros. Science*, vol. xxxviii. p. 325.

Thus a secondary axis, from mouth to anus, is believed by many morphologists to have been acquired in the history of the *Metazoa* (*Heteraxonia*), and accepting this as a theory with a great amount of probability, we may inquire as to the further fate of the four cœlomic pouches.

The acquirement of the secondary axis of symmetry implies a rearrangement of the organs bilaterally about this axis, more especially as the differentiation of an ingestive aperture implies a locomotion in that direction. There are, as far as I can see, only two alternatives with regard to the arrangement of the four cœlomic pouches in relation to the new axis of symmetry. Either the axis will correspond to the septa between the cœlomic pouches and the latter will become symmetrically arranged as a pair of anterior and a pair of posterior pouches, or the axis will correspond to the centre of a pouch, and they will become arranged so that one will be pre-oral, two as paired lateral, and another post-anal.

Of these two alternatives there can be little doubt that the latter is the correct one. So far as I know, all the *Actinozoa* except the *Antipatharia* have the terminal mesenteries arranged in pairs on other side of the main axis, and in the medusæ (*e.g.*, *Aurelia*, *Lucernaria*, etc.), in which the mouth shows a tetramerous symmetry, it is in the axes intersecting the gastro-vascular pouches (per-radial). Thus we have some justification for supposing that upon the assumption of bilateral symmetry about an oro-anal axis the four primitive cœlomic pouches became arranged so that one was pre-oral, two were lateral, right and left, and one was post-anal. In correlation to this, the cœlomate ancestor will present a body of three segments, one pre-oral and two post-oral. Organs arising in these segments will at this stage be symmetrical about the long axis, and others which, like the cœlomic pouches, had their origin prior to the formation of the new axis, will be gradually moulded into bilateral form.

These three segments may be conveniently designated the protomere, mesomere, and metamere, and their cœlomic cavities, protocœle, mesocœles, and metacœle respectively, and although the first and last are primarily unpaired, yet the animal being now bilateral there will be a constant tendency for them to assume a paired or, at least, bilateral condition.* Thus the primitive opening of the protocœle to the exterior may become paired, as in the 'proboscis-pores,' or even the whole protocœle as in *Sagitta*, and the metacœle probably became very early separated into two by the backward progression of the alimentary canal until the anus was terminal.

Primitively, each of the archicœles will open to the exterior by a monocytic egestive opening, in connection with which an ectodermal excretory canal will be secondarily acquired, thus forming primitive nephridia.† On the assumption of bilateral symmetry the protocœlic and metacœlic nephridia will become secondarily paired, and as the gonads become confined to the metacœles (as pointed out below), the metacœlic nephridia alone will continue to function as gonaducts. These three pairs of archi-nephridia are to be found surviving in such varied organs as proboscis-pores, collar-pores, nephridia, and even as oviducts. Those forms which retain the mesocœlic pores (collar-pores of *Balanoglossus*, and *Cephalodiscus*, and stone canal of *Echinodermata*), as a rule, have the metacœlic pores completely metamorphosed into oviducts, whereas those forms which lose their collar-pores (*Brachiopoda*, *Phoronis*), as a rule, retain the metacœlic nephridia with gonaducal functions.

Let us now inquire into the condition of the cœlomic pouches in this Stage III.* The protocœle is pre-oral, and is therefore in the most disadvantageous position for the direct supply of nutrition from the alimentary canal, and the metacœles, enveloping the gut for their whole length at a part where digestion is effected, will be in the most advantageous position. Their walls will therefore be bathed with nutritive fluids supplied direct from the alimentary processes. The mesocœles will in this respect, as in that of location,

* For figures of Stage II., see *Quart. Journ. Micros. Science*, vol. xxxviii. p. 327, and of this Stage III. on p. 328.

† See *Zool. Anzeiger*, Nos. 501, 502, and 503.

be intermediate in character. The reproductive cells will, therefore, be no longer found in the protocœle, but will be confined to the metacœles; whilst, on the other hand, the protocœle will lose the reproductive function and will be entirely devoted to muscular differentiation. In other words, the protocœle will become more 'animal' or katabolic in form and function, the metacœles more anabolic. One curious anatomical peculiarity follows from this—the circulatory vascular system not being as yet fully differentiated the main excretory organ will be in connection with the protocœle, where the metabolism is most active, and not as in the highest animals, in the metacœles, which in them form the 'animal' organs. I have attempted to show elsewhere* that the function of excretion primitively arises in the ectoderm, and in this hypothetical ancestor, the ectoderm of the stomodæum probably came into close connection by an invagination with the protocœle and the vascular sinus in its neighbourhood. This primitive excretory organ may be termed the subneural gland. We have now two important sets of organs the evolution of which are yet to be followed, namely, the vascular and the nervous systems. They may well be taken together, for they are intimately connected. In the paper already referred to, an attempt has been made to show that the vascular system arises phyletically as a system of spaces between the limiting epithelia, ectoderm and endoderm, and the mesoderm, and that the vascular fluid is primarily excretory in function. The nervous areas being, by their very nature, areas of active metabolism, their course is largely followed by the vascular vessels immediately below them.

In Stage I. the nervous system must have been upon a radial principle, and although, as in the present day pelagic *Cœlenterata*, consisting of a diffuse nervous plexus in continuity with the ectodermal epithelium, was probably concentrated in certain areas in relation to the ciliary ingestive and locomotory organs, and had a tetramerous arrangement, four inter-radial nerves meeting a ring round the edge. Where the four radial nerves meet at the aboral pole it is reasonable to suppose, from the analogy of *Ctenophora*, that a central ganglion would be developed. Immediately below this ganglion would be the principal blood-sinus (subneural sinus), and from this would lead four inter-radial sinuses below the nerves. On the assumption of bilateral symmetry, as described, two of the radial nerves, with part of the ring nerve, would form a pre-oral band, the other two between mesocœles and metacœle would form a post-oral band, whilst the rest of the ring-nerve would form ventral cords and a post-anal ring. These three nerve-rings are connected with the formation of three archimeric ciliated bands which, sooner or later, replace the diffuse ciliation of earlier stages; one of these is pre-oral, the prototroch (syn. cephalotroch), and two post-oral, the mesotroch (or branchiotroch) and the metatroch (peri-anal).

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* *Zool. Anzeig.*, 501-503.

These bands do not persist in present-day *Archi-cœlomata*, but are found in their ontogenetic stages. The main ganglion would be moved forward into intimate contact with the main animal organ, the protocœle, and the sub-neural blood-sinus would follow the same course.

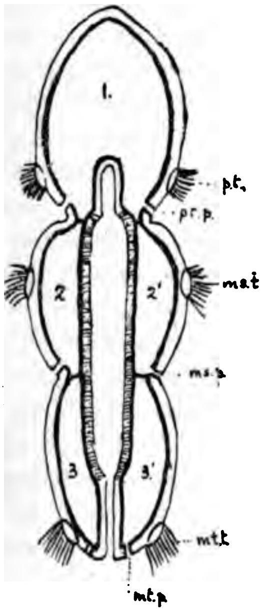

FIG. 14.—Diagrammatic horizontal section through hypothetical ancestor of the *Archi-cœlomata.*

The nerves, to a late stage, remain in connection with the ectoderm.

We have thus been enabled briefly to sketch the leading characters of an ideal ancestor of the *Cœlomata*, as may be derived by general principles of evolution, and the assistance obtained by the acceptance of the theory of the cœlome and that of the blastopore, as stated in the commencement.

Such a type, by the very nature of the case, cannot be found living at the present day, for in giving rise to higher types it has long since ceased to be.*

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* It will be seen that in nearly all these features *Actinotrocha* is the embodiment of this morphological conception of the ancestor from which all the *Archi-cœlomata* have been derived, and it is remarkable how this larva might be, and indeed has been, mistaken for that of nearly all the groups of *Archi-cœlomata* in turn. Cf. *Q. J. M. S.*, vol. xxxviii. p. 282.

We can, however, find evidence of its existence, and more especially of its leading feature, that of archimeric segmentation and its attendant phenomena, in the anatomy and ontogeny of present-day species.

(*B.*) We are thus led to the general characters of the *Archicœlomata* as follows:—

Morphological.

1. Body divided into three more or less clearly defined segments, one pre-oral and two post-oral.

2. Mesoderm forming cœlomic cavities corresponding to the archimeric segments, and primitively opening to the exterior by ciliated ducts. The first or protocœle is essentially muscular, sensory, and locomotive, and is present throughout life or only in early stages,—the second or mesocœle connected with food ingestion and primarily produced into a series of post-oral tentacles which may (*Brachiostoma*) or may not persist (*Balanoglossus*, *Chætognatha*), and may, upon atrophy of the protomere, assume the locomotory function (*Echinodermata*),—the third or metacœle connected primarily with the gonads and the vegetative functions. A chondroid mesoblastic skeleton is of very general occurrence.

3. Nervous system mostly in continuity with the ectoderm, and consisting of a protomeric ganglion (brain), a mesomeric ganglion (sub-œsophageal), and a post-oral, mesomeric, ring connecting them. A diffuse plexus in parts, and more or less prominent metameric bands, which are unsegmented.

4. Vascular system, if present, very simple, consisting of sinuses between the cœlomic epithelia (dorsal and ventral trunks and a post-oral ring-sinus), and a central archimeric heart or sub-neural sinus, in close connection with which there may or may not persist the primitive archimeric excretory organ, or sub-neural gland.

5. No indication of a true metameric segmentation.

6. Gonads arise from wall of metacœles, and in some cases have idiodinic ducts (probably derived from metacœlic nephridia), in others, the metacœlic nephridia function as gonaducts.

Ontogenetic.

1. Those species with larval forms have simple larvæ, with the body divided up into three segments, one pre-oral and two post-oral.

2. The mesoderm arises by archenteric invagination, or by a simple modification of it.

3. There are often in the free swimming larvæ three ciliated bands, one pre-oral and two post-oral.

In order that species retaining this archimeric segmentation should have survived, they must of necessity have adopted certain habitats, in which naturalists are wont to find primitive forms. In this category are pelagic, deep-sea, burrowing and sedentary habitats, and amongst the animals affecting these are found forms agreeing with our type. In the case of sedentary animals, the degeneration and great anatomical modification involved, often disguise, in the adult, the features we wish to find, but the ontogeny comes to our assistance.

Chætognatha (Pelagic).

The isolated position and primitive characters of this group are usually acknowledged, and in their anatomy we find the cœlome divided into paired protocœles, mostly muscular, paired mesocœles, and paired metacœles (fig. 1). The mouth is terminal, and the protocœles are greatly reduced. The mesocœles are predominant, and contain the female sexual elements, which may be correlated with the post-anal position of the metacœles, in which the male (or katabolic) sexual elements arise. This condition may be a primitive survival of Stage II., but is more likely a return to this condition from Stage III. by a movement formed of the alimentary canal and anus, correlated with the muscular development of the metacœles as a means of locomotion through the tail-fin, as is paralleled in the fishes.

The nervous system presents the central ganglion in the protomere, the post-oral ring, and the mesomeric ganglion, all in continuity with the ectoderm.

The ontogeny is usually regarded as direct, but the cœlome still arises by archenteric diverticula. The protocœles are derived from these diverticula by constriction,* and how the mesocœles and metacœles are separated from each other does not appear to have been clearly made out.

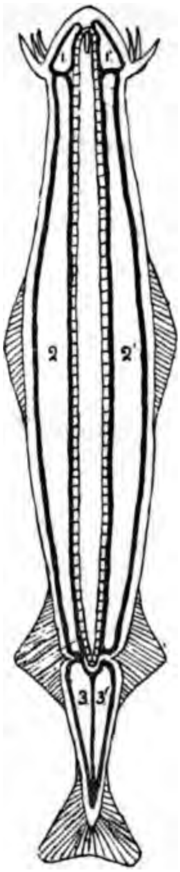

FIG. 1.—Diagrammatic horizontal section through one of the *Chætognatha.*

Altogether, the anatomical evidence is in favour of regarding the *Chætognatha* as animals which have retained a primitive pelagic habitat and a primitive archimeric segmentation,† although both the anatomy and ontogeny show some deviations from the original type, in the addition of setæ, lateral and tail-fins, and sense-organs. It is interesting to note that Gourret ‡ describes an excretory organ in the head.

The peculiar backward extension of the archenteron, resulting in a division of the metacœle into two parts, appears to point to a former extension of the alimentary canal to the posterior end, and agrees with the theoretical explanation given above, of the fact that in the *Archi-cœlomata* the metacœle is frequently paired.

Enteropneusta (*Burrowing*).

This group shows to perfection the pre-oral protomere, the paired mesomeres and metameres (fig. 2). The protocœle is essentially 'animal' and muscular, the metacœles

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[*] Butschli, O. *Zeitschrift Wiss. Zool.*, xxiii., 1873.

[†] "If it is permissible to refer the efferent sexual ducts to metamorphosed nephridia, we should have to ascribe to *Sagittæ* at least two trunk somites, and accordingly explain the *Chætognatha* as forms in which, perhaps in connection with the manner of locomotion, a primitive segmentation of the body has been retained in a degenerated form only."—*Text-Book of Embryology*, by Korschelt and Heider (Translation), p. 371.

[‡] *An. Mus. Nat. Hist.*, Marseille, ii., 1884-5.

contain the sexual elements, and the great elongation of these may be correlated with the burrowing habit. The nervous system is still in connection with the ectoderm, the main ganglion dorsal to the mesomeres, and a post-oral ring gives off a ventral cord. A dorsal cord is superadded along the hypertrophied metameres.

In the protomere is the proboscis-vesicle, which may be the much-modified primitive excretory organ,* and the sub-neural sinus or heart, connected with a primitive vascular system.

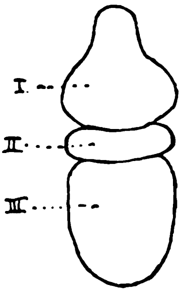

FIG. 8.—Larval *Balanoglossus* (after Bateson).

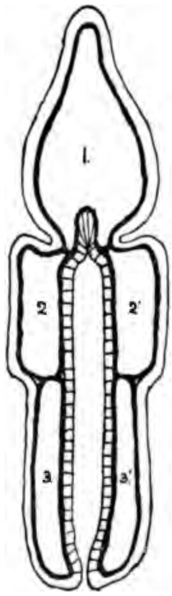

FIG. 2.—Diagrammatic horizontal section through young *Balanoglossus.*

The facts of ontogeny bear out our contention.

Bateson's larva has three distinct segments, each with its ciliated band, one pre-oral and two post-oral, and the cœlomic cavities of these arise, as archenteric diverticula, or by a slightly modified process. In *Tornaria* the three bands—the prototroch, mesotroch, and metatroch—can be discerned, and the development of the five cœlomic elements appears to be reducible to the type of archenteric diverticula. The full significance of *Tornaria* will be discussed with that of the trochosphere. Allowing for the alterations due to a long period of burrowing habit and the consequent adaptations, the anatomy of *Balanoglossus* is very close to that of the ancestral type.

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* *Q. J. M. S.*, August 1897, "On the Diplochorda."

Cephalodiscus (*Deep-Sea*).

The great anatomical resemblance to *Balanoglossus** is well known, and we need only note here that in this form the subneural gland is functional, and retains its primitive connection with the stomodæum. The persistence of mesomeric tentacles preoral in position, and of a reflexed alimentary canal, can be traced to the semi-sedentary habit, and the retention of the protomere and hypertrophy of the pedicle (or ventral sucker) to its peculiar mode of locomotion and attachment.

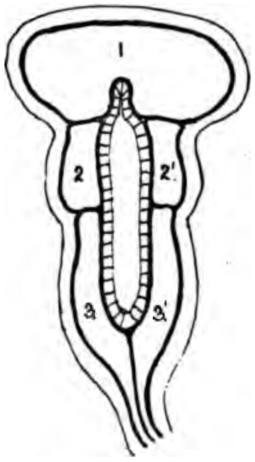

Fig. 3.—Diagrammatic horizontal section through young *Cephalodiscus*.

Fig. 3 shows the young form in horizontal section, with the five archicœles. We have only to add that, as in *Balanoglossus*, the protocœlic pores (proboscis-pores) and mesocœlic pores (collar-pores) are functional, and the metacœlic pores are purely gonaducal.

Brachiopoda (*Sedentary*).

In this archaic group there is no indication of true metameric segmentation. The presence of two pairs of nephridia in *Rhynchonella* cannot be regarded as such, for, apart from other considerations, the archimeric segmentation allows of two pairs of post-oral segments.

The external form of the *Brachiopoda* is profoundly modified by the sedentary habit of the group, so that the most striking evidence for its claims to be regarded as a division of the *Archi-*

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* S. F. Harmer, *Challenger Reports*, vol. xx., and A. T. M., *Q. J. M. S.*, August 1897.

cœlomata will be furnished by ontogeny. The body naturally falls into three parts, consisting of (1) the pre-oral epistome, which is greatly elongated laterally, (2) the rest of the lophophore, produced laterally into two arms, which have cirrhi or tentacles, and (3) the rest of the body (see figs. 4 and 6). These three segments can be directly compared to the three archimeric segments. Each contains a special section of the cœlome, the relationships of which to each other have been carefully worked out in *Crania* by Joubin.[*] In this species the protocœle (*Canal de la lèvre*, Joubin) does not open to the exterior by protocœlic pores, but has secondarily acquired openings into the metacœles through a peri-œsophageal sinus. The mesocœles (*Canal de cirrhes*, Joubin) open together into this sinus, and hence into the protocœle.

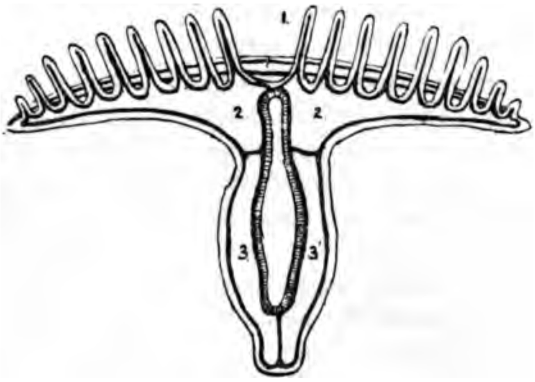

FIG. 4.—Diagrammatic horizontal section through *Phoronis*, *Brachiopoda*, and *Ectoprocta*.

In this manner both the protocœle and the mesocœles no longer open to the exterior, but communicate secondarily with the metacœles, which, on their part, lead to the exterior by the 'nephridia' or metacœlic pores. A similar arrangement is found in *Phoronis*, and some *Ectoprocta*.

Korschelt and Heider, in grouping *Brachiopoda*, *Phoronis*, and *Ectoprocta* together under the old term *Molluscoida*, especially emphasise this division of the cœlome into the epistome-cavity (*grosser Arm sinus der Brachiopoden*), the lophophoral cavity (*Ringkanal der Gymnolämen*, *kleiner Arm sinus der Brachiopoden*), and

[*] Joubin, *Arch. Zool.*, Exper. (2) iv.

the trunk-cavity.* It is probable that these cavities are homologous in the groups mentioned, and the suggestion is here made that they are further homologous to the archimeric segments of the typical *Archi-cœlomata.* Whether these three groups are phyletically related more recently than by means of the archicœlomate pelagic ancestor is at least open to doubt. As regards the other features of the *Archi-cœlomata*, the *Brachiopoda* have

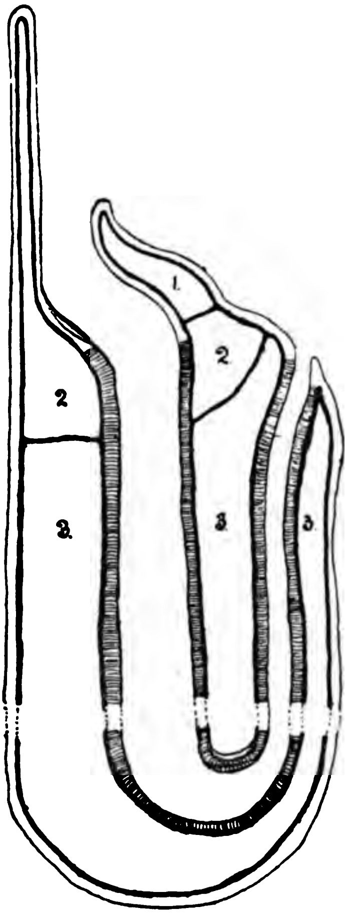

FIG. 6.—Diagrammatic sagittal section through *Phoronis*, *Brachiopoda*, and *Ectoprocta*.

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* *Lehrbuch, d. v. Entwickel, d. Wirb. th.*, p. 1251.

a simple nervous system which at least in some (*Argiope*, Shipley)[*] is in continuity with the ectoderm. It consists, typically, of a protomeric ganglion (supra-œsophageal) a post-oral mesomeric ring with one or two mesomeric ganglia (Van Bemmelen,[†] Joubin), from which the posterior part of the body is supplied with simple nerves. The arms of the lophophore, consisting of epistome (protomere) and mesomere, are supplied by nerves from both the supra- and the sub-œsophageal ganglia (Van Bemmelen).

The vascular system, if existent, is in the rudimentary condition (Blochmann,[‡] Hancock[§]) of simple sinuses between the cœlomic walls, or splits in the mesenteries. In several forms there has been described, between the protocœle and the mesocœle, a complicated peri-œsophageal sinus or plexus of sinuses, which is in direct connection with, and forms the posterior part of, the protocœle, with which the blood system comes into connection. More definite knowledge is required upon this point, but it is possible that further research may reveal a trace of the archimeric heart or subneural sinus in this position. (*Cf.*, specially Shipley, *loc. cit.*, pl. 39, fig. 1, *b.v.*, and Beyer, *Stud. Biol. Lab.*, *J. Hopk Lab.*, 1886.) In the adult, at least, there appears to be no trace of a subneural gland.

The gonads are confined to the metacœles and the metacœlic nephridia act as gonaducts.

Ontogenetic.

Free larval forms are found in the *Brachiopoda*, and all those as yet known present a well-defined division into three segments. This has been especially emphasised by Morse[||] in his attempt to prove the annelid affinities of the group (fig. 12). This comparison of *Brachiopoda* with larval *Annelida* acquires a new significance in the light of the theory here propounded, as will be shown later. *Thecidium* (Lacaze Duthiers) (fig. 11), *Terebratulina* (Morse), (fig. 12), *Argiope* (Kowalewski, Shipley) (figs. 9, 10), present larval

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[*] *Mittheil, Neap. Zool. Stat.*, IV.

[†] *Jenaische Zeitsch.*, 1883.

[‡] *Zoolog. Anzeiger*, viii., 1885.

[§] *Phil. Trans.*, 148, 1858.

[||] *Proc. Boston Soc. N.H.*, xv. p. 315-372.

forms, all showing a well-marked segmentation into one pre-oral and two post-oral segments. The pre-oral portion or protomere appears to become the epistome of the adult,* whilst the mesomere becomes the lophophoral cirrhi and the mantle lobes, into which the metacœlic cavities are later produced. This segment, as in *Annelida*, bears provisional setæ (*m.s.*, figs. 10 and 9). To this we may add that the mesoderm arises by archenteric diverticula.

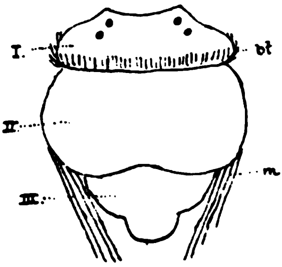

FIG. 9.—Larval *Argiope* (after Shipley).

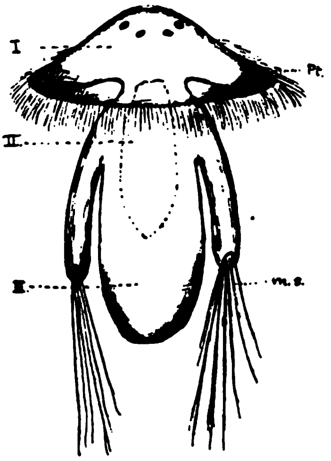

FIG. 10.—Later larval *Argiope* (Kowalevski).

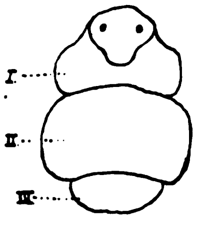

FIG. 11.—Larval *Thecidium* (after Lacaze-Duthiers).

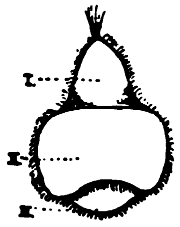

FIG. 12.—Larval *Terbratulina* (after Morse).

There are two difficulties to be mentioned. The cœlomic pouches have not been described as breaking up into four archicœles, and hence the larval segments are not considered as segments by some.

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* *Cf.* Korschelt and Heider, *loc. cit.*, p. 1238, fig. 720.

Such figures as those of Shipley * seem to indicate a later formation of archicœles, and in a larva of *Discina* (?) figured by Joubin,† the mesoderm is distinctly segmented into four archicœles.

It seems likely that further work on these larvæ will show a condition of the mesoderm, as in fig. 13. Apart, however, from these doubtful examples, it has been seen that the mesoderm in the adult is almost completely shut off into five cavities which correspond in major details with the segments derived from the larval segments.

Again, it has been maintained that the larvæ present *four* segments (*cf.*, Shipley, *Argiope*, and Lacaze-Duthiers, *Thecidium*).

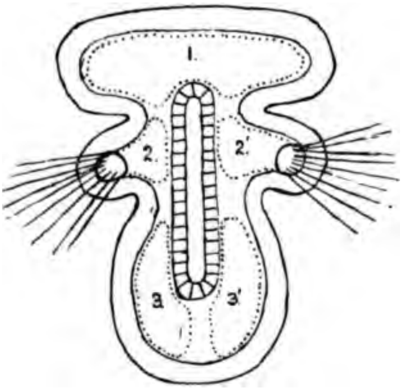

FIG. 13.—Horizontal section through larval *Argiope* (partly after Shipley); the mesoderm is dotted and hypothetical.

The former includes the developing peduncle (fig. 9), which may be regarded as a process of the metacœles not of segmental value, and the latter includes a frontal portion of the protomere which bears the eyes. Neither can be said to have any claim to be regarded as segments.

Lastly, the suggestion that the segments of the *Brachiopoda* are at right angles to the primary oro-anal axis, appears to have even less foundation in fact than that the same is the case in *Phoronis.*‡ In such a larva as that of *Argiope* the segments follow from before backwards along the primary axis as normally as in any chætopod larva, and apart from the fact that the metameres are perhaps bent upon themselves, their inter-relationships are unaltered in the adult. The *Brachiopoda* are still placed by some writers with the *Polyzoa* and *Phoronis*, as the *Molluscoida* (*cf.*, Hatschek), and it is interesting that at least one specialist on the group, Van Bemmelen* regards them as allied to the *Chætognatha.* Mr Shipley,† in criticising his arguments, points out that there are no lophophore, shell, or stalk, in the latter. On the theory here put forward we must assume that the *Chætognatha* are an ancient group which have remained in a free swimming pelagic habitat, and the *Brachiopoda*, another archaic group, which has, for an enormous period, become adapted to a sedentary existence.

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* *Loc. cit.*, pl. xl. figs. 33 and 35.

† *Loc. cit.*, pl. xiv. fig. 20.

‡ *Cf.* Caldwell, *Proc. Roy. Soc.*, 1883, and E. R. Lankester, *Ency. Brit.*, 'Polyzoa,' p. 171.

The presence of a ventral pedicle of attachment, the lophophore and shells may be legitimately connected with the adaptation to a sedentary environment. We are justified in this assumption by a comparison of *Balanoglossus* and *Cephalodiscus*, and their respective habitats.

The Cephalula larva, representing the neo-embryonic stage of Hyatt, reminds one, in many features, of the typical *Archi-cœlomata*, and is evidently its phyletic equivalent. After this stage the development diverges from all other known types, so that we are justified in supposing that the Brachiopod group only meets the rest of the *Cœlomata* at the archi-cœlomate stage.

Echinodermata.

The *Echinodermata* are usually regarded as owing their radial symmetry and some other features to a descent from sedentary ancestors. There appear in the greatly modified present-day forms to be representatives of the right and left metacœles, the left (and right?) mesocœle, and even the protocœle (axial sinus, parietal canal).

In most of the *Echinodermata* there are found the typical characters of the *Archi-cœlomata.* The nervous system is typically in connection with the ectoderm, and in the form of a ring round the mouth (post-oral), and certain lateral cords. The cœlome is simple, and the gonads are confined to the metacœles, possibly to the left metacœle.* Here, as in the *Archi-chorda*, the gonads open directly to the exterior, and the metacœle no longer has paired nephridial openings. The vascular system when present consists of simple inter-cœlomic spaces, and there appears to be a trace of the archi-cœlomate central blood-space, though there is some doubt as to the true nature of the organ.† As in *Phoronis* and *Brachiopoda*, so here the neo-embryonic stage of the free swimming larva (*Dipleurula*) forms a culminating point in the elaboration of organs upon the bilateral archi-cœlomate type. The larval forms will be referred to later, but we need only note here that there is clear evidence of the penta-cœlomic condition of the mesoderm.

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* *Jenaische Zeitschrift*, 1883. *Cf.* Hertwig.

† *Loc. cit.*

The anterior enterocœle or protocœle was first recognised, I believe, by Bury ‡ as such, and the presence of two hydrocœles or mesocœles by Metschnikoff in certain larval forms, whilst the two metacœles have always been easily identified.

Bury,§ however, in the construction of his free ancestor of the *Echinodermata* does not appear to recognise the presence of a well-developed pre-oral lobe or protomere, and indicates the anterior enterocœle as no more prominent in this stage than in some of the present-day *Echinodermata.* Macbride|| has, however, by a happy comparison of the anterior enterocœle and hydrocœle with the proboscis-cavity and left collar-cavity of *Balanoglossus* respectively, shown to my mind the true relationships of the parts in the *Dipleurula*, and Bury's anterior enterocœle then is clearly comparable to the archi-cœlomate protocœle, the hydrocœles to the mesocœles, and the two posterior enterocœles to the metameres (fig. 5). As in so many sedentary *Archi-cœlomata*, the protomere becomes vestigial, the mesomeres (or, more probably, only the left one) become much branched, and serve first as ingestive organs, and later, upon the re-acquirement of a free life, as locomotory

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* E. W. Macbride, *Quart. Journ. Micro. Sci.*, vol. xxxiv. p. 149.

† H. Bury, *Quart. Journ. Micro. Sci.*, vol. xxxviii. p. 129, and G. W. Field, *Q. J. M. S.*, vol. xxxiv.

‡ *Q. J. M. S.*, App. 1889, and *Phil. Trans. Roy. Soc. Lond.*, vol. clxxix.

§ *Q. J. M. S.*, vol. xxxviii.

|| *Proc. Roy. Soc.*, vol. liv.

organs, and the metameres show, as already remarked, the archicœlomate features.

There is no indication at any stage of true metameric segmentation.

In certain of the larvæ, with well-developed protomere, the archi-cœlomate central nervous system is present in this organ. The mesoderm arises typically by enterocœlic invagination, and, according to present knowledge, a single protocœlic pore (water pore) persists, whilst the left mesomere does not open to the exterior, but secondarily into the protomere (stone canal).

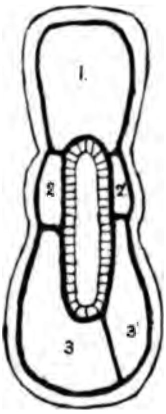

FIG. 5.—Diagrammatic horizontal section through larval *Cribrella*.

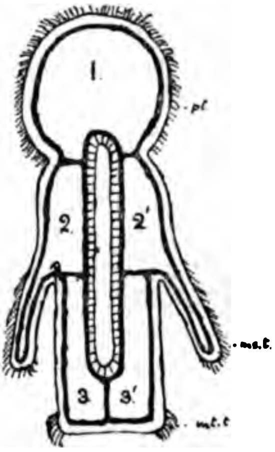

FIG. 7.—Diagrammatic horizontal section through *Actinotrocha*.

The free larval forms will be referred to later.

Phoronis.

This essentially sedentary and tubicolous group has been the subject of my recent work.* Its free larval stage, *Actinotrocha*, exhibits (fig. 7) the typical archimeric arrangement of the cœlome in perfection, besides having a prototroch, mesotroch, and metatroch. In all the other features of the *Archi-cœlomata* it is typical, except that the metacœles do not, in the larval stage, open to the exterior.

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* "On the Diplochorda," *Q. J. M. S.*, August 1897.

In the adult the body-form, as in the *Brachiopoda*, has undergone great modification, and differs little from fig. 4 in general features, whilst fig. 6 shows the inter-relationship of the archicœles. As in *Brachiopoda*, the protocœle communicates secondarily with the mesocœles, and the metacœlic pores are nephridial and gonaducal.

The simple nervous system, in continuity with the ectoderm, with a protomeric ganglion and a post-oral nerve-ring, and the arrangement of the vascular system, are all in close agreement with the archi-cœlomate type. The true zoological position of this group will be discussed in a paper upon its anatomy now in the course of completion, so that its claims to be considered one of the *Archi-cœlomata* can alone be dealt with.

The actual fate of the five archicœles in *Actinotrocha* has not been fully followed, but there is some ground for believing that the epistome-cavity is derived from the protocœle, the 'body-cavity' in front of the septum from the mesocœles, and that behind it from the metacœles (see fig. 4).

Polyzoa.

This very heterogeneous group may yet prove to contain several distinct phyla, and is already held by many to be di-phyletic, the *Ectoprocta* and *Entoprocta* being regarded as convergent.

In Korschelt and Heider's work, and more recently in Parker and Haswell's text-book, the *Polyzoa* (*Ectoprocta*), *Brachiopoda*, and *Phoronis* are placed together under the old title *Molluscoida* (*cf.* figs. 4 and 6).

Their features in common are most probably to be derived as follows:—Firstly, there is an underlying basis of resemblance in fundamental characters, such as archimeric segmentation * and other characters, which have been given here as archi-cœlomate characters, which these groups possess, in virtue of a common descent, from an archi-cœlomate ancestry, and to this extent the groups are genetically connected. Secondly, there is a series of more superficial morphological resemblances, which are to be traced directly to the influence of the sedentary habit which each group has independently acquired. These comprise *the reduplication of the gut* (which is apparently dorsal in *Phoronis*, *Ectoprocta*, and *Brachiopoda*, but ventral in *Entoprocta*), *a reduction of the protomere to a vestige, the epistome, or to extinction, and the forward projection of the tentacular processes of the mesomeres to form a lophophore.* It is not difficult to derive these characters as directly due to the environment of a sedentary animal, by a consideration of types like the *Cirripedia*, *Tunicata*, *Sipunculids*, and the groups under consideration.

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* *Cori. Zeits. f. w. Zool.*, Bd. 51.

Such being the case, the systems of classification which depend upon such characters as these are of the same nature as those in which all triploblastic forms of an elongated type are thrown together as *Vermes.*

Thus all the community of structure in the *Molluscoida*, which is not directly traceable to a similarity of environment, is no more than that which exists in the other phyla of the *Archi-cœlomata* cited here, and this fact, taken in conjunction with the consideration that there are very marked differences in structure in the groups, justifies us in assuming that the *Brachiopoda*, *Ectoprocta*, and *Phoronida* are separate phyla of the *Archi-cœlomata.*

The evidence with regard to the *Entoprocta* is somewhat of the same nature as that in the case of *Rotifera* and *Sipunculids*, to be referred to later.

Ontogeny—Derivation of the Mesoderm and Archicœles.

In connection with the gastrœa theory of Hæckel, there are shown to be in existence a great number of animals, which, extremely diverse in habits and in structure, yet retain the diploblastic structure of the gastrula, as an underlying basis of their organisation. These animals, in their own ontogeny, do not by any means, without exception, show the typical mode of formation of this diploblastic condition, the second layer being formed in different cases by unipolar or multipolar ingression, by delamination or by true embolic invagination.

In the case in point it will not be necessary to the truth of the theory here put forward, that all the forms which have an archimeric segmentation as the fundamental basis of their organisation, should show an ontogeny in which the mesodermal layer or cœlome is formed in the phyletic manner. At the same time, it is remarkable to find that most of the *Archi-cœlomata* agree in one particular method of mesoderm formation.

As early as 1876 Huxley suggested an alliance of the *Enteropneusta*, *Echinodermata*, *Brachiopoda*, and *Chætognatha* together under the title *Enterocœla*, to emphasise the development of the cœlome in these groups by archenteric diverticula.* In *Phoronis* there is said to be a modified form of this method † and the development of *Cephalodiscus* is unknown. In the *Mollusca* the mesoderm is broken up and largely replaced by blood-spaces, and in many of them an archenteric method of mesoderm formation is pursued.

From the preceding portion dealing with the suggested derivation of the mesoderm of the *Archi-cœlomata* from archenteric diverticula, it is evident that for a correct repetition of phylogeny the archimeric segments or their mesodermic elements should arise as follows:—Four diverticula of the gut should arise in a horizontal plane—one pre-oral, two lateral, and one post-anal. The post-anal should then divide into two to form a pair of metacœles.

This method is closely followed by such a type as *Balanoglossus*, in the demersal larva of *B. Kowalevski*.

The other *Archi-cœlomata* do not, as far as is known, derive their mesoderm in such a typical manner, but if this method of formation be assumed to be the primitive and phyletic one, it is not difficult to derive the other methods of mesoderm-formation as modifications due to secondary ontogenetic processes.

According to our theory, the axis of radial symmetry in the primitive cœlomate corresponded with the central axis of the archenteron, and the four archimeric cœlomic pouches arose symmetrically to this axis and at the aboral end.

On the assumption of bilateral symmetry, the main axis of the gut by an elongation in one direction came to lie in a horizontal plane, and hence at right angles to the archimeric axis of symmetry.

In a truly phyletic ontogeny this change should be brought about by an elongation of the blastopore in the direction at right angles to the axis of the gastrula and the closure of its intermediate portion to form a pair of apertures, the mouth and anus.

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* Huxley, T. H., *Journ. Linn. Soc.*, vol. xii.

† Caldwell, W. H., *Quart. Journ. Micro. Soc.*, 1885.

In ontogeny, especially in larval forms, the assumption of the secondary axis of the gut is greatly hastened, so that the blastopore instead of becoming closed in this manner, becomes carried over to form the mouth or anus alone, the other aperture (anus or mouth respectively) being formed later by secondary invagination. The natural result of this acceleration is that, whilst the archenteron is only just sufficiently differentiated to give off its cœlomic diverticula at the distal or aboral end, its main axis has already become parallel to the bilateral axis and the proximal end is already formed into the mouth or anus. If the former takes place, then the cœlomic pouches will arise from the posterior end of the gut as two diverticula, which will move *forwards* in the horizontal plane, and will give rise to *paired* rudiments of the protocœle, then the paired mesocœles, and, finally, the metacœle (fig. 18). On the other hand, if the latter takes place, *i.e.*, the blastopore becomes precociously the anus alone, then the mesoderm will arise from the distal extremity as before as paired diverticula, which will move *backwards* and will give rise distally to paired metacœles, paired mesocœles, and, lastly, an unpaired protocœle (fig. 17).

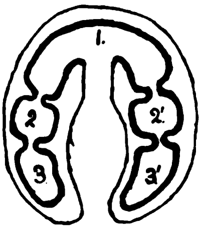

FIG. 17.—Horizontal section through embryo (typical) of *Echinoderm* to show formation of mesoderm. Blastopore forms anus.

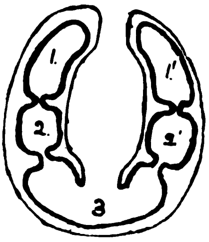

FIG. 18.—Horizontal section through embryo in which blastopore becomes mouth, or in which it closes, and the stomodæum is early formed, as in *Sagitta*.

The former method of archicœle formation is apparently pursued in the *Chætognatha* and the *Mollusca* (*Patella*),* and the latter in a great number of the *Echinodermata*.

Lastly, if the blastopore closes completely, and the archenteron loses all connection with the epiblast, then it will not be affected by the distorted sequence of events, and will give rise to the archicœles in the primitive manner as separate diverticula. The same result will occur if the blastopore forms neither mouth nor anus, and in each of these cases there is apparently no essential reason why the archicœles should not arise in quite as typical a manner as in the phyletic ontogeny, in which the blastopore becomes both mouth and anus.

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* Patten, W., *Arb. Zool. Inst. Wien*, 1886.

The facts again agree with this deduction, for in *Balanoglossus*, which has been cited as a form in which the archicœles arise in a primitively independent manner, the archenteron loses all connection with the epiblast. Again, in *Antedon* the archenteron becomes a free vesicle in the interior of the embryo,* and here there is a formation of the archicœles, which is usually regarded as of a modified type, but which appears to me to be not only closely comparable to that of *Balanoglossus* but to be primitive. With the exception of the absence of a right mesocœle (right hydrocœle) in *Antedon* the types are identical. It is scarcely necessary to point out that any difference in point of time in the formation of the protocœle, mesocœles, or metacœles is immaterial, although the synchronous formation is probably the more primitive. Thus, in this case the archenteric vesicle first gives off the

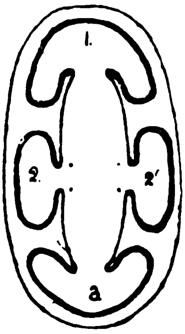

FIG. 16.—Horizontal section through embryo of *Balanoglossus* to show formation of mesoderm.

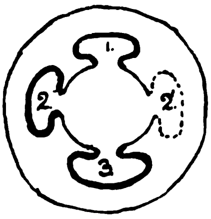

FIG. 21.—Section through *Antedon* embryo to show formation of mesoderm. (The mesodermic pouches are represented as synchronous in origin.)

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* Bury, H., *Phil. Trans. Roy. Soc. Lond.*, vol. clxxix.

metameric (posterior enterocœles), then the left mesocœle (hydrocœle), and, lastly, the protocœle (anterior enterocœle).

In the other *Echinodermata* the open part of the blastopore becomes the anus or opens at the posterior end of the body till after the mesoderm has commenced to arise (anus of bilateral larva). Thus in these, in accordance with the law stated above, the archicœles arise from the distal (front) end of the archenteron, and give rise backwards to metacœles, mesocœles, and protocœle. The main irregularity consists in the fact that the right mesocœle (right hydrocœle) may be absent or vestigial, whilst there are in the various groups variations with respect to the relative time of division of, *e.g.*, the metacœle into its two portions, and of the metacœle from the other archicœles, and, in addition, the formation of the protocœle has not been so clearly followed in some as in others.

It is usual for a pair of vaso-peritoneal vesicles to be formed, which each divides up into mesocœle (hydrocœle) and metacœle (enterocœle).*

In *Synapta* † the metacœle is at first unpaired, and only later divides into two, but, in addition, the assumption by the archenteron of the secondary axis on the one hand, and the formation of the protocœle pore on the other hand, are both so accelerated that the archenteron at one time appears to have two apertures to the exterior, the future anus, and the water-pore.

It can scarcely be maintained that the hydrocœles and enterocœles of the *Echinodermata* are not respectively homologous throughout the group, and their variations in development appear to be inexplicable except as modifications of a primitive type of archicœle development, such as suggested here.

The archimeric segmentation may, therefore, be described in its ontogenetic characters as consisting primarily of four archicœles, sometimes secondarily divided into five or six, which arise primitively as four archenteric diverticula, but which may by a simple modification arise as a pair of diverticula, which later divide up into the archicœles themselves.

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* In certain pelagic larvæ, *e.g. Bipinnaria*, the functional œsophagus is early formed although not actually part of the blastopore. In these the two lateral cœlomes move forwards later to form the protocœle.

† Salenka, *Jena. Zeits.*, xxii., 1888.

The Larval Forms.

One outstanding feature about the *Archi-cœlomata* is the constant occurrence of free pelagic larval forms, and the comparison of the demersal larva of *Balanoglossus Kowalevski* with a *Tornaria*, and a *Bipinnaria* or *Brachiolaria* with the demersal type of larva, such as that of *Asterina gibbosa*, enables one to conjecture to a large extent what are the secondary or cœnogenetic features of these pelagic larvæ. We have assumed that the archi-cœlomate ancestor was pelagic in its habits, and with this fact in view the pelagic habits of so great a number of larvæ in the *Archi-cœlomata* acquire a phyletic significance. In other words, the vast majority of the *Archi-cœlomata* pass through a pelagic stage in their ontogeny, because their common ancestor was pelagic. Amongst these larvæ there are endless instances of the archimeric segmentation into three parts, protomere, mesomere, and metamere, whether exemplified by actual division of the body into three parts by constrictions, as in *Actinotrocha* (fig. 7), larval *Balanoglossus* (fig. 8), the larval of the *Brachiopoda* (figs. 9, 10, 11, 12), or whether indicated by the structure of the three ciliated bands, prototroch, mesotroch, and metatroch, as in *Tornaria*, *Echinoderm* larvæ, *Actinotrocha*, and *Polyzoan* larvæ.

This constant occurrence of archimeric segmentation, in however modified a manner it appears, must indicate a common descent from a form with this archimeric segmentation fully developed.

(*C.*) *Evidence may be derived from morphology and from ontogeny of the metamerically segmented groups that a secondary segmentation (metameric) has been phyletically superposed upon the primary (archimeric). Thus, these animals may be shown to pass through ontogenetic stages which closely resemble the archi-cœlomate type, and to possess in their morphology more or less vestigial traces of the archimeric segments.*

More than two years ago I remarked[*]:—"I believe that these resemblances will all eventually find expression in the constitution of one primitive group . . . . having the mesoderm divided up into one pre-oral and two pos-toral cœlomic pouches, all, primitively,

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[*] *Proc. Royal Soc. Edin.*, 1895-96, p. 63.

opening to the exterior by ciliated pores, the earliest condition of nephro-gonaducts. . . . . From this type, *by segmentation of the trunk or third segment*, are derived the *Annelida* and *Chordata.*"

Later on* these views were given in detail, and the groups of *Archi-chorda* and *Eu-chorda* were shown to differ, among other features, in the fact that, in the latter, the third segment or trunk became secondarily bilaterally segmented, and that the other archimeric segments became reduced by a migration forwards of the newly formed metameric segments. The justification for this view lay in the presence of a pre-oral archenteric pouch in *Amphioxus* larva, which early divides into two, and which has been compared by many writers to the proboscis-cavity of *Balanoglossus*, the rest of the mesoderm being formed by a series of somites arising from the posterior end of the archenteron.

The main difficulty in the comparison lay in the absence of collar-cavities in *Amphioxus*, or its larva. This difficulty has now been removed by the discovery in this form of a pair of archenteric pouches which have been identified with the collar-cavities of *Balanoglossus.*† The author remarks—"The whole process of mesoderm formation is, therefore, referable to the type formed in *Balanoglossus*, the main difference being that the pouch corresponding to the trunk cœlome of *Balanoglossus* becomes segmented," and his work is, therefore, a direct confirmation of the views to some extent put forward by Morgan, and later, but independently, by myself.

Professor Macbride's work will undoubtedly give strong support to "the theory of the descent of the Vertebrates from a form somewhat like *Balanoglossus*"; but, from the views expressed elsewhere, I look to *Actinotrocha* as a proximate morphological presentment of the "form somewhat like *Balanoglossus*" rather than *Tornaria.*

The archimeric segments persist in the adult as the head-cavity, and the pre-oral pit, representing the protocœle, and the first myotome and metapleural cavities, which are the mesocœles (see figs. 19 and 20).

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* *Quart. Journ. Micro. Science*, August 1897.

† E. W. Macbride, *Quart. Journ. Micros. Science*, Jan. 1898.

In the *Holochorda* or *Vertebrata* the premandibular head-cavities, forming most of the eye-muscles, have already been compared by several writers * to the paired elements of the protocœle, whilst the mandibular cavities are under suspicion as the collar-cavities, though further knowledge is required concerning their development. Their innervation (see below) agrees with this interpretation.

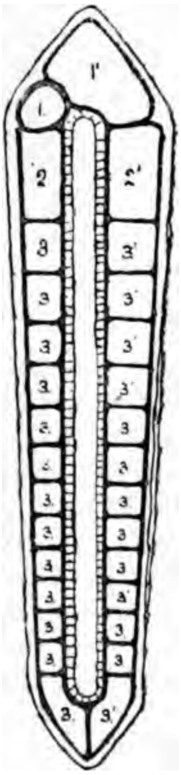

Fig. 19.—Horizontal section of *Amphioxus* larva (diagrammatic). The identification of the mesocœles (2) is due to M'Bride.

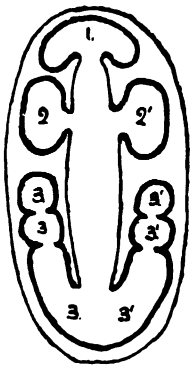

Fig. 20.—Horizontal section of *Amphioxus* embryo diagrammatic (after M'Bride) to show formation of mesoderm.

In the *Archi-chorda* the archimeric segments are clearly defined externally, but in *Amphioxus* they have become no longer definable, except in their mesodermic elements.

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* *Cf.* A. Willey, *Amphioxus and the Ancestry of the Vertebrates*, pp. 128–129, and 279.

Owing probably to the fact that *Amphioxus* has degenerated to a considerable extent, especially in the nervous system, one can find more traces of the archimeric segmentation in the *Holochorda* or *Vertebrata.* Here the neural tube, very typically and very early, shows a segmentation into three primary vesicles, the explanation of which has been obscure. In the light of the theory here put forward, the fore-brain corresponds to the protomere, and is evolved from the protomeric ectoderm. Such being the case, it is not surprising to find that its actual wall gives rise to sense-organs (olfactory and optic), the protomere of the *Archi-cœlomata* being typically sensory.

Again, the mid-brain corresponds to the mesomeric or collar nervous ganglion, with its two nerves (III. and IV. cranial), supplying, respectively, the protocœle (pre-mandibular somite) and the mesocœle (mandibular somite), just as nerves direct from the collar-ganglion or archimeric central nervous system supply the protocœle and mesocœle in *Actinotrocha* or *Balanoglossus* (muscles of proboscis and post-oral ring).

Posterior to this is the elongated nervous tube or neurochord with its front end forming the hind-brain. This corresponds with the greatly specialised dorsal ectoderm of the metamere, a subordinate part of the body in *Archi-chorda* supplied with simple chords, but forming almost the whole of the body in the *Eu-chorda.* These relationships are even better brought out by a comparison of other organs in *Actinotrocha* and in the lowest *Eu-chorda.* This comparison is justified, because *Cephalodiscus, Phoronis, Balanoglossus, Tunicata,* and *Amphioxus* all have either sedentary or burrowing habits, and just to the extent that they are adapted for these habits do they differ in structure from the archi-vertebrate, which, at least for the greater part of its history, must have been pelagic from *Actinotrocha* onwards.

This cannot be further followed out here, but enough has been written to show that there is good evidence for believing that the *Eu-chorda* have been derived from the *Archi-chorda* by secondary segmentation of the third archimeric segment or metamere and gradual atrophy of the protomere and mesomere, as closed cœlomic pouches, though their walls persist as muscles.

In the other groups of segmented animals, namely, the *Annelida* and *Arthropoda*, it is possible to make out a case of the same nature. The *Arthropoda* are greatly modified, especially in regard to their mesodermic organs; but as they are very commonly regarded as being derived from a segmented 'worm' of some kind, it will be sufficient for our present purpose to take the *Archi-annelida* and *Polychæta* as the lowest truly segmented worms.

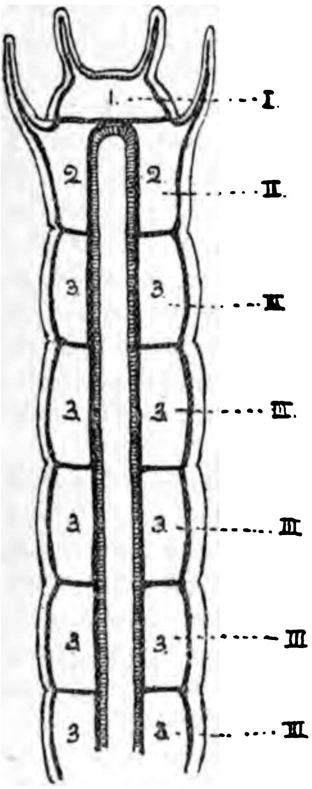

FIG. 22. — Horizontal section through typical Annelid, showing the arrangement of the mesoderm.

In the morphology of these there is a constantly recurring pre-oral 'segment,' the prostomium, derived from the pre-oral lobe of the larva, intimately associated with which is the supra-œsophageal nerve-ganglion. This segment is unpaired and may bear special appendages, such as palpi and antennæ (or tentacles). It is lined by a mesodermic wall, and in certain forms (*e.g.*, *Magelona*) it comes to very closely resemble the epistome of some of the *Archi-cœlomata* (*e.g.*, *Rhabdopleura*). Its homology with the protomere of the archimerically segmented forms is supported by its structure and development (fig. 22, *1*).

The segment immediately following the prostomium, the peristomium or buccal segment (fig. 22, *2*), differs in many respects from the segments following after. It contains the post-oral nerve-ring and the sub-œsophageal nerve-ganglion, has a definite cœlomic space, and, in many cases, has certain processes (tentacular cirri) which recall to mind the post-oral tentacles of the *Archi-cœlomata*. This is especially the case in certain of the sedentary or tubicolous forms, such as *Magelona*. In the *Archi-annelida* the peristomium is simple, but well-defined and prominent (fig. 22, *2*).

Thus, morphologically, the peristomium differs markedly from the hind-segments, and ontogeny shows a still greater difference. This 'segment' I would homologise with the mesomere of the *Archi-cœlomata*, whilst all the segments posterior to it are secondarily formed from a bilateral segmentation of the metamere or last segment (fig. 22, *3*).

In the young of *Nerine* and other species, the peristomium is found to bear long bristle-like provisional setæ, which are also found in adult fossil forms. The suggestion has been made that they are survivals of organs belonging to an unsegmented ancestral stage.* For 'unsegmented,' we may say *archimerically segmented*, and they may then be at once compared with the mesomeric setæ of the Brachiopod larvæ. On this view, the three segments of the latter are morphologically equivalent to the prostomium, peristomium, and the rest of the body of the Annelid. Thus, both the *Brachiopoda* and *Chætognatha*, as also *Balanoglossus*, may, in a sense, be described as 'worms' with three segments only, but these three are not equivalent to the first three of the Annelid. We must rather regard the Annelid as consisting of three archimeric segments, in which the third is hypertrophied and segmented.

In the ontogeny of the most primitive *Annelida* (*e.g.*, *Polygordius*), the larval trochosphere has a pre-oral, a post-oral, and a peri-anal band of cilia, which we may regard as the prototroch, mesotroch, and metatroch respectively.

The first two have nerve-rings in connection with them, and the post-oral nerve-ring apparently forms the peri-œsophageal ring of the adult (fig. 15).

The trochosphere is thus to a large extent divided into one pre-oral segment, the pre-oral lobe, and two post-oral (fig. 15, I., II., III.).

Internally, the cavities of the first two appear to be continuous, because the mesoderm in these segments is in the monocytic (mesenchymatous) condition. It is only later that the definite cœlomic lining of the pre-oral lobe or prostomium and that of the

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* Agassiz, A., *Annals Lyc. Nat. Hist., New York*, vol. viii.

peristomium are formed. The nephridia of the protocœle are absent, but the mesomeric are present in the early condition of 'flame-cell organs,' and are usually known as the 'head-kidneys.' The mesodermic bands arise in the third segment (peri-anal), and by a process, first of growth in length and then of bilateral segmentation, they form the cœlomic cavities of the adult.

It is clearly shown in the ontogeny of *Polygordius* that the 'worm' is formed from the larva *by an immense growth in length of the posterior part* (*metamere*), *immediately behind the mesomeric nephridia, and by a secondary segmentation of the two mesodermic bands filling this part.*

We might expect that the mesoderm of the protomere and mesomere (prostomium and peristomium) should arise as separate archenteric diverticula from the gut, but a consideration of the precocity of ectoderm and endoderm (compared with mesoderm) in pelagic larvæ (*cf.*, *Tornaria* and Bateson's larva) accounts for the condition seen in *Polygordius*, whilst in *Lopadorhynchus* the whole mesoderm is said to arise in this way.* Therefore the typical trochophore larva must be regarded as the 'larvalised' embodiment of the archi-cœlomate type, and, mainly for this reason, we must regard the *Mollusca* as elaborations of the archi-cœlomate type, in which the mesoderm of the protomere and mesomere never forms true cœlomic spaces. The continuous epithelial condition of a cœlome is only an intermediate stage in its

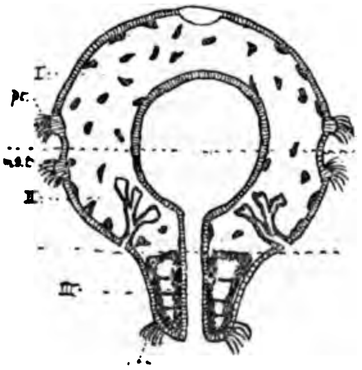

FIG. 15.—Horizontal section through typical trochophore.

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* Kleinenberg, N., *Zeitsch. wiss Zool.*, vol. xliv., 1886.

phyletic history, and in both its early stages (monocytic) and in its latest (muscles, etc.) its walls are discontinuous, and its enclosed space is confluent with the blastocœle (or hæmocœle).

Just as the history of *Echiuridæ* indicates that they may be metamerically segmented forms, which have secondarily lost their metamerism, so that of the *Rotifera* points to the conclusion that they may be descended from archimerically segmented forms, and have lost the archimeric segmentation of their mesoderm.

The recognition of this archimeric segmentation in larval annelids could be followed up more fully through the 'collar' of *Psygmobranchus*, *Spirorbis*, and *Pileolaria*, which evidently belongs to the mesomere, and through the peculiar prostomial organs found in so many species (fig. 23).

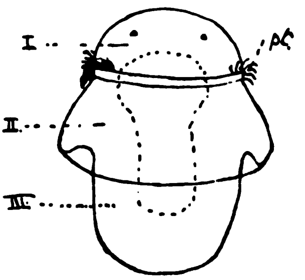

FIG. 23.—Larva of *Spirorbis* (after Fewkes), showing archimeric segmentation.

As regards the nervous system of the *Annelida*, it is evident that the supra-œsophageal ganglion belongs to the protomeric segment, and the sub-œsophageal to the mesomere. As the mesomeric ring disappears in the *Euchorda*, the supra-œsophageal ganglion is the only part of the annelid nervous system which is represented in that of the *Eu-chorda.*

In the higher *Annelida*, the *Arthropoda*, and the *Eu-chorda*, there is the same tendency to a reduction of the first two archimeric segments, and a gradual migration forwards of the metameric segments, and at the same time these tend to become grouped into regions in which the segments themselves become more or less unrecognisable. Thus, the archimeric, or primitive pre-oral, head disappears altogether, and its place is taken by a metameric secondary head, and in the same way the archimeric thorax or mesomere becomes replaced by a metameric thorax.

It will thus be seen that if the trochophore be held to be the larvalised representative of the archi-cœlomate type, then the *Rotifera*, *Entoprocta*, and some smaller groups, such as *Echinoderes*, can claim to be placed in the *Archi-cœlomata*, and the same applies to some at least of the Sipunculids, though more knowledge is required with regard to the origin and arrangement of the cœlome in these types.*

These groups are all placed with those which have been above referred to as one group, *Vermidea*, quite recently by Professor Delage,† and it would be superfluous to add here his arguments for this grouping. For the present, I would not weaken the argument by definitely claiming to be included in the *Archi-cœlomata*, other than the groups previously dealt with, as showing direct indications of an archimerically segmented mesoderm, with the other primitive characters cited.

We now have to inquire what bearing the above facts can have upon classification.

The *Platyhelminthes*, *Nemertea*, and *Nemathelminthes*, all are groups or phyla of the *Triploblastica*, which show more or less a monocytic condition of the mesoderm, and there is at present no evidence for regarding them as having degenerated from cœlomate forms. The body-cavity of the *Nematoda* approaches in some respects a cœlome, but more evidence is required. For convenience sake, they may be placed together as *Pseudo-cœla*. The rest of the *Triploblastica* give evidence of either being archimerically segmented or of having passed through an archimeric stage.

Those which remain with the primitive characters narrated above may be included in one group, the *Archi-cœlomata*, which would comprise the *Echinodermata*, *Archi-chorda*, *Chætognatha*, *Brachiopoda*, *Ectoprocta*, and possibly the *Endoprocta* and *Rotifera*. From forms allied to the *Archi-chorda*, the *Eu-chorda* have been further differentiated by metameric segmentation and further elaboration of structure, whilst from forms allied possibly to the *Chætognatha* and *Brachiopoda* have arisen the *Annelida* and *Arthropoda*, the further elaboration of structure in this case also involving metameric segmentation. Apart from this feature, the *Archi-annelida*

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* *Cf.* Shipley, A. E., *Quart. Journ. Micros. Sci.*, xxxi. There are difficulties in the way of this author's comparison of *Phoronis* and *Phymosoma*. If we accept his comparison in figs. 31 and 32, then the pre-oral lobe must be looked for, not as indicated by Mr Shipley, on the anal side of the tentacles, but on the oral side, where the epistome is found in *Phoronis*.

† *Traité de Zoologie Concrète*, vol. v.

show many archi-cœlomate characters. Lastly, from forms possibly somewhat allied on the one hand to the ancestors of the worms, and partly to the *Endoprocta*, the *Mollusca* have arisen. These have not become metamerically segmented; they retain the metacœles as simple cœlomic sacs with nephridia, but the protocœle and mesocœles are progressively broken up.

In 1876 Huxley wrote "A mollusk appears to me to be essentially an Annelid which is only dimerous, or trimerous, instead of polymerous" (*loc. cit.*). In this respect the *Mollusca* are archicœlomic but their high differentiation causes them to differ from almost every other feature of the *Archi-cœlomata*, so that they must be regarded as having progressed above this group, but without the aid of metameric segmentation. If the presence of archimeric segmentation without the metameric be the sole criterion, then they would rank as *Archi-cœlomata.*

The relationships of the *Archi-cœlomata* may be indicated by the accompanying Table, and they would be further expressed as follows:—

Triploblastica.
1. Pseudocœla.
2. Cœlomata.
    1. *Archicœlomata.*
        - A. Echinodermata.
        - B. Archi-chorda.
        - C. Chætognatha.
        - D. Brachiopoda.
        - E. Ectoprocta.
        - F. Endoprocta?
        - G. Rotifera?
        - H. Sipunculoidea.
    2. *Annulata.*
    3. *Mollusca.*
    4. *Eu-chorda.*

	Protomere.			Mesomere.		
	Protomere.	Protocœle.	Protocœlic pore.	Mesomere	Mesocœle.	Mesocœlic pore.
Hemichorda,	Proboscis.	Proboscis cavity.	Proboscis pore.	Collar.	Collar cavities.	Collar pores.
Diplochorda—						
a. *Cephalodiscus,*	Buccal shield.	Cavity of buccal shield.	Proboscis pore.	Collar or tentacular segment.	Collar cavities.	Collar pores.
b. *Phoronis,*	Epistome.	Cavity of epistome.	.. (Opens into mesocœle.)	Lophophoral segment.	Lophophoral cavity.	.. (Opens into metacœle)
Chætognatha,	Head.	Head cavities.	?	Trunk.	Trunk cavities.	Oviducts. ?
Echinodermata,	..	(Ampulla.) (Axial sinus.) (Parietal canal.)	Madreporite.	Water-vascular system (L)	Cavity of water-vascular system.	Opens into protocœle.
Brachiopoda,	Epistome.	Epistomial sinus.	(Opens into mesocœle.)	Lophophoral segment.	Lophophoral cavity.	(Opens into metacœle)
Ectoprocta,	Epistome.	Epistomial sinus.	(Opens into mesocœle.)	Lophophoral segment.	Lophophoral cavity.	(Opens into metacœle)
Larval *Phoronis,*	Pre-oral lobe.	Cavity of pre-oral lobe.	Proboscis pores.	'Collar.'	Collar cavity.	Nephridia (Larval).
Larval *Brachiopoda,*	'Head' segment.	..	..	Trunk segment.	..	..
Larval *Echinoderm,*	Pre-oral lobe.	Anterior b. c.	Water pore.	Water-vasc. system.	Hydrocœles.	Open into protocœle
Larval *Balanoglossus,*	Pre-oral lobe.	Anterior b. c.	Proboscis pore.	Collar.	Collar cavities.	Collar pores.
Larval *Polygordius,*	Pre-oral lobe.	Cavity of p. o. lobe.	..	Post-oral region.	Cavity of post-oral region.	Larval nephridia
Amphioxus,	Pre-oral lobe.	Pre-oral head cavities.	Pre-oral pit.	Post-oral region and metapleural folds.	Cavity of 1st myotome and metapleural cavities.	?
Annelida,	Prostomium.	Prostomial cavity.	..	Peristomium.	Cavity of peristomium.	?
Vertebrata,	..	Pre-mandib. head cavities.	..	..	Mandibular head cavities.	..

Metamere.			Larval Bands.			Nervous System.			Archimeric Heart.
ta- re.	Metacœle.	Metacœlic pore.	Proto-troch.	Meso-troch.	Meta-troch.	Protomeric Part.	Mesomeric Ganglion.	Metameric Part.	
nk.	Trunk cavities.	?	..	..	..	Nerves of proboscis.	Collar nerve-ganglion and post-oral ring.	Dorsal and ventral chords.	'Heart.'
nk.	Trunk cavities.	Gona-ducts. ?	..	..	..	Nerves of buccal shield.	Central nerve-ganglion and post-oral ring.	Lateral and ventral chords.	Sub-neural sinus.
nk.	Trunk cavity.	Nephridia	..	..	..	Nerves of epistome.	Central nerve-ganglion, post-oral nerve-ring.	Lateral chords.	..
il.	Tail cavities.	Vasa deferentia ?	..	..	..	Supra-œsophageal ganglion.	Trunk-ganglion and post-oral ring.	..	—
y of ult.	Body cavities.	Gona-ducts. ?	..	..	..	..	Nerve-ring.	Radial chords.	?
nk.	Trunk cavities.	Nephridia	..	..	..	Supra-œsophageal ganglion.	Sub-œsophageal ganglion and nerve-ring.	Nerve-chords.	(Heart.)
nk.	Trunk cavities.	Nephridia	..	..	..	..	Central nerve-ganglion.	..	..
nk.	Trunk cavities.	?	Pre-oral band of cilia.	Post-oral band with tentacles.	Peri-anal band.	Apical plate pre-oral ring.	Central nerve-ganglion post-oral ring.	Dorsal and ventral chords.	Sub-neural sinus.
dal ent.	..	..	Pre-oral band of cilia.	Mantle folds with setæ.	..	..	..	..	..
nk.	Entero-cœles.	Open into mesocœle.	Pre-oral band of cilia.	Post-oral band.	..	Apical plate.	Nerve-ring, post-oral.	..	..
nk.	Trunk cavities.	?	Pre-oral band.	Post-oral band.	Peri-anal band.	Apical plate.	Collar nerve-ganglion and ring.	..	..
nk.	Cavities of meso-blastic bands.	Nephridia	Pre-oral band.	Post-oral band.	Peri-anal band.	Apical plate and nerve ring.	Post-oral nerve-ring.	Ventral chords.	..
nk.	Cavities of meso-blastic somites.	Nephridia	..	..	..	Brain.		Dorsal neuro-chord.	Sinus of left aortic arch.
nk.	Cavities of meso-blastic bands.	Nephridia	..	..	..	Supra-œsophageal ganglion.	Post-oral ring and sub-œsophageal ganglion.	Ventral nerve-chords.	..
ly.	Cœlom.	Nephridia (kidneys).	..	..	..	Fore-brain.	Mid-brain.	Hind-brain and neuro-chord.	Trace in elasmo-branch embryo.

ABBREVIATIONS.

I. = protomere.
II. = mesomere.
III. = metamere.
1. 1′. = protocœle.
2. 2′. = mesocœle.
3. 3′. = metacœle.

pt. = prototroch.
ms.t. = mesotroch.
mt.t. = metatroch.
pt.p. = protocœlic pore.
ms.p. = mesocœlic pore.
mt.p. = metacœlic pore.

Addendum.—Since the above was read I have, through the courtesy of Professor Schinskewitsch, had my attention called to a paper read by him before the International Congress at Moscow in 1892, *Sur les Relations génétiques de quelque groupes des Metazoaires.* In this he attempts to bring together very much the same groups as I have done above, and in a similar manner he lays emphasis upon the divisions of the cœlome. He, however, derives them from a common ancestor, the *Enterocœlula* with one pair of "sacs cœlomiques," and regards the subsequent divisions into two (*Tentaculiger*), and later into three, as secondary, whereas the essential part of the above theory is that the divisions are primary and comprise a form of segmentation which the *Metazoa* owe to their descent from axially (radially) symmetrical animals. I have been able to add a number of morphological and ontogenetic facts which have become known since 1892, and feel justified in leaving the term *Archi-Cœlomata* as emphasising the views here expressed of archimeric and metameric segmentation.

On a supposed Resemblance between the Marine Faunas of the Arctic and Antarctic Regions. By D'Arcy Wentworth Thompson, C.B.

(Read May 2, 1898.)

The view that a peculiar likeness exists between the northern and southern extra-tropical faunas, and particularly between those of the Arctic and Antarctic regions, was suggested by Théel in discussing the remarkable deep-sea group of the *Elasipoda*, whose discovery we owe to the Challenger Expedition. A somewhat similar view is hinted at or referred to more than once in other Reports of the same Expedition. It was afterwards stated in an ampler way by Pfeffer (*Versuch über die Erdgeschichtliche Entwickelung der jetzigen Verbreitungsverhältnisse unserer Thierwelt*, 1891), and has of late been dealt with in great detail, and in relation to the antecedent causes that might have led to such a phenomenon, by Sir John Murray. On the other hand, Dr Ortmann, considering the hypothesis from the point of view of our knowledge of the distribution of Crustacea, has rejected it entirely ("Uber Bipolarität in der Verbreitung mariner Thiere," *Zool. Jahrb.*, 1896; *cf.* also "Marine Organismen und ihre Existenzbedingungen," *ib.*, 1897), and Dr Chun, dealing with the pelagic fauna ("Die Beziehungen zwischen dem arktischen und antarktischen Plankton," Stuttgart, 1897), while showing how in truth a certain small number of forms are common to far northern and far southern seas, holds that the facts are sufficiently accounted for by the continuous distribution or gradual intermixture of forms in the depths of the intervening oceans under present conditions, without our needing to have recourse to an explanation of the phenomenon in the different conditions of a former age.

The mere circumstance that so simple an issue should be open to question as the correspondence between the Arctic and Antarctic faunas on the whole, and the existence or non-existence of a large proportion of actually identical species in the two, is in itself clear proof of the inadequacy of our knowledge; and the important theoretical consequences that would follow from proof of the fact, or indeed from any other relation that could be clearly demonstrated between the polar faunæ and specific lines or areas of distribution in the other oceans, exemplify on the zoological side the magnitude of the general problems that deep-sea exploration has yet to solve.

But we are impatient to know, and we are tempted, even in the absence of definite or adequate knowledge, to discuss, how far this Antarctic faunæ extends into other seas, to what degree it is kindred to the marine fauna of the north, how temperature, ocean-current, or other causes may distribute or circumscribe it, and whether existing phenomena are adequate, or past conditions of the globe are necessary, to account for the facts we find. I, for my part, hold with Ortmann, that an actual community of forms is not proven, save for a very few forms, some peculiar to the extreme depths of the sea, and others that inhabit the surface of the ocean in colder latitudes while represented in the deeper and colder waters of tropical seas; and I hold with Chun that the necessity has not been shown for invoking, in explanation of such community and intermixture of forms as actually exists, the hypothesis of a remote community of origin under different conditions in pre-tertiary times.

Sir John Murray has given in his paper, "On the Deep and Shallow Water Marine Fauna of the Kerguelen Region of the great Southern Ocean" (*Trans. Roy. Soc. Edin.*, xxxviii., 1895), a list of nearly one hundred species of animals that are recorded as identical from northern and southern waters, though absent from tropical seas. Had we anything like so great a list to deal with of well-marked and unmistakably identical forms, the case would be indeed a remarkable one. But though Sir John Murray has shown that the forms tabulated in his list were sometimes recorded not as identical but as distinct varieties, and that in other cases the identification was admittedly dubious, I do not think that he has at all fully allowed for the extent to which the basis of his theory is weakened by the aggregate of dubiety in these and other individual cases. I think that we should be very reluctant indeed to rest so weighty a conclusion on facts drawn from fragmentary specimens, the doubtfulness of whose identification the describers have frankly acknowledged, or from minute species in polymorphic groups, where species are distinguished by minute characters and very often by characters drawn only from a shell without examination of the animal itself. Of the forms which remain after deduction of such as these, it seems to me very important to discriminate between those whose affinities appertain to the fauna of the North Pacific and of the North Atlantic respectively. For the fauna of the North Pacific presents many unknown problems to us; but this we do know, that it contains in part a northern circumpolar fauna, and in part a fauna very distinct from that of the North Atlantic and peculiarly linked to the fauna of the Southern Ocean. And we know a little, though not much, of the manner in which this continuity is established, along the western shores of the American continent, in waters singularly cold for the latitudes under which they lie. There are not a few forms that seem to come into the same category as the genus *Serolis*, the penguins, the sea-elephant, the sea-lions, and the fur-seals—I might add the giant sea-weed *Macrocystis*—that, from a circumpolar habitat in the Antarctic, seem to creep up to varying distances far along the Western American coast, to the Galapagos, to California, or even to the northern islands and Japan. The work of American naturalists, and, in particular, the explorations of the "Albatross," have of late years added very largely to our knowledge of this area of distribution.

In this paper I have attempted firstly to weigh the evidence for the particular cases that Murray has quoted as examples of species represented in both northern and southern seas, though absent from the intervening tropical areas: and I think that the evidence so weighed persuades us to recognise that many, if not most, of those alleged cases of identity were extremely dubious, even in the eyes of those who first described them, while others, in regard to which the describers expressed no doubt, seem dubious at least to me, by reason of the insignificant size or comparative poverty in well-marked discriminative characters of the organisms themselves. We know much more than we did when the *Challenger Reports* were written of the difficulty attending the identification of such forms for instance as the Ostracods or the Polyzoa, two groups that figure largely in Dr Murray's list. And if we needed (as we probably did need) another object-lesson to teach us the danger of identifying and classifying species from inadequate material, or from inspection of external characters alone, we have lately been taught that lesson in a very forcible way by Mr Moore's research on the Molluscs of Lake Tanganyika; where he has shown that many Molluscs which had before been identified by their shells alone, and had, on such evidence, been ascribed to lacustrine types, come, in the light of anatomical study, to yield the clearest evidence of affinities and of sources unsuspected before.

The facts here brought together also show, chiefly on the authority of American naturalists and of the explorers of the Bay of Bengal, that in several cases the known range of particular species or genera has been of late extended, so that in some groups of which before we knew only northern and southern representatives, we now discern something more of their cosmopolitan extension across the depths of the tropical sea.

I have also sought to show, but very briefly, how, in the cases that are left to us, we have apparently to deal with facts for which one general explanation will not suffice, but which deserve to be examined and if possible explained one by one according to the special characters or circumstances of each.

Lastly, leaving aside the examples chosen by Sir John Murray, I have tried to bring together what we know regarding the Antarctic fauna in certain groups, especially the Fishes and the Crustacea, groups in regard to which our knowledge is of considerable extent, and in which specific characters are comparatively plain and unmistakable. In these groups I can find no trace of evidence in support of the "bipolar hypothesis."

AN EXAMINATION OF SPECIES ALLEGED TO BE COMMON TO THE NORTHERN AND SOUTHERN EXTRA-TROPICAL SEAS.

Aulocalyx irregularis, F. E. S.—I can find no record of this species except those from the Antarctic given in the *Chall. Rep.*, Hexactinellida, pp. 174, 176. The types taken off Marion Island, 46° 41′ S., 38° 10′ E., consisted of "several much injured and partially macerated specimens," of which "the fragments obtained were only from 3–4 cm. in height," while "of the soft parts only some very small fragments remained." In addition to these, "several fragments" with "completely macerated skeletons" were obtained in lat. 46° 10′ S., long. 48° 27′ E. The former station is off Marion Island, the latter between Marion Island and the Crozets.

Chonelasma lamella, F. E. S.—This sponge is known from some fragments, which, though small, are said to be well preserved, dredged in 630 fathoms off the Kermadec Islands, lat. 29° 45′ S., long. 178° 11′ W.; and also from a macerated specimen got in 550 fathoms, in lat. 46° 53′ S., long. 51° 21′ E. The only evidence that I can find of a more northern distribution is the statement (*Chall. Rep.*, Hexact., p. 321) that a "skeletal fragment," dredged in lat. 32° 8′ 45″ N., long. 64° 59′ 35″ W., from a depth of 1075 fathoms, "appears to belong to the same species." No figures or description are given of the structure of this last specimen, which appears to be again alluded to on p. 397, where the localities of the species are enumerated as Kermadec Islands, 520 fathoms (?630 fathoms, *cf.* "Summary of Results," p. 612); west of Kerguelen Islands (?off the Crozets, *cf.* "Summary," p. 457), 550 fathoms; Bermuda Islands, 1705 fathoms (?1075 fathoms, *cf. Rep.*, "Hexactinellida," p. 321). It is plain accordingly that our knowledge of this species is scanty, and that the fragments which are assigned to it, both from the Southern Ocean and from the West Indian, were in both cases only conjecturally assigned to the same species as the more typical fragment from the neighbourhood of the Kermadecs. As regards the generic distribution of this form, we have well-marked and well-known species from Japan, while fragments are assigned to the same genus (Schultze, *l.c.*, p. 327) from the Gulf of Mexico, the Island of St Thomas, and from the coast of Portugal. Such a distribution has various parallels among other Hexactinellids.

In regard to the Monaxonid and Calcareous Sponges, of which altogether six European species are recorded by Carter (*Phil. Trans.*, 168, p. 286 *et seq.*) from the Kerguelen region, I propose to attempt no discussion in detail, for it is plain that they were studied after methods that have passed into disrepute. They seem, in my opinion, to deserve careful re-examination, and in certain cases (*cf.* Murray, *Trans. Roy. Soc. Edin.*, xxxviii. p. 443) Mr Carter's identifications have already been questioned.

Eudendrium rameum, Pallas, is recorded from Kerguelen Island from a specimen in which no gonosome is present, and Allman remarks that "the want of this essential part of the colony renders the identification of the species less certain than it would otherwise be."

Campanularia cylindrica, Allman, is described from Kerguelen (*Phil. Trans.*, 168, p. 284), where Allman says, "Beyond our knowledge of the situation and the external form of the gonangia, we know nothing of the gonosome, and therefore the reference of this species to the genus *Campanularia* is merely provisional. A form which cannot be distinguished specifically from this has more recently been dredged by H.M.S. "Valorous" from 60 fathoms in Baffin's Bay."

Obelia geniculata (L.) is a northern form, not only European, but found both on the east and west coasts of the United States, and recorded in the *Challenger Report* from Kerguelen and the Falklands.

Liponema multiporum, R. Hertwig. — This is a remarkable sea-anemone, partially described by Hertwig from a "much-mangled" specimen, taken in 1875 fathoms, lat. 34° 37′ N., long. 140° 32′ E., to the south-east of Japan. Two other specimens are described in the *Supplementary Report*, pt. lxxiii. p. 17, one from 120 fathoms in the Straits of Magellan, the other from 1600 fathoms, lat. 46° 16′ S., 48° 27′ E., off Hog Island, between Marion Island and the Crozets.

Cereus spinosus, R. Hertwig.—Of this species we have one specimen recorded from 1950 fathoms, lat. 53° 55′ S., long. 108° 35′ E., and four from 1875 fathoms, in lat. 34° 37′ N., long. 140° 32′ E., off the Japan coast. As is the case of the *Liponema*, the northern specimens were so preserved as to be, according to the describer, "of very little practical use" (*Report*, p. 77). Under these circumstances, and bearing in mind the extreme difficulty of *specific* identification of sea-anemones when preserved, however well preserved they may be, we may here again, I think, hesitate to accept the evidence before us; while, in so far as we may accept it, it would merely serve to extend the distribution of these two Actiniæ to the extreme south of the Japanese region.

Flabellum apertum, Moseley.—Recorded in the *Challenger Report* from Prince Edward Island near Marion Island, and from the coast of Portugal. The many very similar species of this genus come from Patagonia, Japan, Australia, the Philippines, the North Atlantic, and elsewhere.

Holothuria Thomsoni, Théel.—This species is represented by a single incomplete specimen from 1875 fathoms south-east of Yokohama Bay, and three others from 2900 fathoms, lat. 35° 22′ N., long. 169° 53′ E., due east of the same locality. The latter specimens are said to "deviate in some degree from the diagnosis, and in some respects, they are more nearly allied to the preceding species (*H. lactea*)." A single small specimen obtained from 1800 fathoms, lat. 50° 1′ S., long. 123° 4′ E. (due south of Australia), is described in regard to certain of its peculiarities under the name of *H. Thomsoni*, var. *hyalina*, var. nov. (*Rep.*, p. 185), by Dr Théel, who says that he proposes "for the present to consider the specimen in question as a variety of *H. Thomsoni.*"

Professor H. Ludwig, in a paper, which I have not seen ("Hamburger Magalhaenische Sammelreise," Holothuriden, 1890, p. 90), quoted by Ortmann (*Science*, viii. p. 516, 1898), says, in regard to the supposed bipolarity of the Holothurians, that "not a single species of the Antarctic fauna is represented in the Arctic fauna," and that "there is not even a resemblance of both faunas, but a great dissimilarity."

Elpidia glacialis, Théel.—Of this species, well known and even abundant in the Kara Sea and the North Atlantic, the "Challenger" brought home a single specimen from 2600 fathoms, in lat. 42° 42′ S., long. 134° 10′ E. The species has a great range in depth, from 50 to 2600 fathoms, as well as in geographical distribution. "The southern form differs in various points from the northern ones, but the difference is of very little importance" (Théel).

Euphronides depressa, Théel.—One specimen from 1090 fathoms, lat. 35° 47′ N., long. 8° 23′ W., off Gibraltar; two specimens from 1375 fathoms, 33° 42′ S., 78° 18′ W. This is another very remarkable form of a very remarkable group. The southern specimens are said to have differed considerably from the single northern one; they were nearly three times as large, and for other differences deserved and received from the describer a lengthy separate description (*Rep.*, "Holothuroidea," pt. i. p. 95).

Kolge nana, Théel.—Several specimens recorded from Station 50, lat. 42° 8′ N., long. 63° 39′ W., that is to say, a little south of Halifax; and one incomplete individual from Station 152, lat. 60° 52′ S., long. 80° 20′ E., between Heard Island and the Antarctic ice. In regard to this latter specimen the describer says (*Rep.*, p. 40):—"It differs in some ways from those of the first-mentioned station, and will, when compared with individuals in a more complete state, possibly prove to belong to another species differing from this one."

Lœtmogone Wyville-thomsoni, Théel.—This species is recorded from several localities, all of them in very deep water in the Southern Ocean with the exception of one in lat. 35° 11′ N., long. 139° 28′ E., south of Yokohama Bay, at a depth of 345 fathoms. The describer says that this latter "was in such an incomplete state no close examination is possible; it is only 25 mm. long [as against an average of about 200 mm.], and has 13 developed tentacles and rudiments of a fourteenth [15 being given in the definition of the genus]; it is most probable that this form is a distinct species from *L. Wyville-thomsoni*, and this seems still more likely on considering the nature of the sea bottom (sandy mud instead of globigerina-ooze) and the comparatively inconsiderable depth at which the animal was found living." The other species brought home by the "Challenger" were *L. spongiosa*, Th., from 565 fathoms just south of Yokohama Bay, and a very similar form, *L. violacea*, Th., got by the "Challenger" off Sydney, and reported also (Théel, *Rep.*, p. 79) from Dr John Murray's collections between the Faeroe Islands and the coasts of Scotland. We may note here again a very wide distribution in the case of this singular and deep-water form, and we may further note that the single imperfect specimen of the southern or Antarctic species which is recorded from north of the line was obtained at the mouth of Yokohama Bay, in the precise locality to which another species is as yet peculiar.

Pourtalesia phiale, Wyville Thomson.—Station 156, lat. 62° 26′ S., long. 95° 44′ E., 1975 fathoms. Taken first by the "Porcupine" in Rockall Channel, at a depth of 1215 fathoms.

The two small specimens were immature, and "their characters too undefined for satisfactory description" (*Trans. Roy. Soc.*, 1874 (pt. 2), p. 749).

Sir Wyville Thomson (*l.c.*) speaks of this "singular little urchin to which I have given provisionally the name *Pourtalesia phyale.*" The single specimen collected by the "Challenger" measured only 18 mm. long; it came from 1975 fathoms in lat. 62° 26′ S., long. 95° 44′ E. The original description by Wyville Thomson is exceedingly brief, and is accompanied by no structural figures. The genus is one of the most remarkable of the Spatangoid Echinoidea, with peculiarly interesting affinities to certain fossil forms. The type species, *P. rimanda*, was described by Agassiz from 349 fathoms off the Tortugas, while other species are known, all from deep or very deep water, in various parts of the world: one, for instance, *P. rosea*, in the equatorial Pacific; another, *P. laguncula*, from the North, South, and equatorial Pacific; *P. hispida*, *P. ceratopyga*, and *P. carinata*, all from the Southern Ocean in from 34° to 62° south latitude. In regard to Wyville Thomson's specimens no information is given as to their size, but he apparently leaves it to be understood that his figure, which measures nearly two inches long, is of natural size. This is in sharp contrast to the dimensions of the "Challenger" specimen from the Southern Ocean, and, together with the inadequacy of the original description, seems to render the specific identity of the two a very doubtful matter. The genus, however, is manifestly ancient, and is, in fact, a typical example of a form that is at once ancient, abyssal, and widely distributed through northern, equatorial, and southern latitudes.

Phormosoma hoplacantha, Wyville Thomson.—

Station	300,	lat. 33° 42′ S.,	long. 78° 18′ W.,	1375	fms.
,,	164*a*,	,, 34° 13′ S.,	,, 151° 38′ E.,	410	,,
,,	235,	,, 34° 7′ N.,	,, 138° 0′ E.,	565	,,

Pontaster forcipatus, Sladen.—Of this form typical specimens are described from localities off the east coast of Nova Scotia and the United States, at depths ranging from 1240 to 1700 fathoms. These specimens are said (*Rep.*, p. 46) to be "remarkably constant in general character." Another Pontaster from 1375 fathoms, lat. 46° 46′ S., long. 45° 41′ E., between Marion Island and the

Crozets, is assigned by the describer to the same species, var. *echinata*, var. nov. Of this he says:—"The close affinity of this form to its far distant type in the North Atlantic is especially remarkable; whilst the manner in which some of its characters approach even more nearly that of *P. mimicus* from the North Australian sea are very striking, and, at the same time, highly suggestive of the genetic connection of the three forms." *P. mimicus* is also a deep-water asteroid, from 800 fathoms, north-west of the Aru Islands.

Dytaster exilis, Sladen.—This asteroid belongs to a widely-distributed genus, all whose species come from deep water (in one case only from so little as 800 fathoms) from the North and South Atlantic, North and South Pacific, and the Eastern Archipelago. In regard to the species cited, the type came from 1375 fathoms between Valparaiso and Juan Fernandez. A form was got in 1900 fathoms off Tristan d'Acunha, in regard to the single example of which the describer says (*Rep.*, p. 68):—"This I propose to rank provisionally as a variety of this form, although it may afterwards be found to merit recognition as a distinct species"; while, lastly, specimens were taken in 1700 fathoms in the North Atlantic, off Maryland, which are described as a variety considerably nearer than the other to the type-species. We have to do, in short, with a very widely distributed form, very variable, or rather comprising forms only provisionally linked together; while, even if we still do link them, the range of distribution is not shown to be greater than 35–37 degrees of latitude to south and north.

Ophioglypha irrorata, Lym.—I have not found a reference to the occurrence of this species in the northern hemisphere.

Ophiocten sericeum, Lym.—A specimen doubtfully referred to this species was got off Marion Island, 50–75 fathoms. This is a small but well-known and common northern species, nevertheless. Mr Lyman, in describing the "Challenger" specimens, of which there were two (*Rep.*, p. 79), calls attention to their variations and to the difficulty of distinguishing them from allied forms; and indeed marks with a query the identifications at the head of his paragraph. He closes the paragraph by citing without further remark as a locality "off Marion Island, 50–75 fathoms."

Ophiacantha rosea, Lym.—Described from

Off Marion I.,	lat. 46° 40′ S.,	long. 37° 50′ E.,	310 fms.
Magellan Sts.,	„ 50° 10′ S.,	„ 74° 42′ W.,	175 „
Near Yokohama,	lat. 34° 58′ N.,	long. 139° 30′ E.,	420–775 fms.

The genus is one of very numerous (about 40) species, and is very widely distributed.

Ophiocten hastatum, Lym.—

Station	78,	lat. 37° 24′ N.,	long. 25° 13′ W.,	1000 fms.
„	146,	„ 46° 46′ S.,	„ 45° 31′ E.,	1375 „
„	168,	„ 40° 28′ S.,	„ 177° 43′ E.,	1100 „

Ophioglypha bullata, Wyv. Thoms.—

Lat. 38° 34′ N.,	long. 72° 10′ W.,	1240 fms.
„ 34° 51′ N.,	„ 63° 59′ W.,	2650 „
„ 34° 54′ N.,	„ 56° 38′ W.,	2850 „
„ 35° 41′ S.,	„ 20° 55′ W.,	1900 „

On p. 44 of Mr Lyman's Report, *O. bullata* is referred to as "the extreme" of the deep-sea species of the genus. The genus includes 57 species.

Ophiernus vallincola, Lym.—The only species of the genus.

From lat.	37° 24′ N.,	long. 25° 13′ W.,	1000 fms.
„	46° 46′ S.,	„ 45° 31′ E.,	1375 „
„	62° 26′ S.,	„ 95° 44′ E.,	1975 „

Antedon abyssicola, P. H. Carpenter.—This little crinoid is reported from 2600 fathoms, lat. 42° 42′ S., long. 134° 10′ E., off Melbourne, and from 2900 fathoms, lat. 35° 22′ N., long. 169° 53′ E., between Japan and the Sandwich Islands. It is a very tiny species, with a disc about 3 mm. across, and a spread of about 5 cm. It is the only Comatula described in the *Challenger Reports* from a depth greater than 2000 fathoms. One specimen was obtained from the northern, and two from the southern locality. An inspection of the figures (pl. xxxiii. figs. 1 and 2) suggests a grave doubt as to the precise identity of the two species; the basal plates, in particular, are shown to be much broader in shape in the northern example. But if the two be accepted as identical, I see nothing very remarkable in the presence of the same abyssal form at two stations in the region of the Western Pacific, off the Japanese and the Australian coasts.

Professor Jeffrey Bell tells me that the three specimens of *A.*

abyssicola are "so small and so delicate and so much broken, that he would be a bold man who would ascribe to them specific characters at all."

A closely allied species, *A. alternata*, P. H. C., is described from a chain of localities—north-east of New Zealand, north of Papua, near the Kermadecs, and south of Japan. Several other species range from Queensland or Torres Strait to Japan, or to the China Sea, *e.g.*, *A. Coppingeri*, Bell; *A. multiradiata*, L.; *A. variipinna*, Carpenter. Some of the older or better known species have a very wide distribution, especially in longitude, *e.g.*, *A. tenella*, from the Kara Sea to the coasts of Portugal and New England, or *Actinometra pulchella*, Pourt., from the Caribbean Sea, St Paul's Rocks, and the European coast; while in certain other cases, *e.g.*, *A. carinata*, Carpenter, we have an alleged distribution of a very extraordinary, circum-tropical kind, including Brazil, Venezuela, Chili, Ceylon, Seychelles, Zanzibar, Madagascar, the Red Sea, and St Helena. We must remember also that the genus Antedon contains somewhere about 50 species, and is cosmopolitan in its range.

Drepanophorus serraticollis, Hubrecht.—In regard to the collection of Challenger Nemertines, Dr Hubrecht, in reporting upon it, calls attention to the extreme care with which the broken fragments of these fragile animals had been preserved, and to their excellent state for purposes of microscopical examination, although "it looked far from promising from a systematist's point of view," colour and outward form being alike lost. The two specimens, both broken, assigned to *D. serraticollis* (which is a common Mediterranean species), came from shallow water in Bass's Straits. In so assigning them, Professor Hubrecht writes as follows:—"It needs no comment, that it is at the least rather hazardous to identify with a Mediterranean species a specimen in which the proboscis as well as its armature is absent. Still the transverse sections offer such a very close resemblance to those of actual specimens of *D. serraticollis* that it would be again hazardous to establish a new species for the fragments." . . . . "I have, moreover, hazarded the identification with the foregoing specimens of a third fragment collected in the Kerguelen waters, of which not only the proboscis but also the head was absent. Here, too, the internal characters enabled me to refer the specimen to the genus *Drepanophorus* (the transverse cœca of the proboscidean sheath being in this case the guiding feature). The systematic position of this specimen thus only rests upon the similarity of the transverse sections and on the general yellow hue of the fragment, darker on the dorsal than on the ventral surface" (*Rep.*, p. 18). While such evidence was no doubt reasonably valid as grounds for treating together the several fragments of the worm, I cannot imagine that it will really be relied upon as evidence of specific identity between the nemertine faunas of the northern and southern seas.

Bürger (*Fauna and Fl. des Golfes v. Neapel*, Nemertinen, p. 572), including *D. serraticollis* (as Hubrecht had already suggested might be necessary) in *D.* (*Cerebratulus*) *crassus* of Quatrefages, adduces from various authorities, as localities for the species, not only many European stations, but also Madeira, Mauritius, Samoa and Panama, and states that it "lässt wahrscheinlich nur die arktischen Meere frei."

Pelagonemertes Rollestoni, Moseley.—This very remarkable, transparent pelagic nemertine was taken by the "Challenger" "near the southern verge of the South Australian current," lat. 50° 1′ S., long. 123° 4′ E., and was again recorded in the mouth of Yokohama Bay. It seems to be closely allied to the *Pterosoma plana* of Lesson, obtained in great abundance between the Moluccas and New Guinea during the voyage of the "Coquille."

In the more recent monograph of the group by Bürger (*F. and F. d. Golfes v. Neapel*, Nemertinen, p. 596) attention is called to very marked differences, both external and internal, between the specimens of Pelagonemertes from the northern and southern localities, and that writer, without hesitation, erects the two forms into separate species, retaining the name *P. Rollestoni*, Moseley, for the Australian form, and giving the new name *P. Moseleyi* to that from Japan. Bürger points out as diagnostic characters that in *P. Moseleyi* the body is about as long as broad, is constricted in front, behind, and in the middle, and that the gut possesses five pairs of diverticula; whereas in *P. Rollestoni* the body is twice as long as broad, is constricted neither before nor behind, and the gut has thirteen pairs of diverticula. He points out further that the smaller form (from Yokohama) which had been assumed by

Moseley to be a young individual of *P. Rollestoni*, is manifestly adult, possessing ovaries and ova in an advanced stage of development.

Artacoma proboscidea, Malmgren.—Grube ("Anneliden-Ausbeute von S.M.S. 'Gazelle,'" *Monatsbericht d. k. Akad. d. Wiss.*, Berlin, 1877) records this northern species from Kerguelen. Murray, in a footnote (*Trans. Roy. Soc. Edin.*, xxxviii. 445), points out that "his specimens possibly belong to *A. challengerii*, M'Intosh, which seems to be very frequent in Kerguelen waters." M'Intosh (*Chall. Rep.*, "Annelida," p. 477) points out various differences between the northern and southern species, and cites Kinberg as an authority for the occurrence of the latter also at Rio de Janeiro.

Eunice Oerstedi, Stimpson.—Of this species, to which Dr M'Intosh assigns a mark of interrogation in identifying it with Stimpson's original description, a single specimen is said to have been taken by the "Challenger" in 1240 fathoms off the coast of New York, another smaller one in 85 fathoms somewhat further north off the coast of Nova Scotia, while several small specimens about 90 mm. in length are recorded from 69 fathoms off Marion Island. M'Intosh calls attention (*Rep.*, p. 273 *et seq.*) to several minor differences between these latter and the former examples. He states that "in the foreign (southern) example, the branchia of the 10th foot has two divisions, that of the 20th four, the 30th four, and the 40th none," and adds, "this *Eunice* seems to come near the *E. Oerstedi* of Stimpson, the chief difference being the number of divisions in the branchiæ, which Stimpson states is five." He further calls attention to the close resemblance of the form to *E. norvegica* (L.) and also to *E. macrochæta*, Schmarda, from the southern shores of Jamaica.

Eupista Darwini, M'Intosh.—Trawled in 2225 fathoms, lat. 34° 7′ S., long. 73° 56′ W., off the west coast of America south of Valparaiso. The existence of this form in the northern hemisphere rests upon "a softened fragment closely approaching the foregoing," trawled in 2750 fathoms in mid-Atlantic, lat. 35° 29′ N., long. 50° 53′ W., midway between the Azores and Bermuda. M'Intosh (*Chall. Rep.*, "Annelida," p. 459) states that in this latter specimen "the characters of the cephalic region are indistinguishable, and the branchiæ are absent." He cites it as *Eupistes Darwini*, var., saying that, from that species, "sufficient materials are not at hand to establish a reliable distinction if such exist."

Terebellides Strömi, M. Sars.—The specimens of Terebellides taken by the "Challenger" at Kerguelen are grouped by M'Intosh with the northern species under the name of *T. Strömi*, var. *Kerguelenensis*.

Placostegus ornatus, Sowerby.—This little tubicolous annelid is recorded in the *Challenger Report* from 2900 fathoms, lat. 35° 22′ N., long. 169° 53′ E., in the deeps of the North Pacific; again, from 3125 fathoms further west in the same area, and from 2375 fathoms, lat. 32° 36′ S., long. 137° 43′ W., south of the Paumotu group. Other species of the same genus inhabit the British area, the West Indies, the deep water of the Mid-Pacific, etc.

Ostracoda.—Eight species of Ostracoda are enumerated in Dr Murray's list on the strength of the Challenger collections. In regard to two of these, *Krithe tumida*, Brady (*Rep.*, "Ostracoda," p. 115), and *Xestoleberis expansa*, Brady (p. 129), I cannot discover any reference to their occurrence in the north. In regard to most of the others, considerable dubiety is expressed by Dr Brady in regard to their identity; for example, in reference to *Paradoxostoma abbreviatum*, G. O. Sars (*op. cit.*, p. 150), he says, "As no very decided characters appear, and as no sufficient series of specimens of the Kerguelen form is at hand for comparison, it seems best to identify them, for the present at least, with the European species." *Cythere sühmi*, Brady, and *Sclerochilus contortus*, Norman, were represented in their southern localities only by imperfect specimens or separated valves. *Xestoleberis depressa*, G. O. Sars, is a northern species recorded by Brady from Kerguelen, with the remark, "it is to be borne in mind, however, that the distinctions between this and the next species (*X. setigera*, Brady, from Kerguelen, Heard Island, etc.) if valid at all, are very slight, and it is not unlikely that the two may prove to be identical."

Furthermore, it behoves us to remember that the study of the Ostracods is now conducted in a totally different way, and stands on a different basis, compared with the period when the Challenger monograph was prepared. The Challenger species were identified by the form and sculpture of the shells alone, without reference to or examination of the animals within; and such identifications are dismissed in a very summary manner by Müller in his "Monographie der Ostracoden" (*F. and F. d. Golfes v. Neapel*, 1894), who deals with the Ostracoda by characters drawn from the limbs, precisely as with other groups of Crustacea.

Calanus finmarchicus, Gunner.—This abundant northern form is said to have been taken in the surface net in lat. 35° 9′ S., long. 45° 30′ E. Also reported by Brady (*Chall. Rep.*, Copepoda, p. 32), from off Cape Howe, Australia. After calling attention to certain apparent differences, Dr Brady says, "the only reasonable course is to consider both the northern and the southern forms as belonging to one species, probably the most abundant and most widely distributed of all the *Copepoda*." The genus, of which Giesbrecht enumerates fourteen species (*F. and F. d. Golfes v. Neapel*, Pelagische Copepoden, p. 89), is cosmopolitan. *Calanus valgus*, Brady, which Giesbrecht assigns to *C. minor*, Claus, *C. gracilis*, Dana, and *C. propinquus*, Brady, are instances of other species whose extended distribution ranges north and south of and between the tropics. It is *C. propinquus* that at Kerguelen replaces *C. finmarchicus* in the same abundance as the latter occurs in Arctic seas.

Harpacticus fulvus, Fischer.—Recorded by Brady from pools above high-water mark at Kerguelen (*Phil. Trans.*, 168, p. 215). This species, which occurs in similar localities in Europe, is also stated by Brady to have been got in 35 fathoms off the Yorkshire coast (*Monogr. Brit. Copepods*, vol. ii. p. 151). The latter specimens had been formerly described as *H. crassicornis* (*B. A. Rep.*, 1875, p. 196).

Scalpellum velutinum, Hoek.—In regard to this species Dr Hoek (*Chall. Rep.*, Cirripeds, p. 96) says, "This beautiful species is represented by a single specimen (from 1425 fathoms north of Tristan d'Acunha). Provisionally there must be referred to the same species three smaller specimens which were dredged near the southern point of Portugal; yet I am not quite sure that they belong really to the same species. This species is nearly related to *S. regium, S. Darwinii, S. gigas, S. robustum*, etc.," and this group, like the genus as a whole, covers an immense range of distribution.

Typhlotanais kerguelenensis, F. E. B.—Recorded from 127 fathoms off Kerguelen and from 2050 fathoms in the middle of the North Pacific. The describer says (*Chall. Rep.*, Isopoda, p. 122), "I find it impossible to distinguish this individual (from the North Pacific) from those dredged at Kerguelen by any very distinctly marked characters; at the same time the condition of the specimen does not enable me to speak with great certainty, which is all the more to be regretted, as the occurrence of the same species in very deep and in shallow water is a rare occurrence."

Neotanais americanus, F. E. B.—A small species, of which one specimen came from 1900 fathoms off the River Plate, and another from 1240 fathoms south-east of New York. Beddard states that the latter "presents certain slight differences having reference to the proportionate length of the thoracic segments. Seeing, however, that the two specimens come from widely distant localities, it appears to be unnecessary to found a specific distinction between the two individuals at least for the present."

Eurycope fragilis, F. E. B.—To this species are referred several specimens from very deep water off Marion Island and other stations still farther south in the Southern Ocean, and one of much larger size, from 1875 fathoms, off Yokohama. Other species are known from the North and Middle Atlantic, from New Guinea, New Zealand, and the Southern Ocean, while Hansen has recently described (*Harvard Bull.*, 1897, p. 100) a species, *E. pulchra*, from near the Galapagos, which he says is closely allied to *E. fragilis*, Beddard. Sars has added a species from the Mediterranean to the nine others that are as yet only known from the Scandinavian coasts.

Lophogaster typicus, M. Sars.—This remarkable northern species was twice taken by the "Challenger" close to the Cape of Good Hope.

Boreomysis scyphops, Sars.—This species, known also from the Arctic Sea to the north of Norway, is recorded, on the indisputable authority of Sars, from great depths in three localities in the Southern Ocean (*Chall. Rep.*, Schizopoda, p. 182). Another species, *B. arctica*, is very closely allied to one from the Gulf of California, described by Ortmann as *B. californica* (*Harvard Bull.*, xxv. p. 106, 1894). Other species occur in the deep water of the North Atlantic and North Pacific.

Gnathophausia gigas, Willemoes Suhm.—While in the hands of Professor Sars a very little bit may be expected to yield very clear indications, yet we may note that the only record of this species from the Southern Hemisphere is based upon "the recently moulted skin of the outer part of the tail of another specimen apparently belonging to the same species. This skin was brought up along with specimens of *Boreomysis scyphops* in the Southern Ocean between Kerguelen and Australia."

Glyphocrangon rimapes, Spence Bate.—Under this species Mr Spence Bate groups a specimen from 1875 fathoms off Yokohama, one from 1715 fathoms in the South Atlantic between Buenos Ayres and Tristan d'Acunha, and two from 1375 fathoms near Juan Fernandez. The genus is a typical deep-sea one, and other species come from many localities in the North Atlantic, West Indies, Indian Ocean, and North and South Pacific.

Lithodes Murrayi, Henderson.—This species is referred to by Murray (*l.c.*, pp. 406 and 456) as the southern representative of *L. maia*, to which, according to Henderson, it is apparently most closely allied. Ortmann (*Zool. Jahrb.*, 1896, p. 584) deals at length with this case, and, in part following Faxon (*"Albatross" Crust.*, p. 51), brings *L. Murrayi* into closer relation with *L. turritus*, Ortmann, from Japan, and *L. panamensis*, Faxon, from the west coast of America. "Zum mindesten geht hinaus hervor dass die Gruppe des *L. Maia* und *Murrayi* sowohl in Japan als auch an der Westkuste Amerikas Vertreter besitzt, und hierzu kommt noch eine verwandte Art an der Südspitze Amerikas der *L. antarcticus*, Jacq. et Luc. Durch diese Daten wird eine continirliche Verbreitung längs der amerikanischen Westküste für die Gattung wahrscheinlich gemacht, und dazu kommt noch, dass für die ganze Familie der *Lithodidæ* es längst bekannt ist, dass ihre Hauptverbreitung im nördlichen Pacific liegt und von dort längs der Westküste Amerikas bis zur Antarktischen Zone geht."

M. Bouvier, in his recent monograph of the Lithodidæ (*Ann. Sci. Nat.*, *Zool.* (8), v. 1896), likewise gives no support to the view that *L. Maia* is the nearest ally of *L. Murrayi.* He, on the other hand, brings into close relation with the latter, two species, *L. tropicalis*, A.M.E., and *L. ferox*, A.M.E., both from deep water off the west coast of Africa; and we seem accordingly to have in the deep tropical waters of both the Atlantic or the Pacific, forms which, if they are not intermediate between, are at least closely allied to the Arctic and Antarctic species. The distribution of the Lithodidae is, as M. Bouvier remarks, extremely curious and interesting, and his account merits close attention; it gives no support at all to a theory of bipolar distribution.

Munidopsis Antonii, M. E.—Specimens from 1800 fathoms south-west of Australia, and from 1375 fathoms west of Valparaiso, are identified by Henderson with the *Galathodes Antonii*, Milne Edwards, taken by the "Talisman" in 4000 meters off the north-west coast of Africa.

Munidopsis subsquamosa, Henderson.—Described from 1875 fathoms off Yokohama. Two specimens, one from 1450 fathoms off the west coast of Patagonia, and the other from 1375 fathoms between Marion Island and the Crozets, are described as var. *aculeata* of the same species (Henderson, *Chall. Rep.*, Anomura, p. 153). A variety of striking characters in which the variety differs from the type species are pointed out by the describer, and an inspection of his figures reveals a striking difference in aspect. Other species of the same genus are described from Patagonia, the Philippines, the West Indies, the north-east coast of the United States, etc., and another variety of the same species is described by Alcock from 1800 fathoms in the Indian Ocean (*A. and M. N. H.* (6), xiii. p. 331, 1894). Mr Faxon (*Harvard Mem.*, xviii. p. 86) considers this variety to be closely allied to *M. crassa*, Smith, from the east coast of the United States.

More recent students of the group, for instance Bonnier ("Galathéides des côtes de France," *Bull. Sc. France et Belgique* (3), i., 1888), find it necessary to describe and figure very many of the appendages for the identification even of the commoner European species.

Mytilus edulis, (L.).—Recorded from Kerguelen (*Phil. Trans.*, 168, p. 189), as well as from New Zealand, Falkland Islands, River Plate, etc. Smith says (*Chall. Rep.*, Lamellibranchiata, p. 272):—"This common species has become widely distributed, and differs considerably in form, colour, and size." In the Kerguelen Report, Mr Smith says:—"After a careful consideration of this species, I cannot arrive at any other conclusion but that the Kerguelen shells undoubtedly are specifically the same as the common edible mussel." Dall (*Bull. U.S. Nat. Mus.*, 1876, iii. p. 41), with whom Smith differs, says, in assigning the Kerguelen form to *M. canaliculus*, Hanley, that "the shell of this species closely resembles some varieties of *M. edulis*, but the soft parts are quite different." It is plain that the allied forms or species from Chili, New Zealand, etc., are very similar.

Glomus nitens, Jeff.—Recorded in deep water from various stations in the North Atlantic, and also from 1900 fathoms off the Rio de la Plata (E. A. Smith, *Chall. Rep.*, Lamellibranchiata, p. 248). There are several closely-allied species of these little shells in the West Indian region.

Kellia suborbicularis, Mont.—A European species found also in the Canary Islands, and of which two specimens are recorded (Smith, *l.c.*, p. 201) from Kerguelen. "One of them exhibited a very trifling difference in the hinge-plate, which was not, however, maintained by the second example." Three other minute species are recorded from Kerguelen, besides others from the Australian region, the Straits of Magellan, etc.

Puncturella noachina, (L.).—This small Fissurellid is recorded in one variety, v. *princeps*, Migh, from several stations near Marion Island, and in another, v. *galeata*, Gould, from the Straits of Magellan. Mr Watson says that he has found it impossible to separate the southern form, which is unquestionably the *P. princeps*, from the species of Linné. He further groups together a number of species or varieties, giving to the unified form a distribution including North Greenland, Spitzbergen, Sea of Okhotsk, Northern Japan, Oregon, Straits of Magellan, and the Falkland Islands. Of the few other, not dissimilar, species, some are from the North Atlantic, several from the West Indies, and one from the Australian region.

Trochus (*Margarita*) *infundibulum*, Watson.—Recorded by the "Challenger" from 1375 fathoms off Marion Island, and 1075 fathoms off Bermuda. The species, which belongs to a group of many similar forms, is said to be very like *T. ottoi*, Phil., a form found fossil at Messina, and identified with the *M. regalis*, Verrill, abundant off the New England coast.

Dentalium keras, Watson.—Dredged by the "Challenger" in

2050 fathoms, lat. 36° 10′ N., long. 178° 0′ E., in the middle of the North Pacific, and again in 2160 fathoms a little west of Valparaiso; recorded also by Dall from 1568 fathoms in the Gulf of Mexico. On the grounds for accepting the West Indian specimen as identical with those from the Pacific, see Dall, quoted by Watson, *Chall. Rep.*, Scaphopoda, p. 4.

Janthina rotundata, Leach.—This well-known pelagic shell of the North Atlantic was also taken by the "Challenger" in the middle of the South Atlantic, lat. 35° 41′ S., long. 20° 55′ W.

Natica groenlandica, Beck.—One specimen from 75 fathoms off Heard Island. Watson (*Rep.*, p. 448) says:—"On comparing this *Natica* with G. O. Sars' specimens from Norway, I am not quite satisfied, and yet I cannot part them." He recites several differences between the forms, but states that "*N. groenlandica* varies in all these respects, and the study of that species leaves the impression that the differences I have mentioned above might be found filled up." It is notorious that the identification of the smaller *Naticæ* is a matter of no little difficulty. At least three other closely allied species, *N. xantha*, *N. prasina*, and *N. fartilis*, Watson, all come from the same region, and the shell of the last named is noted by the describer as "so closely resembling *N. affinis*, G. (*N. clausa*, Brod. and Sow.), that I have hesitated very much to separate them, and have been glad to be strengthened in so doing by the opinion of Professor von Martens and of Mr E. A. Smith." There are many other allied species of this cosmopolitan genus in southern waters.

Homalogyra atomus, (Philippi).—A single, slightly weathered specimen dredged by the "Challenger" in 140 fathoms off Marion Island. This tiny shell, whose length is only ·0125 of an inch, is widely distributed from Norway to the Mediterranean and Madeira. Dr Watson remarks (*Rep.*, "Gasteropoda," p. 121) that it "is extremely abundant in Madeira, and careful search will probably supply many additional localities for its dwelling."

Odostomia rissoides, Hanley.—A common northern form recorded from the neighbourhood of Marion Island. Dr Watson remarks (*Rep.*, "Gasteropoda," p. 481):—"I give this species on the authority of Dr Gwyn Jeffreys. I had remarked the shell's great resemblance in form to *O. rissoides*, but the distinct and strong spiral sculpture which characterises it, coupled with the locality, prevented my referring it to that species."

Doris tuberculata, Cuv.—This species is recorded in the "Report of the Antarctic Expedition" (E. A. Smith, *Phil. Trans.*, vol. 168, p. 183). "A Nudibranch brought from Kerguelen Island by the Antarctic Expedition has been identified as a variety of this common European species by Mr P. S. Abraham, who has recently been studying the species of this genus in the national collection. He says that it possesses no characters of specific distinction from *D. tuberculata*, and differs from it only in a few slight and unimportant particulars attributable to mere variation." Murray (*Trans. Roy. Soc. Edin.*, vol. xxxviii. p. 448) notes that Studer doubtfully records *D. tuberculata* from Kerguelen, but Bergh considers the form referred to as probably identical with his *Archidoris kerguelensis*, and distinct from the northern form.

I do not know that any conchologist has ascribed importance to these few and somewhat doubtful instances of identity between northern and southern shells. Mr E. A. Smith, in reporting on the Mollusca of Kerguelen, while calling attention to the apparent similarity of certain species to northern forms, dwells upon the great resemblance of the fauna as at present known to that of the Falkland Islands and South Patagonia; while the Rev. A. H. Cooke (*Cambr. Nat. Hist.*, "Mollusca," p. 367) states that "the Mollusca of Kerguelen Island and the Marion and Crozets groups show relationship partly with South America, partly with the Cape, and partly with South Australia and New Zealand, thus showing some trace of a circumpolar antarctic fauna corresponding to, but not nearly so well marked as, that of the circumpolar arctic sub region."

Kinetoskias cyathus, Wy. Th.—Recorded in the *Challenger Report* from 1525 fathoms off the coast of Portugal, and also from 2650 fathoms, lat. 36° 44′ S., long. 46° 16′ W. This very remarkable Polyzoan is supposed by Koren and Danielssen to be identical with their *K. Smittii* from the north of Norway. Another species, *K. pocillum*, Busk, from near Pernambuco, lat. 9° 5′ S., long. 34° 49′ W., and from 2160 fathoms off Valparaiso, is in like manner similar to Koren and Danielssen's *K. arborescens* from Norway.

Crisia eburnea, (L.).—Busk refers a specimen in the Challenger collection from Kerguelen Island to this common northern species under the name of *C. eburnea* var. *laxa.* Two other species of the same genus are described from Kerguelen Island by Busk in "Rep. Antarctic Exp.," *Phil. Trans.*, vol. clxviii. p. 197, and various species are known from all parts of the world. The difficulty of identifying the various forms is well illustrated by Mr Harmer's elaborate paper, "On the British Species of Crisia," *Q. J. M. S.*, 1891, pp. 127–181.

Diachoris magellanica, Busk.—This species is known from Kerguelen, Straits of Magellan, and the Australian region, and has been identified by Mr Busk with *D. Buskii,* Heller, from the Adriatic (*Bry. Adriatica,* 1867, p. 93).

Escharoides verruculata, (Smit).—Brought by the "Challenger" from the neighbourhood of Heard Island, and identified with *Cellepora verruculata,* Smit, from the West Indies. The species is also said to occur in the Mediterranean (Waters, *A. and M. N. H.* (5), iii. 193).

Hornera violacea, Sars.—This is another northern species with which Busk identifies a specimen from 75 fathoms in the neighbourhood of Heard Island.

Hornera lichenoides, (L.).—This widely distributed northern species is recorded by Busk from 600 fathoms off the mouth of the River Plate.

Pustulipora delicatula, Busk.—Recorded from Kerguelen and from Australia, and, with a mark of interrogation, also from Madeira (*cf. B. M. C.*, "Polyzoa," pt. iii. p. 20).

Pustulipora proboscidioides, Smit.—A West Indian species with which Busk identifies a specimen from Kerguelen Island.

Pustulipora proboscidea, M. E.—A Mediterranean and Atlantic form with which is identified a specimen from Heard Island.

Pustulipora deflexa, Smit.—A European species with which is identified a form from the neighbourhood of Heard Island. Mr Busk says:—"On the whole it seems extremely doubtful what name should be assigned to the form here described, with respect to which all that appears to me to be certain is, that the specimens, mere fragments, in the Challenger collection are identical with the form described by Prof. Smit (from Florida) as *Entalophora deflexa.*" But there seems to be very considerable doubt as to the synonymy or identity of this latter species.

Membranipora galeata, Busk.—Described from Kerguelen and from the Falkland Islands (Busk, "Rep. Antarctic Exp.," *Trans. Roy. Soc.*, vol. clxviii. p. 195). A form described as var. *multifida* is recorded both from Simon's Bay and from the Azores (Busk, *Chall. Rep.*, pt. xxx. p. 64).

Membranipora crassimarginata, Hincks.—A variety *incrustans* from Tristan d'Acunha is identified by Busk with Mr Hincks' species from Madeira, certain differences being noted between the two, and also, with a mark of interrogation, with Smit's *M. lacroixii* from the West Indies.

Cribrilina monoceros, Busk.—Recorded from Port Jackson, 35 fathoms; from off Marion Island; from 1325 fathoms in lat. 45° 31′ S., long. 78° 9′ W.; from 55 fathoms, lat. 52° 20′ S., long. 68° 0′ W.; from 12 fathoms, lat. 51° 40′ S., long. 57° 50′ W.; and also from 3125 fathoms, lat. 38° 9′ N., long. 156° 25′ W.

Platydia anomioides, Scacchi.—Several specimens of this little Brachiopod were got by the "Challenger" off Marion Island. The species was formerly known from the Mediterranean Sea and the coast of Portugal.

Pyrosoma spinosum, Herdman.—This species is represented in the Challenger collections by a few small fragments only, obtained in the South Atlantic 400 miles west of Inaccessible Island, and again in the North Atlantic to the west of the Azores.

In regard to the Tunicata as a whole, Professor Herdman has lately expressed his opinion as follows:—"The Tunicata instanced by Dr Murray, both in his 'Challenger Summary,' and in his paper on the 'Marine Fauna of the Kerguelen Region,' help to swell lists that assume rather imposing dimensions; but when I examine the case of these species and genera of Tunicata individually, I find that the records of occurrence have to be added to or modified in such a way as to entirely change the nature of their evidence, and show that there is no such close resemblance between the northern and southern polar faunas as Dr Murray and others have supposed" (*Trans. Liverpool Biol. Soc.*, xii. p. 251, 1898).

Halosaurus macrochir, Günther.—Recorded by the "Challenger" from 1375 fathoms near Marion Island. The species is common in the deep waters of the Atlantic, and allied species come from the Philippines and Japan.

Synaphobranchus bathybius, Günther.—Was got in very deep water in the middle of the North Pacific, again off Yokohama Bay, and lastly off Marion Island. Another species, *S. pinnatus*, is still more widely distributed, occurring in the North and South Atlantic, off Japan, and south of the Philippines, and another, *S. brevidorsalis*, occurs both off New Guinea and Japan.

Stomias boa, Cuv.—With this Mediterranean fish Dr Günther (*Chall. Rep.*, Deep-Sea Fishes, p. 204) identifies a specimen got south of Australia (St. 158) in 1800 fathoms. Peters (*Monatsber. d. K. P. Akad. d. Wiss.*, Berlin, 1876, p. 46) records also a specimen from the South Pacific caught in lat. 42° 56′ S., long. 149° 26′ W. Dr Günther remarks:—"I must not omit to mention that none of the authors referred to have given the number of luminous spots along the abdomen, and that, not having a specimen from the Mediterranean, I am consequently unable to assert the agreement of our fish in this respect. Also that Valenciennes has counted 72 scales along the side of the body, whilst our Antarctic specimen possesses 88." The genus is known from Greenland and from all the oceans.

We discover from the preceding notes that the following *species recorded from southern localities are dubiously or in some cases more than dubiously identified with the corresponding northern forms:—*

Aulocalyx irregularis, F. E. S.
Chonelasma lamella, F. E. S.
Eudendrium rameum, Pallas.
Campanularia cylindrica, Allman.
Holothuria Thomsoni, Théel.
Euphronides depressa, Théel.
Lætmogone Wyville-Thomsoni, Théel.
Kolge nana, Théel.
Pourtalesia phiale, Wy. Th.
Ophioglypha irrorata, Lyman.

Ophiocten sericeum, Lj.
Dytaster exilis, Sladen.
Antedon abyssicola, P. H. C.
Drepanophorus serraticollis, Hubrecht.
Pelagonemertes Rollestoni, Moseley.
Artacama proboscidea, Malmgren.
Eunice Oerstedi, Stimps.
Eupista Darwini, M'Int.
Scalpellum velutinum, Hoek.
Typhlotanais kerguelenensis, F. E. B.
Neotanais americanus, F. E. B.
Gnathophausia gigas, W. S.
Munidopsis subsquamosa, Hend.
Mytilus edulis, (L.).
Odostomia rissoides, Hanley.
Natica groenlandica, Beck.
Doris tuberculata, Cuv.
Pustulipora delicatula, Busk.
„ *deflexa*, Smit.
Membranipora crassimarginata, Hincks.
Crisia eburnea, (L.).
Pyrosoma spinosum, Herdman.

Of the remaining forms quoted as common to far southern as well as to northern latitudes, there are a very considerable number in regard to which I for my part have no right to question their identification, yet they seem to me to fall into a category of evidence comparatively valueless in support of a theory, by reason of their minute size, the paucity of specimens examined, the similarity of allied forms, or from the circumstance that their characters are drawn from hard parts alone, in most instances from a shell, with no reference to the animal within. It must be manifest that while such records swell the lists of apparently discontinuous distribution, yet that a multitude of them would not carry the same conviction as one well-marked and unmistakable Decapod or Cephalopod or Fish. In this category I should be inclined to place about five and twenty other species cited by Murray, including the eight species of Ostracods, most of the Polyzoa, and a number of mostly very small molluscan shells:—

Obelia geniculata, (L.).
Flabellum apertum, Moseley.
Placostegus ornatus, Sow.
The eight species of Ostracods.
Glomus nitens, Jeff.
Kellia suborbicularis, Mont.
Dentalium keras, Watson.
Trochus infundibulum, Watson.
Puncturella noachina, (L.).
Homalogyra atomus, Phil.
Pustulipora proboscidioides, Smit.
Hornera lichenoides, (L.).
„ *violacea*, Sars.
Membranipora galeata, Busk.
Escharoides verruculata, (Smit.).
Diachoris magellanica, Busk.
Platydia anomioides, (Scacchi).

We next come to a considerable number of forms in which the northern specimens identified (often dubiously enough) with the southern come, not from the North Atlantic, but only from the North Pacific, and mainly from Japan.

Southern Forms not represented in the North Atlantic but in the North Pacific.

Cereus spinosus, R. Hertwig.	Japan.
Liponema multipora, R. Hertwig.	Japan.
Holothuria Thomsoni, Théel.	Japan (?).
Lætmogone Wyville-Thomsoni, Théel.	Japan.
Phormosoma hoplacantha, Wy. Th.	Japan.
Ophiacantha rosea, Lym.	Japan.
Eurycope fragilis, F. E. B.	Japan.
Glyphocrangon rimapes, Sp. B.	Japan.
Munidopsis subsquamosus, Hend.	Japan.

Cribrilina monoceros, Busk. Japan.
Synaphobranchus bathybius, Günther. Japan.

These coincidences are, to my mind, important, but they are far from proving the "bipolar hypothesis." Information is slowly growing in regard to the affinities of the Japanese fauna with that of the western American coast, and this faunistic relation between the Japanese, the western American, and the far southern forms is, in my opinion, an important and indisputable one.

After considering and deducting the forms enumerated above, the list with which we started shrinks into little space. There remains, in the first place, the little Copepod *Harpacticus fulvus*, found in the brackish pools of Kerguelen Island, which rather belongs to the question of the distribution of fresh-water animals, a problem totally distinct from that of the marine. We next have a single Annelid, *Terebellides Strömii*, M. Sars. The few remaining forms belong either to the pelagic or to the abyssal fauna of the ocean. In the former group we have two instances, namely,

Ianthina rotundata, Leach.
Calanus finmarchicus, Gunner.

The latter of these is the commonest and most widely distributed of all the Copepods. It does not seem to be recorded from further south than the Cape, its place being taken at Kerguelen by *C. propinquus*, Brady, which occurs there in the same abundance as *C. finmarchicus* in Arctic seas. The former is widespread over the Atlantic; and neither of them is recorded, to my knowledge, from further south than 35° S. lat.

We are left with the following list of deep-water or abyssal species:—

Elpidia glacialis, Théel.
Euphronides depressa, Théel.
Ophioglypha bullata, Wy. Th.
Ophiocten hastatum, Lym.
Ophiernus vallincola, Lym.
Pontaster forcipatus, Sladen.
Dytaster exilis, Sladen.
Kinetoskias cyathus, K. and D.

Boreomysis scyphops, Sars.
Lophogaster typicus, Sars.
Stomias boa, Cuv.
Halosaurus macrochir, Günther.

These are the forms in regard to which the best evidence exists, and it is by no means equal or adequate in them all, of specific identity between examples from the northern and southern seas. In two of them, *Pontaster forcipatus* and *Dytaster exilis*, the southern form is described as a variety, and in the latter case as perhaps a separate species. Differences or doubts are also mentioned in the case of *Euphronides depressa*, *Elpidia glacialis*, and *Stomias boa*, and the small Ophiurids probably deserve to be re-examined before they bear the weight of a theory. But, setting these minor doubts aside, we have in this last list a little assemblage of species that, brought together for one purpose, has at the same time another interest of its own. It is framed to include the safe and sure residuum of forms common to the North Atlantic area and to the southern or Antarctic seas. The species, it is true, of which it consists differ much in their known range, several going no further south than 35° S. lat., and some having a great range in longitude as well as in latitude; and even the restricted list contains instances by no means so safe and sure in their identification as we would desire. But such as it is, the list comprises a collection of deep-water or abyssal species not only remarkable for their distribution, but on the whole (apart from its Ophiuroids) conspicuous as an assemblage of peculiar and aberrant forms. I submit that the facts are entirely inadequate to prove, even for these species, or for the groups to which they belong, a principle of bipolar distribution. They are in the main ancient types, the meaning of whose wide distribution has to be studied in each case by itself.

ON THE FISHES AND ISOPOD CRUSTACEA OF THE ANTARCTIC FAUNA.

I propose, in the next place, to examine the characters of the Antarctic fauna as illustrated by the Fishes and the Isopods, without restricting the discussion in these cases to forms quoted in support of the bipolar hypothesis.

The following is a list of all the Fishes known from the Kerguelen area:—

TRACHINIDÆ.

Notothænia.—Seven species known from Kerguelen Island, and one other from Marion Island. One of the Kerguelen species is known also from the Aucklands, and about five from the Falklands, one of which ranges also to New Zealand.

Harpagifer.—A single species only, known from the Falklands, Cape Horn, Marion and Kerguelen Islands.

Chænichthys.—A single species only, from Kerguelen.

Dr Günther speaks of these forms as representing the Cottoids of the north; but it is clear that this suggestion is meant to be interpreted in the most general way, merely as we sometimes say, for instance, that the Humming-birds of the New World are represented, we might somewhat better say replaced, by the Sunbirds of the Old; the deer of northern regions by the antelopes predominant in Africa; the dog, the beaver, the bear, and the mole of Europe by the Australian Thylacine, Wombat, Koala, and Notoryctes. Dr Günther says, immediately afterwards (*Chall. Rep.*, Shore Fishes, p. 14), that such resemblance is an external one, and that "there is no such relation between the representative forms as might be considered to be genetic." The two groups in question belong to totally different families of fishes, and it will be observed, moreover, that not only the species but the genera of Trachinidæ enumerated have, so far as we know them at present, a clearly circumscribed and definitely antarctic or subantarctic habitat.

SCORPÆNIDÆ.

Zanclorhynchus.—A genus, with a single species, peculiar to Kerguelen.

GADIDÆ.

Murænolepis.—A peculiar genus, with a single species, known only from Kerguelen.

ELASMOBRANCHII.

Raia.—Of this widely distributed genus, two species are known from, and peculiar to, Kerguelen. No characters are recorded in which these resemble European species more than those of the south and west of the American continent or of Japan.

It appears from this, that of thirteen fishes that we know from Kerguelen, *not one fish, and, with the exception of the cosmopolitan skate, no genus of fishes, extends its northern range beyond New Zealand and the Falklands.*

The resemblance which Murray, following Günther, has drawn between the fishes of the northern and the southern *temperate* zones is another problem altogether. Without entering further on this problem, I may refer at least to the two particular instances quoted by Murray, the genera *Trachichthys* and *Polyprion.*

Of *Polyprion* the northern species is exceedingly similar to, though less elongate than, the southern form, and the two are united in a single species by Goode and Bean. One or other occurs on both sides of the North Atlantic, at the Cape, at Juan Fernandez, and in the Australian region. The southern form of this deep-water fish has thus an immense range, and of the northern, Dr Günther says (*A. M. N. H.* (5), xx. p. 237, 1887), "we may well expect that *P. cernium* will be met with far beyond the limits of the North-eastern Atlantic." Its discovery off Newfoundland (Goode and Bean, p. 239) is a beginning of this anticipated extension of its range.

Murray cites from Günther the genus *Trachichthys*, "by which the family *Berycidæ* is represented in the southern temperate zone," as being "much more nearly allied to the northern than to the tropical genera." But the genus itself is both tropical and northern, for one species, *T. Darwinii*, Johnston, is from Madeira, and is said to be identical with *T. japonicus*, St. and D., from Japan, while *T. intermedius*, Hector, described from New Zealand, has more lately been discovered by Alcock in the Bay of Bengal (*A. M. N. H.* (6), iv. p. 380, 1889).

Again, when the two genera, *Sebastes* and *Agonus*, are quoted as Arctic genera represented in the Patagonian region, we must remember that the latter is also represented in Chili and Vancouver, as well as in the North Pacific, and that *Sebastes* is described by Dr Günther (*B. M. Cat.*, Fishes, ii. p. 95) as an "inhabitant of nearly all the seas, but not yet found on the Atlantic shores of tropical America and on the east coast of Africa."

Murray again quotes Dr Günther in regard to the reappearance in the south of the northern genus *Lycodes*, of which a single specimen of a new species was got in Magellan Strait. Of late years a good many more closely allied forms of this deep-water fish have been described, and go a good way to bridge over its area of distribution. The expedition of the "Travailleur" obtained one form, *L. albus*, Vaillant, and American expeditions have obtained various species in many localities, especially off California, and from as far south as 24° north latitude. The not very remote *Melanostigma gelatinosum*, Günther, has been taken in the Straits of Magellan, and somewhat abundantly off California.

We happen to have a considerable list of Isopoda from the Kerguelen area, and of these the following is an epitome giving their geographical range as at present known:—

APSEUDIDÆ.

Apseudes antarctica, F. E. B., and
A. spectabilis, Studer,
are described from and as yet limited to Kerguelen.

TANAIDÆ.

Tanais Willmoesii, F. E. B. Kerguelen.
T. hirsutus, F. E. B. Crozets.
Paratanais dimorphus, F. E. B., from Kerguelen.

Sars (*Isopoda Norway*, p. 16) restricts the genus *Paratanais* to include only the type-species *P. elongatus*, Dana, from the Sooloo Sea, and *P. Batei*, G. O. Sars, from Norway and the Mediterranean.

Leptognathia australis, F. E. B.

A single specimen from Kerguelen is referred by Beddard to this northern genus.

ANTHURIDÆ.

Paranthura neglecta, F. E. B.

This species is represented by a single immature specimen from Kerguelen. Other species are described from Australia and New Zealand. There is some uncertainty as to the limitations of the genera in this family, which as a whole is cosmopolitan.

GNATHIIDÆ.

Anceus gigas, F. E. B. Kerguelen.
A. bathybius, F. E. B. 900 fathoms, the Azores.

The genus *Anceus*, of which several northern forms are known, was originally named in error from male individuals only, and is now replaced in synonymy by the name *Gnathia* of Leach.

Stebbing calls attention to the great differences between Beddard's two southern forms and the type, and institutes the genus *Euneognathia* for the reception of the Kerguelen species. With regard to *A. bathybius*, Beddard, he states that it "will no doubt require to be transferred to another new genus, but the species, being founded on a fragment of a specimen, may wait for a new generic name till fuller material is obtained" (*Crustacea*, p. 338).

AEGIDÆ.

Aega semicarinata. Kerguelen.

SEROLIDÆ.

Serolis latifrons, White. Kerguelen, Aucklands.
S. Bromleyana, W. S. South of Kerguelen.
S. septemcarinata, White. Crozets.
S. trilobitoides, Eights. S. Shetlands, Patagonia.
S. cornuta, Studer. Crozets, Kerguelen.

SPHŒROMIDÆ.

Stebbing (*Crust.*, p. 360) says:—"This family is at present in a state of confusion. Genera and species have been established on characters which, it has since been ascertained, are, at least in some instances, only marks of age or sex."

Sphœroma gigas, Leach. Kerguelen, New Zealand.
Dynamene Eatoni, Miers. Kerguelen,
resembles *D. Dumerilii*, Aud., Natal.
Cassidina emarginata, G. M. Kerguelen, Patagonia, Falklands.

Studer describes a *C. maculata*, also from Kerguelen, while G. M. Thomson has another from New Zealand. The original description of *C. typa*, M. E., in the *Histoire des Crustacés*, iii. p. 224, has no locality.

Cymodoce Darwini, Cunningham.

IDOTEIDÆ.
Arcturides cornutus, Studer. Kerguelen.

The only species of its genus.

ARCTURIDÆ.
Arcturus brunneus, F. E. B. 1600 fathoms, Crozets.
A. furcatus, Studer. Heard Island and Kerguelen.
A. glacialis, F. E. B. South Ocean.
A. spinosus, F. E. B. Marion Island.
A. Studeri, F. E. B. Kerguelen.
A. Stebbingi, F. E. B. Kerguelen.

Not one of these species extends beyond the Southern Ocean. Other species are known from the Arctic, the Australian region, the Moluccas, the West Indies, and Patagonia.

Astacilla marionensis, F. E. B. Kerguelen.

This more shallow water genus, to which the British "Arcturi" belong, has an almost world-wide distribution.

JANIRIDÆ.
Jaera pubescens, Dana. Kerguelen and Tierra del Fuego.
Semi-parasitic on *Spheroma gigas* and from the similar *S. lanceolatum* in Tierra del Fuego.
Jaeropsis marionis, F. E. B. Marion Island.
Belongs to a genus of minute forms, of which one species is known from New Zealand and another from Sark.

Iolanthe acanthonotus, F. E. B. South of Kerguelen.
The single species of its genus.

MUNNIDÆ.

Munna pallida, F. E. B. Kerguelen.
M. maculata, F. E. B. Kerguelen.

Besides the northern species of the genus, Chilton has described *M. Neozealanica* from New Zealand.

Pleurogonium albidum, F. E. B., and
P. serratum, F. E. B.,
are each described from a single specimen from Kerguelen. There is another species from Tristan d'Acunha, and three more from the North Atlantic.

Astrurus crucicauda, F. E. B., and
Neasellus kerguelenensis, F. E. B.

The single species of their genera, as yet known only from Kerguelen.

DESMOSOMIDÆ.

Ischnosoma bacillus, F. E. B. Lat. 50° S., south-west of Melbourne.

Other species, all from deep water, come from Valparaiso, the Azores, the North Pacific, Norway, and the Arctic Ocean.

MUNNOPSIDÆ.

Munnopsis australis, F. E. B. Crozets.

Beddard describes a species from New Zealand, and another doubtfully appertaining to the same genus from Japan. Hansen has one also from the German Plankton Expedition; but, according both to Stebbing (*Crustacea*, p. 384) and to Sars (*Isopoda, Norway*, p. 133), it is questionable whether any of these species really belong to the same genus as *M. typica*, the North Atlantic form.

I. quadrispinosa, Beddard, from Kerguelen, is, according to Sars, markedly different from the North Atlantic *Ilyarachna*, and is more properly to be regarded as the type of a separate genus (*Isopoda of Norway*, p. 135).

Eurycope fragilis, F. E. B. Marion Island, Southern Ocean, Yokohama.

Eurycope, of which three or four species are from the Southern Ocean, has nearly twenty other species from New Zealand, Sandwich Islands, New Guinea, the Azores, Japan, the east and west sides of the North Atlantic, and the west coast of Central America, all from deep water.

Acanthocope spinicauda, F. E. B. 1800 fathoms, lat. 50° S., 123° E.

Acanthocope has two species from the Southern Ocean and Valparaiso, which two species are markedly different (Stebbing, *Crustacea*, p. 387). Certain smaller genera of *Munnopsidæ* are still only known from the North Atlantic.

It appears from this list that *every known Isopod from the Antarctic area* (with the single exception of *Eurycope fragilis*, known also from Japan) *is confined to that area*, or the immediately adjacent seas, one species only extending so far as Valparaiso; while those genera that are not limited to the southern seas are cosmopolitan in their distribution.

Without dealing in similar detail with the Amphipods, we may note that of fifty-eight species recorded from the Kerguelen region six only are recorded from elsewhere, four of these are also Australian, and of the remaining two, which alone are identical with northern species, Stebbing says (*Chall. Rep.*, "Amphipoda," p. 1135), in reference to one, namely, *Podocerus falcatus* (Mont.):—"There is the possibility, as I have elsewhere suggested, that these creatures may have travelled out from our own waters along with the vessel to the southern latitudes in which they were captured." The specimen of this species recorded from the Cape of Good Hope is mentioned as having been taken from the screw of the ship. We may also add that in this genus the pronounced sexual dimorphism and the great changes undergone in the different stages of growth have given rise to considerable confusion as to the exact limits of many of the species, which confusion has not yet been altogether removed. While in regard to the other, *Eusirus longipes*, Sars says (*Amphipoda, Norway*, p. 421):—"The form recorded by the Rev. Mr Stebbing under this name from the

'Challenger' Expedition is scarcely identical with Boeck's species."

The difficulty and uncertainty attending the examination of forms, even so highly organised as these, may be illustrated by the following instance.

The species now called *Polycheria antarctica*, originally described by Stebbing from a dried specimen taken from a sponge in the Antarctic Ocean under the name *Dexamine antarctica* (*A. M. N. H.*, (4) xv., p. 184), later transferred by him to the genus *Atylus* (*A. M. N. H.*, (5) ii., p. 370), and now regarded by him as identical with the form which he afterwards described (*Chall. Rep.*, Amphipods) from Kerguelen as *Tritata kergueleni* (under which name it is cited in Murray's recent list as one of the species peculiar to Kerguelen), is in all probability identical with the forms described from Australia and New Zealand under the names *Polycheria tenuipes*, Haswell, *P. brevicornis*, Haswell, and *P. obtusa*, Thomson. In other words, this single species has been described by so great and careful an authority as Mr Stebbing as two species under three generic names, now replaced by a fourth, and by other writers as three other distinct species.

Of the genus to which this species is now ascribed, namely, *Polycheria*, one other species only is known, from Puget Sound, in Western North America.

SUMMARY.

The foregoing analysis shows that of some ninety species quoted by Murray as common to northern and southern localities while absent from the intermediate zone, there are more than one-third in which grave doubt as to their identification was expressed by the original describers, or in which the identification has been doubted or denied by later writers.

In somewhat more than another third the evidence of identity is inconclusive or even inadmissible by reason of the nature of the examination to which the specimens were subjected (as in the case of the horny and calcareous sponges), or by reason of the small size of the objects and lack of adequate marks of characterisation (as in the case of the minute Ostracod and Molluscan shells).

Of the remaining forms, about a dozen find their northern representatives in the Japanese seas, where they form part of a fauna predominantly southern in its relations, and where at least the occurrence of any particular form cannot be taken, *ipso facto*, as evidence of a boreal centre of distribution.

Both these last forms and the remnant of equal number that are quoted as occurring in the North Atlantic as well as in or near the Southern Ocean, are for the most part deep-water species, and have in a large proportion of cases peculiar characters of their own. We cannot say at present that they are forms characteristic of any particular geographical province, and their specific area of distribution has in some cases been greatly extended since the date of their original discovery.

Turning, in conclusion, to particular groups, we find the bipolar hypothesis specifically rejected by Prof. Herdman in the case of the Tunicata, by Prof. Ludwig in that of the Holothurians, and by Dr Ortmann in that of the Crustacea; and (to limit ourselves to the groups that we have particularly discussed) we have found no single species of fish, of Decapod, of Isopod, no certain one out of a large fauna of Amphipods, to inhabit at once the Arctic and Antarctic Oceans, or the regions adjacent thereto.

Before leaving this subject in the meantime, it is proper to admit that the question of *specific* identity between Arctic or sub-Arctic and Antarctic or sub-Antarctic forms is not the only evidence, and not necessarily the most important evidence, by which to decide the truth of the "bipolar hypothesis." In the distribution of the land mammalia, the existence of a northern circumpolar or circumterrestrial region is unaffected by questions of the specific identity of, among many other examples, the European and American bison, beaver, elk, or reindeer. And though, on the one hand, the separate existence in the Arctic and Antarctic of one or more well-marked types of fish or higher invertebrates, of whose specific identity there could be no question, would be the simplest proof of a sound basis in the bipolar theory, yet, without any such cases of specific identity, the theory might still find firm support if it could be shown that similar and truly allied forms gave to the two regions a common facies. We have seen that, as regards the fish, even such a likeness as this is certainly absent; even such a likeness is explicitly denied by Ortmann for the Crustacea, and by Ludwig for the Holothurians; nor do I myself, in the case of any other group, see signs of its existence.

Note.—Since the above was written, I have read Mr I. C. Thompson's paper on "Antarctic Plankton" in the *Trans. Liverpool Biol. Soc.*, vol. xii., 1898. Mr Thompson considers the common large Antarctic *Calanus* to be identical with the *C. hyperboreus* of the Arctic. But the latter is closely allied to, if not merely a large variety of *C. finmarchicus*, which is known to occur off the Canaries in 30° N. lat., as well as off Australia in 37° S. lat., and which, according to Thompson, is also present in the Antarctic together with the genus *hyperboreus*; it is therefore not "bipolar" but cosmopolitan (*cf. supra*, p. 326).

Note on Dew Bows. By C. G. Knott, D.Sc., and R. A. Lundie, M.A., B.Sc., M.B., C.M.

(Read December 19, 1898.)

A dew bow differs from a fog bow in being formed by minute globules of water resting on the ground; but these globules may or may not have been produced by the usual process of dew formation. In the *Proceedings* of the Society (vol. vii., 1870), Clerk Maxwell has published a note on a bow seen on ice. With the exception of this record, we have been unable to find any distinct account of observations of the phenomenon, although no doubt it must have been occasionally observed soon after sunrise.

The peculiarity of the bows we observed was that they were seen by night, the sources of light being the gas lamps and electric lights of the city. It was on the evening of Friday, November 11, after several days of thick foggy weather. The tiny fog particles seem to have gradually settled down in the still air, the fog steadily clearing the whole time. These globules, in spite of their contact with the damp ground, must have retained their spherical form almost perfectly; for they were able to throw back to the eye, with a rainbow deviation of approximately 42°, the light incident upon them.

In the neighbourhood of every lamp where the surface of the ground had been undisturbed by traffic the bows were seen as bright streaks, which shifted position with the observer. Any disturbance of the surface, a wheelmark, a footprint, a finger drawn across the pavement, obliterated the bright streak; and it was not seen on the surface of puddles. On stooping, we saw the bow run from us along the ground; and as we moved, it moved with us. With the gas lamp as a source of light the streak was of one colour; but with the more powerful electric light as the source it was possible to distinguish rainbow tints.

Every drop of water that is efficient in producing the bow must be the vertex of a triangle, whose base is the line joining the source of light and the eye of the observer, and whose vertical angle is about 42°. Hence, all possible bow-producing drops must lie on or near the surface described by the revolution about this line of the circle which circumscribes any one of these triangles. The plane sections of this toroidal surface give the various forms of dew bow that may be seen on level ground. Many of these have a heart shape; and if the eye of the spectator were on the level of the ground the two halves of the curve, which is symmetrical with regard to the vertical plane containing the source and the eye, would form a cusp-like point as they met at the eye.

The equation of the dew bow referred to the stance of the observer as origin is easily shown to be

$$(r^2 - ar\cos\theta + h\mathrm{H})^2 = \cos^2\alpha(r^2 + h^2)(r^2 - 2ar\cos\theta + a^2 + \mathrm{H}^2)$$

where r, θ are the usual polar co-ordinates, H the height of the source of light, h the height of the observer's eye, a the horizontal distance between the eye and the source of light, and α the angle of maximum deviation of any particular ray which has suffered two refractions and one reflection in the drop. By choosing different values for r, and solving for θ, we can very rapidly trace any particular case by points. The particular form of the curve will of course depend upon the relative values of h, H, and a.

A set of curves for different values of these constants may be traced out with great ease by means of a triangular frame with two grooves cut on it so as to meet at the proper angle of 42°, the grooves being made to slide simultaneously on the tops of two pillars, of which the one represents the lamp post and the other the observer. The point where the grooves meet will trace out the required curve on any chosen plane surface, and indeed on any surface whatsoever.

Clerk Maxwell, in his note on the bow observed on ice, says:—"How a drop of water can lie upon ice without wetting it and losing its shape altogether, I do not profess to explain."

In the phenomenon described in this paper a similar difficulty meets us. We have all seen dew drops of different sizes more or less distorted from the spherical form; but not usually on the damp surface of the ground. What we have to postulate for the formation of dew bows is a multitude of minute drops of nearly the same size, and of so small a diameter that the sphericity is not appreciably disturbed. Here we must suppose that the impact of the drops on the ground itself, or on the drops already lying there, was so extremely gentle that it did not break through their surface-films. The well-defined character of the bows observed at distances of 10 or 20 yards from a dim gas lamp proves at once a great uniformity in the size of the drops, and a practically innumerable multitude of them. The only meteorological conditions under which such uniformity could be attained are what we have already described, viz., a calm, still, foggy air, permitting the fog particles to settle gently down upon the ground with a terminal speed of, at most, an inch per second.

December 23.—Since the above note was read, we have heard that the phenomenon was observed by some of the night policemen on the two previous evenings of November 9 and 10, while the fog was still dense, as well as on the 11th, when we noticed it. The bows are said to have been much brighter on the last night than on the two previous ones; and on that night to have increased in brilliancy towards morning.

On the Eliminant of a Set of Quadrics, Ternary or Quaternary. By E. J. Nanson. *Communicated by* Professor CHRYSTAL.

(Read January 9, 1899.)

1. There are two methods for expressing in determinant form the eliminant of three ternary quadrics. If u, v, w are the quadrics and J their Jacobian we may eliminate dialytically the ten quantities x^3, y^3, z^3, y^2z, yz^2, z^2x, zx^2, x^2y, xy^2, xyz from the ten cubics xu, yu, zu, xv, yv, zv, xw, yw, zw, J. This is in effect the process given by Sylvester in 1841.* The other method is to eliminate dialytically the six quantities x^2, y^2, z^2, yz, zx, xy from u, v, w and the three differentials of the Jacobian.†

2. The eliminant of four quaternary quadrics may be found in determinant form by a process having points in common with each of these two methods. Multiplying each of the quadrics by the variables x, y, z, w in turn we get sixteen quaternary cubics, and we have also the four differentials of the Jacobian. Thus we have in all twenty quaternary cubics, or just sufficient to eliminate dialytically the twenty expressions $x^3, \ldots, x^2y, \ldots xyz, \ldots$ The result is a determinant of order 20, and of the correct degree, viz., 8, in the coefficient of each quadric.

It does not appear to be possible to extend this process so as to obtain, in determinant form, free from extraneous factors, the eliminant of a set of quadrics in more than four variables.

3. As an illustration of the two methods of forming the eliminant of a set of ternary quadrics consider the equations

$$\left.\begin{aligned} ax^2 + fyz + gzx + hxy &= 0 \\ b'y^2 + f'yz + g'zx + h'xy &= 0 \\ c''z^2 + f''yz + g''zx + h''xy &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

discussed by Sylvester ‡ and Muir. §

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* *Cambridge Mathematical Journal*, vol. ii. p. 235. See also Muir, *History of Determinants*, p. 233.

† Salmon, *Higher Algebra*, § 90, p. 85.

‡ *Loc. cit.*, p. 233.

§ *Proc. Roy. Soc. Edin.*, vol. xxi. pp. 220–234.

The Jacobian of these quadrics is

$$\begin{vmatrix} 2ax+hy+gz & hx+fz & gx+fy \\ h'y+g'z & h'x+2b'y+f'z & g'x+f'y \\ h''y+g''z & h''x+f''z & g''x+f''y+2c''z \end{vmatrix}.$$

Expanding and rejecting the factor 2 we get

$$\left.\begin{array}{c} -aFx^3-b'G'y^3-c''H''z^3 \\ +b'(2c''h+H')y^2z+c''(2b'g+G'')yz^2 \\ +c''(2af'+F'')z^2x+a(2c''h'+H)zx^2 \\ +a(2b'g''+G)x^2y+b'(2af''+F')xy^2 \\ +(\Delta+4ab'c'')xyz \end{array}\right\} \quad . \quad . \quad (2)$$

where Δ is the determinant

$$\begin{vmatrix} f & g & h \\ f' & g' & h' \\ f'' & g'' & h'' \end{vmatrix}$$

and F, G, etc., are the co-factors of f, g, etc., in Δ.

4. Eliminating x^2, y^2, z^2, yz, zx, xy dialytically from (1) and the three differentials of (2), we obtain for the eliminant of (1) the determinant

$$\left|\begin{array}{cccccc} a & \cdot & \cdot & f & g & h \\ \cdot & b' & \cdot & f' & g' & h' \\ \cdot & \cdot & c'' & f'' & g'' & h'' \\ -3aF & b'(2af''+F') & c''(2af'+F'') & \Delta+4ab'c'' & 2a(2c''h'+H) & 2a(2b'g''+ \\ a(2b'g''+G) & -3b'G' & c''(2b'g+G'') & 2b'(2c''h+H') & \Delta+4ab'c'' & 2b'(2af''+ \\ a(2c''h'+H) & b'(2c''h+H') & -3c''H'' & 2c''(2b'g+G'') & 2c''(2af'+F'') & \Delta+4ab' \end{array}\right.$$

Taking out the factors a, b', c'' from columns 1, 2, 3 and then reducing to zero all the elements common to rows 1, 2, 3 and columns 4, 5, 6 by subtracting from columns 4, 5, 6 the proper multiples of columns 1, 2, 3, we find for the eliminant $4^3ab'c''$ times the determinant

$$\begin{vmatrix} a(b'c''-f'f'')+fF & a(c''h'-f'g')+gF & a(b'g''-h''f')+hF \\ b'(c''h-fg'')+f'G' & b'(c''a-g''g)+g'G' & b'(af''-gh'')+h'G' \\ c''(b'g-h'f)+f''H'' & c''(af'-g'h)+g''H'' & c''(ab'-hh')+h''H'' \end{vmatrix} \quad (3)$$

5. If A, B′, C″ are the co-factors of a, b', c'' in Δ' where

$$\Delta' = \begin{vmatrix} a & h & g \\ h' & b' & f' \\ g'' & f'' & c'' \end{vmatrix}$$

the eliminant may also be expressed as $4^3ab'c''$ times the determinant

$$\begin{vmatrix} aA+fF & h'B'+g'(gh''-af'') & g''C''+h''(g'h-af') \\ hA+f(h''f'-b'g'') & b'B'+g'G' & f''C''+h''(h'f-b'g) \\ gA+f(f''g'-c''h') & fB'+g'(fg''-c''h) & c''C''+h''H'' \end{vmatrix} \quad (4)$$

6. If with Sylvester and Muir we reject the special factor $4^3ab'c''$ the eliminant of (1) is the third order determinant (3), or what is the same thing, the third order determinant (4).

From (3) it is obvious that when

$$a=b'=c''=0$$

the eliminant reduces to $FG'H''\Delta$ and from (4) it is obvious that when

$$f=g'=h''=0$$

the eliminant reduces to $AB'C''\Delta'$.

7. Returning now to the other method of elimination we find, on eliminating x^3, x^2y, etc., dialytically from the Jacobian J and the nine equations obtained from (1) by multiplying each equation by x, y, z, for the eliminant of (1), the determinant

$$\begin{vmatrix} a & \cdot & \cdot & \cdot & \cdot & \cdot & g & h & \cdot & f \\ \cdot & b' & \cdot & f' & \cdot & \cdot & \cdot & \cdot & h' & g' \\ \cdot & \cdot & c'' & \cdot & f'' & g'' & \cdot & \cdot & \cdot & h'' \\ \cdot & \cdot & \cdot & f & \cdot & \cdot & \cdot & a & h & g \\ \cdot & \cdot & \cdot & \cdot & f & g & a & \cdot & \cdot & h \\ \cdot & \cdot & \cdot & b' & f' & g' & \cdot & \cdot & \cdot & h' \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & g' & h' & b' & f' \\ \cdot & \cdot & \cdot & \cdot & \cdot & c'' & g'' & h'' & \cdot & f'' \\ \cdot & \cdot & \cdot & f'' & c'' & \cdot & \cdot & \cdot & h'' & g'' \\ -aF & -b'G' & -c''H'' & (4) & (5) & (6) & (7) & (8) & (9) & (10) \end{vmatrix}$$

where

$$(4)=b'(2c''h+H'),\quad (6)=c''(2af'+F''),\quad (8)=a(2b'g''+G)$$
$$(5)=c''(2b'g+G''),\quad (7)=a(2c''h'+H),\quad (9)=b'(2af''+F')$$
$$(10)=\Delta+4ab'c''.$$

Taking out the factors a, b', c'' from columns 1, 2, 3, and then reducing to zero all the elements common to rows 1, 2, 3 and columns 4, 5, 6, 7, 8, 9, 10 by subtracting from the last mentioned columns the proper multiples of columns 1, 2, 3, we find for the eliminant $ab'c''$ times the determinant

$$\left|\begin{array}{ccccccc} f & . & . & . & a & h & g \\ . & f & g & a & . & . & h \\ b' & f' & g' & . & . & . & h \\ . & . & . & g' & h' & b' & f' \\ . & . & c'' & g'' & h'' & . & f'' \\ f'' & c'' & . & . & . & h'' & g'' \\ (71) & (72) & (73) & (74) & (75) & (76) & (77) \end{array}\right| \qquad (5)$$

where

$$\begin{aligned} (71) &= b'(2c''h + \mathrm{H}') + f'\mathrm{G}' \\ (72) &= c''(2b'g + \mathrm{G}'') + f''\mathrm{H}'' \\ (73) &= c''(2af' + \mathrm{F}'') + g''\mathrm{H}'' \\ (74) &= a(2c''h' + \mathrm{H}) + g\mathrm{F} \\ (75) &= a(2b'g'' + \mathrm{G}) + h\mathrm{F} \\ (76) &= b'(2af'' + \mathrm{F}) + h'\mathrm{G}' \\ (77) &= \Delta + 4ab'c'' + f\mathrm{F} + g'\mathrm{G}' + h''\mathrm{H}''. \end{aligned}$$

Rejecting, as before, the special factor $ab'c''$, the eliminant of (1) is (5). Multiplying now rows 1, 2, 3, 4, 5, 6 of (5) by

$$\mathrm{G} - 2b'g'', \ \mathrm{H} - 2c''h', \ \mathrm{H}' - 2c''h, \ \mathrm{F}' - 2af'', \ \mathrm{F}'' - 2af', \ \mathrm{G}'' - 2b'g,$$

adding to row 7 and dividing by 4 we get

$$\left|\begin{array}{ccccccc} f & . & . & . & a & h & g \\ . & f & g & a & . & . & h \\ b' & f' & g' & . & . & . & h' \\ . & . & . & g' & h' & b' & f' \\ . & . & c'' & g'' & h'' & . & f'' \\ f'' & c'' & . & . & . & h'' & g'' \\ -b'fg'' & -c''h'f & -c''g'h & -af''g' & -ah''f' & -b'gh'' & \lambda \end{array}\right| \qquad (6)$$

where

$$\lambda = \Delta + \Delta' - f'g''h - f''gh'.$$

This form of the eliminant is equivalent to that given by Muir in the last volume of the *Proceedings*, p. 233 (ϵ). In the special cases considered in § 6, all the elements of the last row of (6) except the last vanish. The last element of the last row of (6) reduces to either Δ or Δ', and its co-factor reduces to either

FG′H″ or AB′C″ so that as in § 6 the eliminant reduces to either FG′H″Δ or AB′C″Δ'.

8. An interesting special case is that of the equations

$$\left.\begin{array}{l} fx^2 + ayz - hzx - gxy = 0 \\ gy^2 - hyz + bzx - fxy = 0 \\ hz^2 - gyz - fzx + cxy = 0 \end{array}\right\} \quad . \quad . \quad . \quad (7)$$

associated with Sylvester's classical elimination problem.

To adapt the preceding results to this case we must replace

$$a,\ b',\ c'',\ f,\ g,\ h,\ f',\ g',\ h',\ f'',\ g'',\ h''$$

respectively by

$$f,\ g,\ h,\ a,\ -h,\ -g,\ -h,\ b,\ -f,\ -g,\ -f,\ c.$$

Making this change and denoting by A, B, etc., the co-factors of a, b, etc., in δ where

$$\delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

the determinant (3) or (4) becomes

$$\begin{vmatrix} a\mathrm{A} & b\mathrm{H} & c\mathrm{G} \\ a\mathrm{H} & b\mathrm{B} & c\mathrm{F} \\ a\mathrm{G} & b\mathrm{F} & c\mathrm{C} \end{vmatrix}$$

and is therefore equal to $abc\delta^2$. Hence restoring the literal portion of the factor omitted in § 6 the eliminant of (7) is $abcfgh\delta^2$.

9. Again by making the substitution of the preceding section the determinant (6) becomes

$$\begin{vmatrix} a & . & . & . & f & -g & -h \\ . & a & -h & f & . & . & -g \\ g & -h & b & . & . & . & -f \\ . & . & . & b & -f & g & -h \\ . & . & h & -f & c & . & -g \\ -g & h & . & . & . & c & -f \\ afg & afh & bgh & bgf & chf & chg & \delta - 6fgh \end{vmatrix} \quad . \quad . \quad (8)$$

Multiplying rows 1, 2, 3, 4, 5, 6 by

$$fg,\ hf,\ gh,\ fg,\ hf,\ gh,$$

and subtracting from row 7, the first six elements in row 7 are reduced to zero and the seventh element becomes δ. Hence the determinant (8) is δ times the determinant

$$\begin{vmatrix} a & \cdot & \cdot & \cdot & f & -g \\ \cdot & a & -h & f & \cdot & \cdot \\ g & -h & b & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & b & -f & g \\ \cdot & \cdot & h & -f & c & \cdot \\ -g & h & \cdot & \cdot & \cdot & c \end{vmatrix}$$

and this is found to be $abc\delta$. Hence as before the eliminant of (7) is $abcfgh\delta^2$.

Note on the Compressibility of Solutions of Sugar. By Prof. Tait.

(Read July 18, 1898.)

In continuation of former investigations of the alteration of compressibility of water, which is produced by dissolving various salts in it, I was led to imagine that some instructive results might be furnished by solutions such as those of sugar, whose bulk is nearly the sum of the bulks of their constituents:—for, in them, we might expect little change in compressibility from that of water itself; *i.e.* in accordance with my hypothetical formula, little change in the term regarded as representing the molecular pressure.

The following preliminary results have recently been obtained for me by Mr Shand, Nichol Foundationer, who employed the Fraser gun and the Amagat gauge procured for my "Challenger" work:—and a new set of piezometers of the same (Ford's) glass as that whose compressibility I had determined to be 0·0000026. These have been carefully gauged, but have not as yet been directly compared with those formerly employed.

The solutions experimented on were prepared, in Dr Crum Brown's Laboratory, by Mr W. W. Taylor, M.A., B.Sc., and contained respectively 5, 10, 15, 20 parts, by weight, of sugar to 100 of water. The temperature varied but slightly from 12°·4 C. during the whole course of the experiments.

Average Compressibility per Atmosphere, at 12°·4 *C.*

Sugar per 100 water	0	5	10	15	20
For first ton, . .	0·00004650	4430	4265	4109	3965
,, two tons, .	4520	4316	4160	4013	3875
,, three tons, .	4410	4210	4065	3920	3789

The numbers in the first column were taken direct from the Plate in my second *Challenger* Report, 0·0000026 being (of course) added to each.

The Reciprocals of these numbers are, in order,

2151	2257	2344	2439	2522
2212	2317	2404	2492	2581
2268	2375	2460	2551	2640

Comparing with the formula, we see that these reciprocals should be, in the first column proportional to Π, $\Pi+1$, $\Pi+2$; in the second to $\Pi+5x$, $\Pi+1+5x$, $\Pi+2+5x$; etc., where x is the increase of Π for 1 part sugar in 100 (by weight) of water.

The results are not very concordant, especially in the second and fifth columns (which seem to indicate some error in the gauging of the corresponding piezometers), but they are all fairly satisfied by taking

$$\Pi : 1 : x = 2151 : 58{\cdot}1 : 19{\cdot}2 ;$$

so that the actual value of Π appears to be 37 tons' weight per sq. inch.

Thus it appears that the effect of sugar is, weight for weight, barely one-third of that of common salt in reducing the compressibility of water; for, with common salt, $x=1$ nearly.

Dust Figures of Electrostatic Lines of Force. By David Robertson, A.G.T.C. *Communicated by* Professor Jamieson, M.Inst. C.E., of the Electrical Engineering Department of the Glasgow and West of Scotland Technical College.

(Read December 19, 1898.)

When lecturing on Electrostatics at the Glasgow and West of Scotland Technical College, the author endeavoured to impress upon his students the identity of the laws of force for magnetism and electricity at rest. He thought it would further this idea if he could get pictures of electrostatic lines analogous to those obtained from magnets by means of iron filings, and an analysis of the difference of the conditions in the two cases led to the conclusion that there was no reason why this should not be done.

When we shower iron filings upon a glass plate, or a sheet of paper, under which a magnet is placed, unlike poles are induced at the ends of each filing, and it becomes for the time being a little compass needle. Since it is very short, the total action upon it will be a couple turning it into the direction of the force at its centre, unless it be close to the magnet, in which case there will be, in addition, a translation along a line of force. This latter effect causes the filings to crowd towards the edges of the magnet.

Now, in electrostatics we also have an inductive action, opposite charges being induced in the near and far ends of each particle as it comes into the field, but we have the important difference, that whereas the magnetic poles are attached to the filing and cannot be separated from it, the electric charges will leave a body if they have an opportunity of doing so. To avoid this difficulty as far as possible, the substance we employ should have some insulating qualities to prevent the immediate escape of the charges, and yet should be a sufficiently good conductor to allow enough electrical separation to take place. Further, each particle should be greater in one dimension than in the other two, in order that there may be a tendency to bring it into a definite direction.

An ideal material would consist of little long-shaped grains having a conducting interior within an insulating shell, such as metal filings covered with a thick coating of insulating varnish, but the author has not yet been able to experiment on these lines. He first used fine sawdust, and better results have been obtained by its use than from any other materials he has since tried. Among these are iron filings, magnetic sand (powdered Fe_3O_4), tea (as purchased), oatmeal, and boar's bristles cut into very short pieces. Tea seems a good material, but the curliness of the leaves spoils the appearance of the curves, while oatmeal is rather coarse, although it does show something. The bristles gave no result at all, while the filings and sand were thrown off without forming curves. Probably the former conducts too feebly and the latter too well.

All the photographs are from curves taken with mahogany sawdust coloured with ink except three, for which tea was employed.

As a first trial, one side of each of three glass plates was varnished with shellac, and pieces of tinfoil were stuck on it. One plate had a single circular disc of tinfoil placed at its centre, a second had two discs, and the third had two parallel strips.

The first plate was then supported horizontally by two wooden blocks at its ends with the varnished side downwards, so that a spring of thin wire would make contact with the disc. This spring rested on a glass plate, and was connected to one pole of an ordinary Wimshurst machine. Sawdust was sprinkled on the plate from a muslin bag whilst the machine was running; but, instead of giving pictures of the lines of force, it jumped off again as soon as it touched the glass. The other plates were tried, one piece of tinfoil being connected to each pole of the machine, but with like results.

The author next tried putting the dust on before charging the foil, and then tapping the plate when the machine was started. This proved more satisfactory, for, although the powder flew off as formerly, just before doing so it formed itself partly into definite lines. By putting on larger pieces of tinfoil and stopping the machine the instant the powder began to get thrown outwards, good curves were obtained on all three plates.

The plates were then cleaned and heated, and a thin coating of a mixture of paraffin wax and vaseline was applied to that side of the plate on which there was no tinfoil, *i.e.*, to the side on which the curves were made. When this had set, new curves were made and were fixed by heating them until the wax melted.

Photo. No. 11 is from part of one of these first curves, but with this exception, all the photographs have been taken from curves specially made on clear unvarnished glass, and not fixed. Glasses from photographic plates were used, three of them being whole-plate size and the remainder half-plate. The first illustration shows the apparatus itself, as arranged for getting the later curves. The glass plate lies horizontally in a wooden frame held in a retort stand, while two wires or electrodes, standing on a glass plate, make contact with the tinfoil. Each of these wires is about 24 centimetres long, and carries a little sliding piece at the top, which is pressed upwards by a spring so as to ensure contact. It was found that this method of supporting the plate loosely caused the least confusion from vibration figures.

In order to get a perfect control of the action, the electrodes are connected to the poles of the machine through a pair of thick bare wires lying on the glass table. Another wire is laid across these so as to short circuit the machine until all is ready.

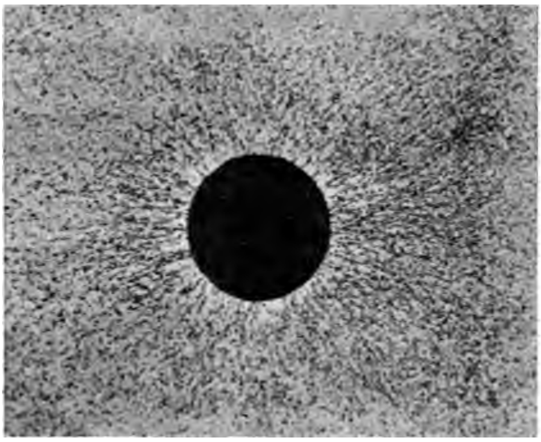
The sawdust, or other material, is spread as uniformly as possible over the plate by means of a sieve or muslin bag, and the Wimshurst is set agoing. The cross wire is lifted, and, while the foil is being charged, the wooden frame (or the arm holding it) should be briskly tapped. The short circuit wire is dropped again whenever the dust begins to move outwards from the tin. This motion is caused by the attracted charge being deposited on the glass, leaving only the repelled charge on the dust. It is so great immediately over the tinfoil that that part is very often cleared of dust altogether. The secret of getting good results lies in stopping the action at the right moment.

When there is only one piece of tinfoil on the plate, only one electrode is used, and when two pieces have to receive like charges, the two electrodes are connected together and to one of the horizontal wires. In both these cases the other horizontal wire is still left connected to its pole of the machine, but is, of course, not allowed to touch the electrodes.

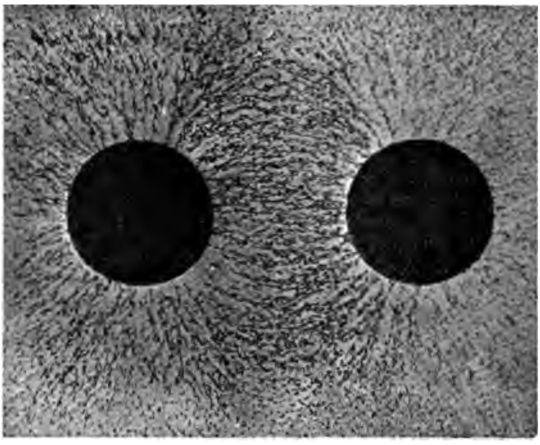
The curves obtained in this manner were photographed in a reducing camera fixed to a wall, so that its axis was vertical and the plates horizontal. The figures were lighted by a curved sheet of white cardboard placed above the camera, and illuminated by a gas jet or by burning magnesium ribbon.

Curves of the same general nature as those drawn by construction or calculation were obtained for the cases shown in the table.

The author is indebted to several of Professor Jamieson's students for assistance in making the curves, and to Messrs Baird & Tatlock for the loan of the apparatus shown at the meeting.



No. 2. Single Charge.



No. 3. Unlike Charges.

ROBERTSON'S DUST FIGURES OF ELECTROSTATIC LINES OF FORCE.

List of Curves and Photographs.

Photo. No.	Conductors.	Charges.	Size of Glass (cms.).	Size of Foil (cms.).	Distance between nearest parts (cms.).	Material Used.	Method of Photographing.	Remarks.
1	The Apparatus, .	...	41 cm. plates on Wimshurst	...	...	...	Camera	
2	Disc, . . .	...	21·5 × 16·5	5 (dia.)	...	Sawdust	Contact	
3	Two discs, . .	Unlike	,,	5 ,,	4·7	,,	Reduced	
4	,, . .	Like	,,	5 ,,	4·7	,,	,,	
5	Unequal discs, .	Unlike	16·5 × 12·0	7·7 & 3·9	2·6	,,	Contact	Ratio of areas = 4:1.
6	Disc in ring, . .	,,	,,	10·2, 7·7 & 2·7	2·5	,,	,,	,, = 6:1.
7	Strip, . . .	...	,,	10 × 1·5	...	,,	,,	
8	Two parallel strips,	Unlike	21·5 × 16·5	12 × 1·5	4	,,	Reduced	
9	,,	Like	16·5 × 12·0	10 × 0·4	2	,,	Contact	
10	Square, . . .	...	,,	5 × 5	...	,,	,,	
—	Disc, . . .	...	24 × 19	5 (dia.)	...	,,	Not photo'd	First three curves fixed on waxed plates.
11	Two discs, . .	Unlike	,,	,,	5	,,	Contact	
—	Two strips, . .	,,	,,	16 × 1·7	3	,,	Not photo'd	
12	Disc, . . .	...	21·5 × 16·5	5 (dia.)	...	Tea	Reduced	
13	Two discs, . .	Unlike	,,	5 ,,	4·7	,,	,,	
14	Two strips, . .	,,	,,	12 × 1	2	,,	,,	

On the Reflexion and Refraction of Solitary Plane Waves at a Plane Interface between two Isotropic Elastic Mediums—Fluid, Solid, or Ether. By Lord Kelvin, G.C.V.O.

(Read December 19, 1898.)

§ 1. "Elastic solid" includes fluid and ether; except conceivable dynamics* of the mutual action across the interface of the two mediums. Maxwell's electro-magnetic equations for a homogeneous non-conductor of electricity are identical with the equations of motion of an incompressible elastic solid,† or with the equations expressing the rotational components of the motion of an elastic solid compressible or incompressible; but not so their application to a heterogeneous non-conductor or to the interface between two homogeneous non-conductors.‡

§ 2. The equations of equilibrium of a homogeneous elastic solid, under the influence of forces X, Y, Z, per unit volume, acting at any point (x, y, z) of the substance are given in Stokes' classical paper "On the Theories of the Internal Friction of Fluids in Motion, and of the Equilibrium and Motion of Elastic Solids," p. 115, vol. i. of his *Mathematical Papers*; also in Thomson and Tait's *Natural Philosophy* [§ 698 (5) (6)]. Substituting according to D'Alembert's principle, $-\rho\ddot{\xi}$, $-\rho\ddot{\eta}$, $-\rho\ddot{\zeta}$ for X, Y, Z, and using as in a paper of mine§ of date November 28, 1846,

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* See *Math. and Phys. Papers*, vol. iii., Art. xcix. (first published May 1890), §§ 14–20, 21–28; and particularly §§ 44–47. Also Art. c. of same volume; from *Comptes Rendus* for September 16, 1889, and *Proc. Roy. Soc. Edin.*, March 1890.

† See *Electricity and Magnetism*, last four lines of § 616, last four lines of § 783, and equations (9) of § 784.

‡ *Ibid.*, § 611, equations (1*). In these put C = 0, and take in connection with them equations (2) and (4) of § 616. Consider K and μ as different functions of x, y, z; consider particularly uniform values for each of these quantities on one side of an interface, and different uniform values on the other side of an interface between two different non-conductors, each homogeneous.

§ *Camb. and Dublin Math. Jour.*, vol. ii. (1847). Republished as Art. xxvii., vol. i. of *Math. and Phys. Papers*.

∇^2 to denote the Laplacian operator $\frac{d^2}{dx^2}+\frac{d^2}{dy^2}+\frac{d^2}{dz^2}$, we find as the equations of motion

$$\left.\begin{aligned} \rho\frac{d^2\xi}{dt^2}&=(k+\tfrac{1}{3}n)\frac{d\delta}{dx}+n\nabla^2\xi, \\ \rho\frac{d^2\eta}{dt^2}&=(k+\tfrac{1}{3}n)\frac{d\delta}{dy}+n\nabla^2\eta, \\ \rho\frac{d^2\zeta}{dt^2}&=(k+\tfrac{1}{3}n)\frac{d\delta}{dz}+n\nabla^2\zeta, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1),$$

ρ denoting the density of the medium, ξ, η, ζ its displacement from the position of equilibrium (x, y, z), and δ the dilatation of bulk at (x, y, z) as expressed by the equation

$$\delta=\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

§ 3. Taking d/dx, d/dy, d/dz of (1), we find

$$\rho\frac{d^2\delta}{dt^2}=(k+\tfrac{4}{3}n)\nabla^2\delta \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

From this we find

$$\nabla^{-2}\delta=\frac{k+\frac{4}{3}n}{\rho}\cdot\left(\frac{d}{dt}\right)^{-2}\delta \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Put now

$$\xi=\xi_1+\frac{d}{dx}\nabla^{-2}\delta\,; \quad \eta=\eta_1+\frac{d}{dy}\nabla^{-2}\delta\,; \quad \zeta=\zeta_1+\frac{d}{dz}\nabla^{-2}\delta\,; \quad (5).$$

These give

$$\frac{d\xi_1}{dx}+\frac{d\eta_1}{dy}+\frac{d\zeta_1}{dz}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6),$$

and, therefore, eliminating by them ξ, η, ζ from (1), we find by aid of (4)

$$\rho\frac{d^2\xi_1}{dt^2}=n\nabla^2\xi_1\,; \quad \rho\frac{d^2\eta_1}{dt^2}=n\nabla^2\eta_1\,; \quad \rho\frac{d^2\zeta_1}{dt^2}=n\nabla^2\zeta_1\,; \quad (7).$$

§ 4. By Poisson's theorem in the elementary mathematics of force varying inversely as the square of the distance, we have

$$\nabla^{-2}\delta=-\frac{1}{4\pi}\iiint^{\infty} d\,(\text{volume})\,.\,\frac{\delta'}{\mathrm{PP}'}\,; \qquad (8),$$

where δ, δ' denote the dilatations at any two points P and P′; *d* (volume) denotes an infinitesimal element of volume around the point P′; and PP′ denotes the distance between the points

P and P′. This theorem gives explicitly and determinately the value of $\nabla^{-2}\delta$ for every point of space when δ is known (or has any arbitrarily given value) for every point of space.

§ 5. If now we put

$$\xi_2=\frac{d}{dx}\nabla^{-2}\delta\,;\quad \eta_2=\frac{d}{dy}\nabla^{-2}\delta\,;\quad \zeta_2=\frac{d}{dz}\nabla^{-2}\delta\,; \qquad (9),$$

we see by (5) that *the complete solution of* (1) *is the sum of two solutions,* (ξ_1, η_1, ζ_1) *satisfying* (6) *and therefore purely distortional without condensation*; *and* (ξ_2, η_2, ζ_2) *which, in virtue of* (9), *is irrotational and involves essentially rarefaction or condensation or both.* This most important and interesting theorem is, I believe, originally due to Stokes. It certainly was given for the first time explicitly and clearly in §§ 5–8 of his "Dynamical Theory of Diffraction." *

§ 6. The complete solution of (3) for plane waves travelling in either or both directions with fronts specified by (α, β, γ), the direction-cosines of the normal, is, with ψ and χ to denote arbitrary functions,

$$\delta=\psi\left(t-\frac{\alpha x+\beta y+\gamma z}{v}\right)+\chi\left(t+\frac{\alpha x+\beta y+\gamma z}{v}\right) \qquad (10),$$

where

$$v=\sqrt{\frac{k+\frac{4}{3}n}{\rho}}\ . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)\,;$$

so that v denotes the propagational-velocity of the condensational-rarefactional waves. By inspection without the aid of (8), we see that for this solution

$$\nabla^{-2}\delta=v^2\left(\frac{d}{dt}\right)^{-2}\left[\psi\left(t-\frac{\alpha x+\beta y+\gamma z}{v}\right)+\chi\left(t+\frac{\alpha x+\beta y+\gamma z}{v}\right)\right] \quad (12).$$

For our present purpose we shall consider only waves travelling in one direction, and therefore take $\chi=0$; and, for convenience in what follows, we shall take $-\left(\frac{d}{dt}\right)^{-1}f$ instead of $v\left(\frac{d}{dt}\right)^{-2}\psi$; f being an arbitrary function. Thus by (12) and (9) we have, for our condensational-rarefactional solution,

$$\frac{\xi_2}{\alpha}=\frac{\eta_2}{\beta}=\frac{\zeta_2}{\gamma}=f\left(t-\frac{\alpha x+\beta y+\gamma z}{v}\right) \quad . \quad . \quad . \quad (13).$$

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* *Camb. Phil. Trans.*, November 26, 1849. Republished in vol. ii. of his *Math. Papers.*

In the wave-system thus expressed the motion of each particle of the medium is perpendicular to the wave-front (α, β, γ). For purely distortional motion, and wave-front still (α, β, γ) and therefore motion of the medium everywhere perpendicular to (α, β, γ), or *in the wave-front*, we find similarly from (7) and (6)

$$\frac{\xi_1}{\alpha \mathrm{A}}=\frac{\eta_1}{\beta \mathrm{B}}=\frac{\zeta_1}{\gamma \mathrm{C}}=f\left(t-\frac{\alpha x+\beta y+\gamma z}{u}\right) \quad . \quad . \quad . \quad (14),$$

where

$$u=\sqrt{\frac{n}{\rho}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15),$$

and so denotes the propagational velocity of the distortional waves; and A, B, C, are arbitrary constants subject to the relation

$$\alpha^2\mathrm{A}+\beta^2\mathrm{B}+\gamma^2\mathrm{C}=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (16).$$

§ 7. To suit the case of solitary waves we shall suppose the arbitrary function $f(t)$ to have any arbitrarily given value for all values of t from 0 to τ, and to be zero for all negative values of t and all positive values greater than τ. Thus τ is what we may call the transit-time of the wave, that is, the time it takes to pass any fixed plane parallel to its front; or the time during which any point of the medium is moved by it. The thicknesses, or, as we shall sometimes say, the wave-lengths, of the two kinds of waves are $u\tau$ and $v\tau$ respectively, being for the same transit-times directly as the propagational velocities.

§ 8. And now for our problem of reflexion and refraction. At present we need not occupy ourselves with the case of purely distortional waves with vibratory motions perpendicular to the plane of the incident, reflected, and refracted rays. It was fully solved by Green * with an arbitrary function to express the character of the motion (including therefore the case of a solitary wave or of an infinite procession of simple harmonic waves). He showed that it gave precisely the "sine law" which Fresnel had found for the reflexion and refraction of waves "polarized in the plane of incidence." The same law has been found for light, regarded as electro-magnetic waves of one of the two orthogonal polarizations, by von Helmholtz, H. A. Lorenz, J. J. Thomson, FitzGerald, and Rayleigh.* None of them has quite dared to say that the physical action represented by his formulas for this case is a to-and-fro motion of the ether perpendicular to the plane of incidence, reflexion, and refraction; nor has any one, so far as I know, absolutely determined whether it is the lines of electric force or of magnetic force that are perpendicular to that plane in the case of light polarized by reflexion at the surface of a transparent medium. For the action, whatever its physical character may be, which takes place perpendicular to that plane, they all seem to prefer "electric displacement," of which the only conceivable meaning is motion of electricity to and fro perpendicular to the plane. If they had declared, or even suggested, definitely this motion of ether, they would have been perfectly in harmony with the undulatory theory of light as we have it from Young and Fresnel. We shall return to this very simple problem of reflexion and refraction of purely distortional waves in which the motion is perpendicular to the plane of the three rays, in order to interpret in the very simplest case the meaning, for a solitary wave, of

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* "On the Reflection and Refraction of Light at the common Surface of two Non-Crystallized Media," *Math. Papers*, p. 258. Also *Trans. Camb. Phil. Soc.*, 1838.

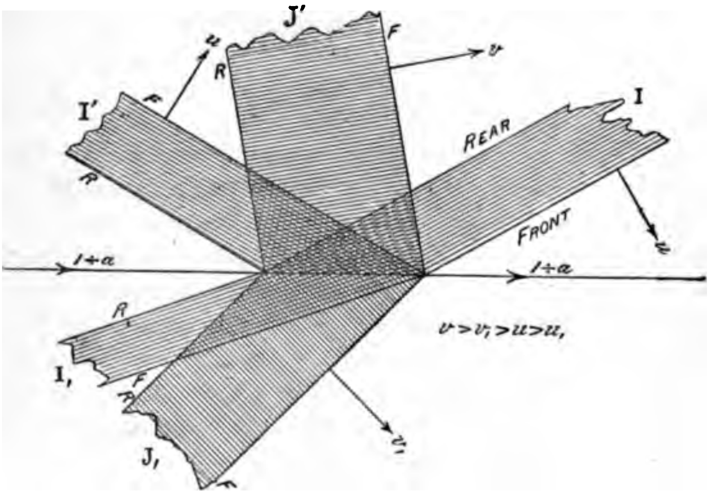

FIG. 1.

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* See Glazebrook's "Report on Optical Theories" to British Association, 1885.

the "change of phase" discovered by Fresnel and investigated dynamically by Green for a procession of periodic waves of simple harmonic motion experiencing "total internal reflexion." (See § 20 below.)

§ 9. Meantime we take up the problem of the four reflected and refracted waves produced by a single incident wave of purely distortional character, in which the motion is in a plane perpendicular to the five wave-fronts. Taking this for XOY, the plane of our diagram, let YOZ be the interface between the two mediums. We shall first consider one single incident wave, I, of the purely distortional character. By incidence on the interface, it will generally introduce reflected and refracted waves I′, I,, of its own kind, that is purely distortional, and J′, J,, reflected and refracted waves of the condensational-rarefactional kind. The diagrams represent, for two cases, sections of portions of the five waves by the plane XOY. F and R show the front and rear of each wave; and the lines of shading belonging to it show the direction of the motion, or of the component, which it gives to the medium. The inclinations of the fronts and rears to OX, being what are ordinarily called the angle of incidence and the angles of reflexion or refraction of the several waves, will be denoted by

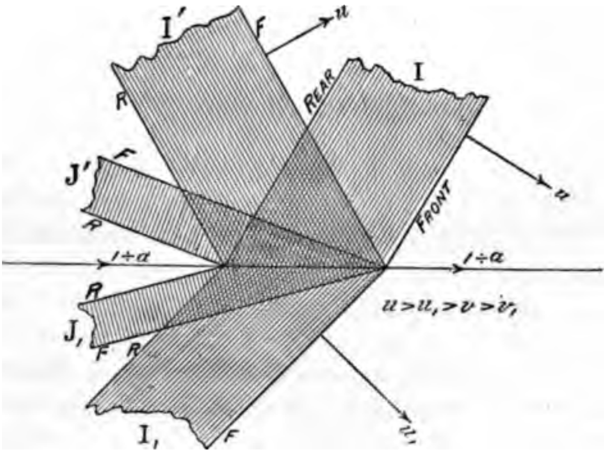

FIG. 2.

i, i', $i_{,}$, j, $j_{,}$. The value of γ for each of the five waves is zero, and the values of α and β are as shown in the following table:—

	α	β
I.........	$+\sin i$	$-\cos i$
I′.........	$+\sin i$	$+\cos i$
$\text{I}_{,}$.........	$+\sin i_{,}$	$-\cos i_{,}$
J′.........	$+\sin j$	$+\cos j$
$\text{J}_{,}$.........	$+\sin j_{,}$	$-\cos j_{,}$

The section of the five waves by OX is the same for all, being expressed by $u\tau/\sin i$ for I, and by corresponding formulas for the four others. Hence if we denote τ-times its reciprocal by a, we have

$$a = \frac{\sin i}{u} = \frac{\sin i_{,}}{u_{,}} = \frac{\sin j}{v} = \frac{\sin j_{,}}{v_{,}} \quad . \quad . \quad . \quad . \quad (17),$$

where u and $u_{,}$ are the propagational velocities of the distortional waves, and v, $v_{,}$, those of the condensational waves in the two mediums. If now we take

$$\begin{aligned} b &= a \cot i = \surd\,(u^{-2} - a^2)\,;\; b_{,} = a \cot i_{,} = \surd\,(u_{,}^{-2} - a^2)\,; \\ c &= a \cot j = \surd\,(v^{-2} - a^2)\,;\; c_{,} = a \cot j_{,} = \surd\,(v_{,}^{-2} - a^2) \qquad (18), \end{aligned}$$

we have for the arguments of f in the five waves

$$t - ax + by\,;\; t - ax - by\,;\; t - ax + b_{,}y\,;\; t - ax - cy\,;\; t - ax + c_{,}y \; (19).$$

§ 10. Following Green * in calling the two sides of the interface the upper and lower medium respectively (and so shown in the diagram), we have for the components of the displacement in the upper medium

$$\left.\begin{aligned} \xi &= b\text{I}f(t - ax + by) - b\text{I}'f(t - ax - by) + a\text{J}'f(t - ax - cy) \\ \eta &= a\text{I}f(t - ax + by) + a\text{I}'f(t - ax - by) + c\text{J}'f(t - ax - cy) \end{aligned}\right\} (20),$$

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* Green's *Math. Papers*, p. 253.

and in the lower medium

$$\left.\begin{array}{l}\xi = b_{,}I_{,}f(t-ax+b_{,}y)+aJ_{,}f(t-ax+c_{,}y)\\ \eta = aI_{,}f(t-ax+b_{,}y)-c_{,}J_{,}f(t-ax+c_{,}y)\end{array}\right\} \quad . \quad . \quad . \quad (21),$$

where I, I′, I$_{,}$, J′, J$_{,}$, denote five constant coefficients. The notation J′ and J$_{,}$ is adopted for convenience, to reserve the coefficient J for the case in which the incident wave is condensational, and there is no incident distortional wave. There would be no interest in treating simultaneously the results of two incident waves, one distortional (I) and the other condensational (J).

§ 11. We may make various suppositions as to the interfacial conditions, in respect to displacements of the two mediums and in respect to mutual forces between them. Thus we might suppose free slipping between the two: that is to say, zero tangential force on each medium; and along with this we might suppose equal normal components of motion and of force; and whatever supposition we make as to displacements, we may suppose the normal and tangential forces on either at the interface to be those calculated from the strains according to the ordinary elastic solid theory, or to be those calculated from the rotations and condensations or dilatations, according to the ideal dynamics of ether suggested in the article referred to in the first footnote to § 1. We shall for the present take the case of no interfacial slip, that is, equal values of ξ, η on the two sides of the interface. Remarking now that where $y=0$, the argument of f for every one of the five waves is $t-ax$ we see that the condition of equality of displacement on the two sides of the interface gives the following equations:—

$$\left.\begin{array}{l}b(I-I')+aJ'=b_{,}I_{,}+aJ_{,}\\ a(I+I')+cJ'=aI_{,}-c_{,}J_{,}\end{array}\right\} \quad . \quad . \quad . \quad . \quad (22).$$

§ 12. As to the force-conditions at the interface, I have already given, for ordinary elastic solid or fluid matter* on the two sides

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* The force-conditions for this case are as follows:—

Normal component force equated for upper and lower mediums,

$$(k-\tfrac{2}{3}n)\delta+2n\frac{d\eta}{dy}=(k_{,}-\tfrac{2}{3}n_{,})\delta_{,}+2n_{,}\left(\frac{d\eta}{dy}\right)_{,};$$

and tangential forces equated,

$$n\left(\frac{d\eta}{dx}+\frac{d\xi}{dy}\right)=n_{,}\left(\frac{d\eta}{dx}+\frac{d\xi}{dy}\right)_{,}.$$

of the interface, a complete solution of the present problem in my paper* "On the Reflexion and Refraction of Light" in the *Philosophical Magazine* for 1888 (vol. xxvi.); nominally for the case of simple harmonic wave-motion, but virtually including solitary waves as expressed by an arbitrary function: and I need not now repeat the work. At present let us suppose the surface-force on each solid to be that which I have found it must be for ether,† if magnetic force is due to rotational displacement of ether, and the lines of magnetic force coincide with axes of rotation of etherial substance. According to this supposition the two components, Q (normal) and T (tangential), of the mutual force between the mediums, which must be equal on the two sides of the interface, are

$$\left.\begin{aligned} Q&=\kappa\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}\right)=\kappa_,\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}\right)_, \\ T&=n\left(\frac{d\eta}{dx}-\frac{d\xi}{dy}\right)=n_,\left(\frac{d\eta}{dx}-\frac{d\xi}{dy}\right)_, \end{aligned}\right\} \quad . \quad . \quad . \quad (23),$$

where κ denotes for ether that which for the elastic solid we have denoted by $(k+\frac{4}{3}n)$, and suffixes indicate values for the lower medium. If we begin afresh for ether, we may define n as $1/4\varpi$ of the torque required to hold unit of volume of ether rotated through an infinitesimal angle ϖ from its orientation of equilibrium, and κ as the bulk-modulus, that is to say, the reciprocal of the compressibility, of ether. Thus we now have as before in equations (15), (11), and (18)

$$\left.\begin{aligned} a^2+b^2&=u^{-2}=\frac{\rho}{n}; & a^2+b_,^2&=u_,^{-2}=\frac{\rho_,}{n_,} \\ a^2+c^2&=v^{-2}=\frac{\rho}{\kappa}; & a^2+c_,^2&=v_,^{-2}=\frac{\rho}{\kappa_,} \end{aligned}\right\} \quad . \quad (24).$$

Using (20) and (21) in (23) with $y=0$ we find

$$\left.\begin{aligned} \kappa(a^2+c^2)J'&=\kappa_,(a^2+c_,^2)J_, \\ n(a^2+b^2)(I+I')&=n_,(a^2+b_,^2)I_, \end{aligned}\right\} \quad . \quad . \quad . \quad (25),$$

whence by (24)

$$\rho J'=\rho_, J_,; \quad \rho(I+I')=\rho_, I_,; \qquad (26).$$

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* In that paper B, A, and ζ denote respectively the n, the $k+\frac{4}{3}n$, and the ρ of the present paper.

† See first footnote to § 1.

By these equations eliminating $\mathrm{I}_,$ and $\mathrm{J}_,$ from (22), we find

$$\left.\begin{aligned} -(b\rho_, - b_,\rho)\mathrm{I} + (b\rho_, + b_,\rho)\mathrm{I}' &= a(\rho_, - \rho)\mathrm{J}' \\ a(\rho_, - \rho)(\mathrm{I} + \mathrm{I}') &= -(c\rho_, + c_,\rho)\mathrm{J}' \end{aligned}\right\} \quad . \quad . \quad (27);$$

and solving these equations for I′ and J′ in terms of I, we have

$$\left.\begin{aligned} \mathrm{I}' &= \frac{(b\rho_, - b_,\rho)(c\rho_, + c_,\rho) - a^2(\rho_, - \rho)^2}{(b\rho_, + b_,\rho)(c\rho_, + c_,\rho) + a^2(\rho_, - \rho)^2}\,\mathrm{I} \\ \mathrm{J}' &= \frac{-2ab\rho_,(\rho_, - \rho)}{(b\rho_, + b_,\rho)(c\rho_, + c_,\rho) + a^2(\rho_, - \rho)^2}\,\mathrm{I} \end{aligned}\right\} \quad . \quad . \quad (28),$$

and with J′ and I′ thus determined, (26) give $\mathrm{J}_,$ and $\mathrm{I}_,$; completing the solution of our problem.

§ 13. Using (18) to eliminate a, b, $b_,$, c, and $c_,$, from (28), and putting

$$\frac{\rho_, - \rho}{\rho_, \cot j + \rho \cot j_,} = h \quad . \quad . \quad . \quad . \quad . \quad (29);$$

we find

$$\frac{\mathrm{I}'}{\mathrm{I}} = \frac{\rho_, \cot i - \rho \cot i_, - h(\rho_, - \rho)}{\rho_, \cot i + \rho \cot i_, + h(\rho_, - \rho)} \quad . \quad . \quad . \quad . \quad (30),$$

and

$$\frac{\mathrm{J}_,}{\mathrm{I}} = \frac{-2h\rho_, \cot i}{\rho_, \cot i + \rho \cot i_, + h(\rho_, - \rho)} \quad . \quad . \quad . \quad . \quad (31).$$

Consider now the case of v and $v_,$ very small in comparison with u and $u_,$; which by (28) makes

$$\cot j \doteq 1/va, \quad \text{and } \cot j_, \doteq 1/v_,a \quad . \quad . \quad . \quad (32).$$

This gives

$$h \doteq \frac{(\rho_, - \rho)\sin i}{\rho_, \frac{u}{v} + \rho \frac{u}{v_,}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33),$$

which is a very small numeric. Hence J′ is very small in comparison with I; and

$$\frac{\mathrm{I}'}{\mathrm{I}} \doteq \frac{\rho_, \cot i - \rho \cot i_,}{\rho_, \cot i + \rho \cot i_,} \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

§ 14. If the rigidities of the two mediums are equal, we have $\rho_, | \rho = \sin^2 i | \sin^2 i_,$, and (34) becomes

$$\frac{\mathrm{I}'}{\mathrm{I}} = \frac{\sin 2i - \sin 2i_,}{\sin 2i + \sin 2i_,} = \frac{\tan(i - i_,)}{\tan(i + i_,)} \quad . \quad . \quad . \quad (35);$$

which is Fresnel's "tangent-formula." On the other hand, if the densities are equal, (34) becomes

$$\frac{\mathrm{I}'}{} = \frac{-\sin(i - i_,)}{\sin(i + i_,)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (36),$$

which is Fresnel's "sine-formula"; a very surprising and interesting result. It has long been known that *for vibrations perpendicular* to the plane of the incident, reflected, and refracted rays, unequal densities with equal rigidities of the two mediums, whether compressible or incompressible, gives Fresnel's sine-law: and unequal rigidities, with equal densities, gives his tangent-law. But for vibrations *in the plane* of the three rays, and both mediums incompressible, unequal rigidities with equal densities give, as was shown by Rayleigh in 1871,* a complicated formula for the reflected ray, vanishing for two different angles of incidence, if the motive forces in the waves are according to the law of the elasticity of an ordinary solid. Now we find for vibrations *in the plane of the rays*, Fresnel's sine-law, with its continual increase of reflected ray with increasing angles of incidence up to 90°, if the restitutional forces follow the law of dependence on rotation which I have suggested † for ether, and if the waves of condensation and rarefaction travel at velocities small in comparison with those of waves of distortion.

§ 15. Interesting, however, as this may be in respect to an ideal problem of dynamics, it seems quite unimportant in the wave-theory of light; because Stokes ‡ has given, as I believe, irrefragable proof that in light polarized by reflexion the vibrations are perpendicular to the plane of the incident and reflected rays, and therefore, that it is for vibrations *in this plane* that Fresnel's tangent-law is fulfilled.

§ 16. Of our present results, it is (35) of § 14 which is really important; inasmuch as it shows that Fresnel's tangent-law is fulfilled for vibrations in the plane of the rays, with the rotational law of force; as I had found it in 1888 § with the elastic-solid-law of force; provided only that the propagational velocities of condensational waves are small in comparison with those of the waves of transverse vibration which constitute light.

§ 17. By (28) we see that when a^{-1}, the velocity of the wave-trace on the interface of the two mediums, is greater than the greatest of the wave-velocities, each of b, $b_{,}$, c, $c_{,}$ is essentially real. A case of this character is represented by fig. 2, in which the velocities of the condensational waves in both mediums are much smaller than the velocity of the refracted distortional wave, and this is less than that of the incident wave which is distortional. When one or more of b, $b_{,}$, c, $c_{,}$ is imaginary, our solution (26) (28) remains valid, but is not applicable to f regarded as an arbitrary function; because although $f(t)$ may be arbitrarily given for every real value of t, we cannot from that determine the real values of

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* *Phil. Mag.*, 1871, 2nd half year.

† See * of § 1.

‡ *Dynamical Theory of Diffraction.* See footnote § 5.

§ See footnote § 14.

$$f(t+\iota q)+f(t-\iota q) \quad . \quad . \quad . \quad . \quad . \quad . \quad (37),$$

and

$$\iota\{f(t+\iota q)-f(t-\iota q)\} \quad . \quad . \quad . \quad . \quad . \quad (38).$$

The primary object of the present communication was to treat this case in a manner suitable for a single incident *solitary* wave whether condensational or distortional; instead of in the manner initiated by Green and adopted by all subsequent writers, in which the realised results are immediately applicable only to cases in which the incident wave-motion consists of *an endless train of simple harmonic waves.* Instead, therefore, of making f an exponential function as Green made it, I take

$$f(t)=\frac{1}{t+\iota\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (39),$$

where τ denotes an interval of time, small or large, taking the place of the "transit-time" (§ 7 above), which we had for the case of a solitary wave-motion starting from rest, and coming to rest again for any one point of the medium after an interval of time which we denoted by τ.

§ 18. Putting now

$$\mathrm{I}=p+\iota q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40);$$

and from this finding I', $\mathrm{I}_{,}$, J', $\mathrm{J}_{,}$; and taking for the real incident wave-motion (§ 10 above)

$$\left.\begin{aligned}\frac{\xi}{b}=\frac{\eta}{a}&=\tfrac{1}{2}\left[\frac{p+\iota q}{t-ax+by+\iota\tau}+\frac{p-\iota q}{t-ax+by-\iota\tau}\right]\\ &=\frac{p(t-ax+by)+q\tau}{(t-ax+by)^2+\tau^2}\end{aligned}\right\} \quad . \quad (41)$$

being the mean of the formulas for $+\iota$ and $-\iota$; we find a real solution for any case of $b_{,}$, c, $c_{,}$, some or all of them imaginary.

§ 19. Two kinds of incident solitary wave are expressed by (41), of types represented respectively by the following elementary algebraic formulas :—

$$\frac{t - ax + by}{(t - ax + by)^2 + \tau^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42),$$

and

$$\frac{\tau}{(t - ax + by)^2 + \tau^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43).$$

The same formulas represent real types of condensational waves with ξ/a and $\eta/(-c)$; instead of the ξ/b and η/a of (41) which relates to distortional waves. It is interesting to examine each of these types and illustrate it by graphical construction: and particularly to enquire into the distribution of energy, kinetic and potential, for different times and places in a wave. Without going into details we see immediately that both kinetic and potential energy are very small for any value of $(t - ax + by)^2$ which is large in comparison with τ^2. I intend to return to the subject in a communication regarding the diffraction of solitary waves, which I hope to make at a future meeting.

§ 20. It is also very interesting to examine the type-formulas for disturbance in either medium derived from (41) for reflected or refracted waves when $b_,$, or c, or $c_,$ is imaginary. They are as follows, for example if $b_, = \iota g$, where g is real;

$$\frac{t - ax}{(t - ax)^2 + (gy + \tau)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44),$$

and

$$\frac{gy + \tau}{(t - ax)^2 + (gy + \tau)^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45).$$

These real resultants of imaginary waves are not plane waves. They are forced linear waves sweeping the interface, on which they travel with velocity a^{-1}; and they produce disturbances penetrating to but small distances into the medium to which they belong. Their interpretation in connection with total internal reflexion, both for vibrations in the plane of the rays, and for the simpler case of vibrations perpendicular to this plane (for which there is essentially no condensational wave), constitutes the dynamical theory of Fresnel's rhomb for solitary waves.

Symmetrical Solution of the Ellipse-Glissette Elimination Problem. By The Hon. Lord M'Laren.

(Read January 23, 1899.)

The publication of Mr Nanson's paper on the glissette elimination problem (*Proc. Roy. Soc.*, vol. xxii. p. 158) has led me to make a further investigation of this interesting point, with the result that the general eliminant is expressed in the form of a single symmetrical bordered determinant. I shall first give the solution for the case where the tracing-point is on the axis of the ellipse, and then extend it to the general case.

I. *Case of the Tracing-point on the Elliptic Axis.*

The equations for the centre of the generating ellipse as tracing-point are—

$$X = \sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)}\,;\ Y = \sqrt{(b^2\cos^2\theta + a^2\sin^2\theta)}.$$

For a point in the line of the major axis, if r be the distance of the tracing-point from the centre, we have

$$\left.\begin{aligned} x - r\cos\theta &= \sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)} \\ y - r\sin\theta &= \sqrt{(b^2\cos^2\theta + a^2\sin^2\theta)} \end{aligned}\right\}$$

whence, by squaring each side,

$$(a^2 - r^2)\cos^2\theta + b^2\sin^2\theta + 2rx\cos\theta - x^2 = 0 \quad . \quad . \quad . \quad (1)$$

$$b^2\cos^2\theta + (a^2 - r^2)\sin^2\theta + 2ry\sin\theta - y^2 = 0 \quad . \quad . \quad . \quad (2)$$

By addition, $$2rx\cos\theta + 2ry\sin\theta + a^2 - r^2 + b^2 - x^2 - y^2 \quad (3)$$

To abridge, these may be written,

$$A\cos^2\theta + C\sin^2\theta + \alpha\cos\theta - x^2 = 0 \quad . \quad . \quad (1)$$

$$C\cos^2\theta + A\sin^2\theta + \beta\sin\theta - y^2 = 0 \quad . \quad . \quad (2)$$

$$\alpha\cos\theta + \beta\sin\theta + \gamma = 0 \quad . \quad . \quad (3)$$

where $$\gamma = A + C - x^2 - y^2 = a^2 - r^2 + b^2 - x^2 - y^2.$$

We have also,

$$\cos^2\theta + \sin^2\theta - 1 = 0 \quad . \quad . \quad . \quad (4)$$

The required determinant, of the 4th order, is formed from (3), together with the three dialytic eliminants got by eliminating successively $\cos^2\theta$, $\sin\theta$ and unity; $\sin^2\theta$, $\cos\theta$ and unity; and $\cos\theta$, $\sin\theta$, $(1)^2$, between (1), (2), and (4). The three partial eliminants are—

$$\begin{array}{c|ccc|l} & [\sin\theta] & [\cos^2\theta] & [1] & \\ (1) & C\sin\theta & A & \alpha\cos\theta - x^2 & \\ (2) & A\sin\theta + \beta & C & -y^2 & =0 \\ (4) & \sin\theta & 1 & -1 & \end{array}$$

$$=\alpha\beta\cos\theta + (A-C)\gamma\sin\theta + \beta(A-x^2) + (A-C)\alpha\cos\theta\sin\theta = 0. \quad (a)$$

$$\begin{array}{c|ccc|l} & [\sin^2\theta] & [\cos\theta] & [1] & \\ (1) & C & A\cos\theta + \alpha & -x^2 & \\ (2) & A & C\cos\theta & \beta\sin\theta - y^2 & =0 \\ (4) & 1 & \cos\theta & -1 & \end{array}$$

$$=(A-C)\gamma\cos\theta + \alpha\beta\sin\theta + \alpha(A-y^2) + (A-C)\beta\cos\theta\sin\theta = 0. \quad (b)$$

$$\begin{array}{c|ccc|l} & [\sin\theta] & [\cos\theta] & [1^2] & \\ (1) & C\sin\theta & A\cos\theta + \alpha & -x^2 & \\ (2) & A\sin\theta + \beta & C\cos\theta & -y^2 & =0 \\ (4) & \sin\theta & \cos\theta & -1 & \end{array}$$

$$=\beta(A-x^2)\cos\theta + \alpha(A-y^2)\sin\theta + \alpha\beta + (A-C)\gamma\cos\theta\sin\theta = 0. \quad (c)$$

The final eliminant is,

$$\begin{array}{c|cccc|l} & [\cos\theta] & [\sin\theta] & [1] & [(A-C)\cos\theta\sin\theta] & \\ (a) & \alpha\beta & (A-C)\gamma & \beta(A-x^2) & \alpha & \\ (b) & (A-C)\gamma & \alpha\beta & \alpha(A-y^2) & \beta & =0 \quad . \quad (\Delta_0) \\ (c) & \beta(A-x^2) & \alpha(A-y^2) & \alpha\beta & \gamma & \\ (3) & \alpha & \beta & \gamma & & \end{array}$$

If we write this bordered symmetrical determinant in the generalized form,

$$\left|\begin{array}{cccc} \alpha\beta & l & m & \alpha \\ l & \alpha\beta & n & \beta \\ m & n & \alpha\beta & \gamma \\ \alpha & \beta & \gamma & \end{array}\right| = 0$$

the solution evidently is of the form,

$$0 = \alpha^2(n^2 - \alpha^2\beta^2) + \beta^2(m^2 - \alpha^2\beta^2) + \gamma^2(l^2 - \alpha^2\beta^2) + 2\alpha\beta(l\alpha\beta - mn) + 2\alpha\gamma(m\alpha\beta - ln) \\ + 2\beta\gamma(n\alpha\beta - lm) \Big\}$$

and, by putting for *l*, *m* and *n* their equivalents, we have

$$\begin{aligned} 0 = \alpha^4(\mathrm{A} - y^2)^2 + \beta^4(\mathrm{A} - x^2)^2 + \gamma^4(\mathrm{A} - \mathrm{C})^2 + 2\alpha^2\beta^2\gamma(\mathrm{A} - \mathrm{C}) &+ 2\alpha^2\beta^2\gamma(\mathrm{A} - x^2) \\ &+ 2\alpha^2\beta^2\gamma(\mathrm{A} - y^2) \\ - \alpha^2\beta^2.\alpha^2 - \alpha^2\beta^2.\beta^2 - \alpha^2\beta^2\gamma^2 - 2\alpha^2\beta^2(\mathrm{A} - y^2)(\mathrm{A} - x^2) &- 2\alpha^2\gamma^2(\mathrm{A} - y^2)(\mathrm{A} - \mathrm{C}) \\ &- 2\beta^2\gamma^2(\mathrm{A} - x^2)(\mathrm{A} - \mathrm{C}) \end{aligned}$$

where each vertically-placed pair of terms corresponds to a double term in the preceding formula.

The 1st and 2nd terms of the 1st line with the 4th term of the 2nd line form a square factor, which I shall put last in order. Then arranging by powers of (A – C), γ, and $\alpha\beta$ the equation is

$$(\mathrm{A} - \mathrm{C})^2\gamma^4 - 2(\mathrm{A} - \mathrm{C})\gamma^2\{\alpha^2(\mathrm{A} - y^2) + \beta^2(\mathrm{A} - x^2)\} + 2(\mathrm{A} - \mathrm{C})\alpha^2\beta^2\gamma \\ - \alpha^2\beta^2(\alpha^2 + \beta^2 + \gamma^2) + 2\alpha^2\beta^2\gamma(2\mathrm{A} - x^2 - y^2) + \{\alpha^2(\mathrm{A} - y^2) - \beta^2(\mathrm{A} - x^2)\}^2 = 0$$

But observing that $\alpha = 2rx$, $\beta = 2ry$, the highest powers of the square term disappear, and the term reduces to $\mathrm{A}^2(\alpha^2 - \beta^2)^2$; whence,

$$\Delta_0 = 0 = (\mathrm{A} - \mathrm{C})^2\gamma^4 - 2(\mathrm{A} - \mathrm{C})\gamma^2\{\alpha^2(\mathrm{A} - y^2) + \beta^2(\mathrm{A} - x^2)\} + 2(\mathrm{A} - \mathrm{C})\alpha^2\beta^2\gamma \\ - \alpha^2\beta^2(\alpha^2 + \beta^2 + \gamma^2) + 2\alpha^2\beta^2\gamma(2\mathrm{A} - x^2 - y^2) + \mathrm{A}^2(\alpha^2 - \beta^2)^2 \Big\}$$

Comparing the complete and the abbreviated forms of the original equations, (1) and (2), we have $\mathrm{A} = a^2 - r^2$; $\mathrm{C} = b^2$; $\alpha = 2rx$; $\beta = 2ry$; $\gamma = \mathrm{A} + \mathrm{C} - x^2 - y^2 = a^2 - r^2 + b^2 - x^2 - y^2$.

If the tracing-point be at the focus, we have

$$a^2 - r^2 - b^2 = \mathrm{A} - \mathrm{C} = 0\,; \quad \gamma = 2\mathrm{A} - x^2 - y^2.$$

The terms in the first line of the expression, $\Delta_0 = 0$, which are all multiplied by (A – C), vanish; and the eliminant becomes, after substituting for γ,

$$\begin{aligned} 0 &= -\alpha^2\beta^2\{\alpha^2 + \beta^2 + (2\mathrm{A} - x^2 - y^2)^2\} + 2\alpha^2\beta^2(2\mathrm{A} - x^2 - y^2)^2 + \mathrm{A}^2(\alpha^2 - \beta^2)^2 \\ &= \alpha^2\beta^2\{(2\mathrm{A} - x^2 - y^2)^2 - \alpha^2 - \beta^2\} + \mathrm{A}^2(\alpha^2 - \beta^2)^2 \\ &= \alpha^2\beta^2\{4\mathrm{A}^2 + (x^2 + y^2)^2 - 4\mathrm{A}(x^2 + y^2) - \alpha^2 - \beta^2\} + \mathrm{A}^2(\alpha^2 - \beta^2)^2 \\ &= \alpha^2\beta^2\{(x^2 + y^2)^2 - 4\mathrm{A}(x^2 + y^2) - \alpha^2 - \beta^2\} + \mathrm{A}^2(\alpha^2 + \beta^2)^2 \end{aligned}$$

(by combining the first and last terms of the preceding expression),

$$= 16r^4x^2y^2\{(x^2 + y^2)^2 - 4(\mathrm{A} + r^2)(x^2 + y^2)\} + 16r^4\mathrm{A}^2(x^2 + y^2)^2$$

The equation is now divisible by $16r^4(x^2+y^2)$; and by putting for A and r their values, the equation for *focus* as tracing-point reduces to the *sextic*, $x^2y^2\{(x^2+y^2)-4a^2\}+b^4(x^2+y^2)=0$, or

$$(x^2+y^2)(x^2y^2+b^4)=4a^2x^2y^2. \quad . \quad . \quad . \quad (f)$$

This is the same locus as is brought out by Mr Nanson (p. 160 of his paper) by a different method of solution. To explain the want of an absolute term Mr Nanson points out that the origin is a conjugate point, which follows from the fact that in the form first given the roots of $b^4(x^2+y^2)$ are imaginary.

These results may be verified by finding the equation of the glissette for *focus* as tracing-point directly. If in the original equations (1) and (2) we put $a^2-r^2-b^2=0$, or $C=A$, these equations reduce to

$$\alpha\cos\theta=x^2-A\,; \qquad \beta\sin\theta=y^2-A$$

Whence after squaring

$$\begin{array}{ccc} [\cos^2\theta] & [\sin^2\theta] & [1] \\ \left|\begin{array}{ccc} \alpha^2 & . & -(A-x^2)^2 \\ . & \beta^2 & -(A-y^2)^2 \\ 1 & 1 & -1 \end{array}\right| =0 \end{array}$$

$$=\alpha^2(A-y^2)^2+\beta^2(A-x^2)^2-\alpha^2\beta^2=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (f')$$

This is identical with the preceding equation when their proper values are given to α, β, A and r^2.

Transforming to axes bisecting the angles at origin, we have for the new X and Y coordinates $x^2+y^2=X^2+Y^2$; $2xy=X^2-Y^2$, whence

$$(X^2+Y^2)(X^2-Y^2)^2-4a^2(X^2-Y^2)^2+4b^4(X^2+Y^2)=0, . \quad . \quad (f'')$$

which is the curve referred to its axis of symmetry. The curve consists of two ovals lying on opposite sides of the new axis of X, although the mechanical construction only covers one of the ovals. By putting $Y=0$ the four points of intersection of these ovals with the axis of X are given by, $X^4-4a^2X^2+4b^4=0$.

I cannot find that the equation Δ_0 is reducible below the 8th degree for any other tracing-point except the focus.

The determinant for a tracing-point on the minor axis is found similarly from

$$\left.\begin{aligned} x - r\sin\theta &= \sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)} \\ y - r\cos\theta &= \sqrt{(b^2\cos^2\theta + a^2\sin^2\theta)} \end{aligned}\right\}$$

II. *Expression of the General Equation of the Glissette as a Symmetrical Determinant.*

The simultaneous equations of the glissette for any tracing-point rigidly connected with the generating ellipse are

$$\left.\begin{aligned} x - r\cos(\theta - \alpha) &= \sqrt{(a^2\cos^2\theta + b^2\sin^2\theta)} \\ y - r\sin(\theta - \alpha) &= \sqrt{(b^2\cos^2\theta + a^2\sin^2\theta)} \end{aligned}\right\} \quad . \quad . \quad (\mathrm{A});$$

where r is the distance of the tracing-point from the centre of the ellipse and α is the inclination of the line, r, to the major axis.

A symmetrical bordered determinant can be got from the equations in this form, but its expression is much simplified by putting $\phi = \theta - \alpha$; $\phi + \alpha = \theta$; or

$$\left.\begin{aligned} x - r\cos\phi &= \sqrt{\{a^2\cos^2(\phi + \alpha) + b^2\sin^2(\phi + \alpha)\}} \\ y - r\sin\phi &= \sqrt{\{b^2\cos^2(\phi + \alpha) + a^2\sin^2(\phi + \alpha)\}} \end{aligned}\right\} \quad . \quad (\mathrm{B})$$

By squaring each side and expanding $(\phi + \alpha)$ we get,

$$\left\{\begin{matrix} a^2\cos^2\alpha \\ + b^2\sin^2\alpha - r^2 \end{matrix}\right\}\cos^2\phi + 2(b^2 - a^2)\cos\alpha\sin\alpha\cos\phi\sin\phi + \left\{\begin{matrix} b^2\cos^2\alpha \\ + a^2\sin^2\alpha \end{matrix}\right\}\sin^2\phi$$
$$+ 2rx\cos\phi - x^2 = 0 \quad . \quad . \quad (1)$$

$$\left\{\begin{matrix} b^2\cos^2\alpha \\ + a^2\sin^2\alpha \end{matrix}\right\}\cos^2\phi + 2(a^2 - b^2)\cos\alpha\sin\alpha\cos\phi\sin\phi + \left\{\begin{matrix} a^2\cos^2\alpha \\ + b^2\sin^2\alpha - r^2 \end{matrix}\right\}\sin^2\phi$$
$$+ 2ry\sin\phi - y^2 = 0 \quad . \quad . \quad (2)$$

By addition,

$$2rx\cos\phi + 2ry\sin\phi + a^2 + b^2 - r^2 - x^2 - y^2 = 0 \quad . \quad . \quad (3)$$

These expressions, with abridged coefficients, are equivalent to,

$$\mathrm{A}\cos^2\phi + 2\mathrm{B}\cos\phi\sin\phi + \mathrm{C}\sin^2\phi + \alpha\cos\phi - x^2 = 0 \quad . \quad (1)$$

$$\mathrm{C}\cos^2\phi - 2\mathrm{B}\cos\phi\sin\phi + \mathrm{A}\sin^2\phi + \beta\sin\phi - y^2 = 0 \quad . \quad (2)$$

By addition, $\alpha\cos\phi + \beta\sin\phi + \gamma = 0$ . . . . (3)

where $\gamma = \mathrm{A} + \mathrm{C} - x^2 - y^2.$

We have also, $\cos^2\phi + \sin^2\phi - 1 = 0$ . . . . . (4)

The partial eliminants are formed as before by excluding successively $\sin\phi$, $\cos^2\phi$, and 1; $\sin^2\phi$, $\cos\phi$, and 1; $\sin\phi$, $\cos\phi$, and 1^2; as thus,

$$\begin{array}{c|ccc|l} & [\sin\phi] & [\cos^2\phi] & [1] & \\ (1) & 2B\cos\phi + C\sin\phi & A & \alpha\cos\phi - x^2 & \\ (2) & -2B\cos\phi + A\sin\phi + \beta & C & -y^2 & =0 \quad (a) \\ (4) & \sin\phi & 1 & -1 & \end{array}$$

$$\begin{array}{c|ccc|l} & [\sin^2\phi] & [\cos\phi] & [1] & \\ (1) & C & 2B\sin\phi + A\cos\phi + \alpha & -x^2 & \\ (2) & A & -2B\sin\phi + C\cos\phi & \beta\sin\phi - y^2 & =0 \quad (b) \\ (4) & 1 & \cos\phi & -1 & \end{array}$$

$$\begin{array}{c|ccc|l} & [\sin\phi] & [\cos\phi] & [1^2] & \\ (1) & B\cos\phi + C\sin\phi & B\sin\phi + A\cos\phi + \alpha & -x^2 & \\ (2) & -B\cos\phi + A\sin\phi + \beta & -B\sin\phi + C\cos\phi & -y^2 & =0 \quad (c) \\ (4) & \sin\phi & \cos\phi & -1 & \end{array}$$

The partial eliminants $(=0)$ are,

$$(a) \quad (\alpha\beta - 2B\gamma)\cos\phi + (A-C)\gamma\sin\phi + \beta(A-x^2) + (A-C)\alpha\cos\phi\sin\phi - 2B\alpha\cos^2\phi.$$

$$(b) \quad (A-C)\gamma\cos\phi + (\alpha\beta + 2B\gamma)\sin\phi + \alpha(A-y^2) + (A-C)\beta\cos\phi\sin\phi + 2B\beta\sin^2\phi.$$

$$\begin{aligned}(c) \quad \{\beta(A-x^2) - B\alpha\}\cos\phi + \{\alpha(A-y^2) + B\beta\}\sin\phi + \alpha\beta + (A-C)\gamma\cos\phi\sin\phi \\ + B\gamma\sin^2\phi - B\gamma\cos^2\phi.\end{aligned}$$

To make these expressions symmetrical a slight transformation is requisite in the last terms of (*a*) and (*b*).

In (*a*) substitute $B\alpha\sin^2\phi - B\alpha\cos^2\phi - B\alpha$ for $(-2B\alpha\cos^2\phi)$.

In (*b*) substitute $B\beta\sin^2\phi - B\beta\cos^2\phi + B\beta$ for $(+2B\beta\sin^2\phi)$.

The three equations $(=0)$ are then,

$$(a) \quad (\alpha\beta - 2B\gamma)\cos\phi + (A-C)\gamma\sin\phi + \{\beta(A-x^2) - B\alpha\} + \alpha\left\{\begin{array}{l}\{(A-C)\cos\phi\sin\phi \\ +B(\sin^2\phi - \cos^2\phi)\}\end{array}\right\}$$

$$(b) \quad (A-C)\gamma\cos\phi + (\alpha\beta + 2B\gamma)\sin\phi + \{\alpha(A-y^2) + B\beta\} + \beta\left\{\begin{array}{l}\{(A-C)\cos\phi\sin\phi \\ +B(\sin^2\phi - \cos^2\phi)\}\end{array}\right\}$$

$$(c) \quad \{\beta(A-x^2) - B\alpha\}\cos\phi + \{\alpha(A-y^2) + B\beta\}\sin\phi + \alpha\beta + \gamma\left\{\begin{array}{l}\{(A-C)\cos\phi\sin\phi \\ +B(\sin^2\phi - \cos^2\phi)\}\end{array}\right\}$$

The same expressions can be got from the equations of addition and subtraction of (1) and (2) along with (4).

In forming the final determinant we eliminate the factor $\{(A-C)\cos\phi\sin\phi + B(\sin^2\phi-\cos^2\phi)\}$ in one column, thus,

$$\begin{array}{c|cccc|l}
 & [\cos\phi] & [\sin\phi] & [1] & \left\{\begin{matrix}(A-C)\cos\phi\sin\phi\\ +B(\sin^2\phi-\cos^2\phi)\end{matrix}\right\} & \\
(a) & \alpha\beta-2B\gamma & (A-C)\gamma & \beta(A-x^2)-B\alpha & \alpha & \\
(b) & (A-C)\gamma & \alpha\beta+2B\gamma & \alpha(A-y^2)+B\beta & \beta & \\
(c) & \beta(A-x^2)-B\alpha & \alpha(A-y^2)+B\beta & \alpha\beta & \gamma & =0 \quad (\Delta_1) \\
(3) & \alpha & \beta & \gamma & &
\end{array}$$

This, it will be observed, is a bordered symmetrical determinant of the 4th order.

By cancelling the terms multiplied by B (the coefficient of $\cos\phi\sin\phi$ in the original equations) we get the determinant already found for an axial tracing-point, as we ought to do.

The developed solution is

$$\begin{array}{ll}
\alpha^2(A\alpha-\alpha y^2+B\beta)^2 & -\alpha^3\beta(\alpha\beta+2B\gamma) \\
+\beta^2(A\beta-\beta x^2-B\alpha)^2 & -\alpha\beta^3(\alpha\beta-2B\gamma) \\
+\gamma(A-C)^2 & -\alpha^2\beta^2\gamma^2+4B^2\gamma^4 \\
-2\alpha\beta(A\beta-\beta x^2-B\alpha)(A\alpha-\alpha y^2+B\beta) & +2\alpha^2\beta^2(A-C)\gamma \\
-2\alpha\gamma^2(A-C)(A\alpha-\alpha y^2+B\beta) & +2\alpha\gamma(\alpha\beta+2B\gamma)(A\beta-\beta x^2-B\alpha) \\
+2\beta\gamma^2(A-C)(A\beta-\beta x^2-B\alpha) & +2\beta\gamma(\alpha\beta-2B\gamma)(A\alpha-\alpha y^2+B\beta)
\end{array}$$

We can form a complete square out of the terms in the first column if we substitute for the 4th term,

$$(2\alpha\beta-\underline{4\alpha\beta})(A\beta-\beta x^2-B\alpha)(A\alpha-\alpha y^2+B\beta).$$

Thence,

$$\begin{aligned}
&\imath(A\alpha-\alpha y^2+B\beta)+\beta(A\beta-\beta x^2-B\alpha)-(A-C)\gamma^2\}^2-\underline{4\alpha\beta}(A\beta-\beta x^2-B\alpha)(A\alpha-\alpha y^2+B\beta)\\
&\quad+(\alpha\beta+2B\gamma)\{2\alpha\gamma(A\beta-\beta x^2-B\alpha)-\alpha^3\beta\}+(\alpha\beta-2B\gamma)\{2\beta\gamma(A\alpha-\alpha y^2+B\beta)-\alpha\beta^3\}\\
&\quad+4B^2\gamma^4-\alpha^2\beta^2\gamma^2+2(A-C)\alpha^2\beta^2\gamma=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\Delta_1)
\end{aligned}$$

The expression found for the glissette having its tracing-point on the axis was not combined precisely in this way, because the reduction to the 6th degree for the tracing-point at focus was more easily effected by keeping the eliminant in its original form.

It would be easy to expand Δ_1 by writing out the coefficients at full length, but such an expansion would serve no useful purpose. In fact, the bordered symmetrical determinant, after substituting $2rx$, $2ry$, for α and β, and $A + C - x^2 - y^2$ for γ, may be considered the definite algebraic expression for the glissette of an Ellipse or Hyperbola.

For certain loci of the tracing-point, the eliminant is simplified by losing a considerable number of its terms. Comparing the original and the abbreviated forms of (1) and (2) of the complete equation (p. 383), we observe that

$$A = a^2\cos^2\alpha + b^2\sin^2\alpha - r^2. \qquad C = b^2\cos^2\alpha + a^2\sin^2\alpha.$$

And (1), for $A = 0$, the locus of the tracing-point is determined by

$$a^2\cos^2\alpha + b^2\sin^2\alpha = r^2.$$

The locus of the tracing-point in this case is the pedal of the generating ellipse from centre as origin of the perpendiculars on tangents, and this includes the case of the tracing-point at the vertex of the ellipse.

(2) For $A - C = 0$, the locus of the tracing-point is determined by

$$(a^2 - b^2)(\cos^2\alpha - \sin^2\alpha) = r^2.$$

When $\alpha = 0$, the tracing-points are the foci. When $\alpha > \frac{\pi}{4}$, r is impossible; and when $\alpha < \frac{\pi}{4}$ the locus of the tracing-point is the pedal of a rectangular hyperbola having the foci of the generating ellipse as vertices.

(3) For $A + C = 0$, we have, $(a^2 + b^2)(\cos^2\alpha + \sin^2\alpha) = r^2$; and the locus of the tracing-point is a circle concentric with the generating ellipse whose radius is $\sqrt{(a^2 + b^2)}$. But as $\sqrt{(a^2 + b^2)}$ also measures the distance of the centre of the ellipse from the origin, it follows that every glissette having its tracing-point on the circle represented by $A + C = 0$, passes through the origin, as is otherwise evident from the fact that when γ is reduced to $-(x^2 + y^2)$ the absolute term of the equation disappears.

The three cases of tracing-point at the vertices, at the foci, and at the extremities of the circumscribing rectangle are noticed in the papers of Dr Muir and Mr Nanson on the Glissette Elimination Problem; but it is only by means of the symmetrical expressions here found that it is known that these are only particular cases of three systems of glissettes, each having a definite locus of the tracing-point, and a definite equation which is less complex than the general equation. The case of the tracing-point on one of the axes represents a fourth system. Except in the unique case of the focus, where $A-C=0$ and $B=0$, the reduced eliminant is not divisible by a factor, but remains an equation of the 8th degree.

III. *Formation of a Symmetrical Determinant from the Original Untransformed Equations.*

As already stated, a bordered symmetrical determinant may also be obtained from the original simultaneous equations without changing θ into $\phi+a$. (A, p. 383.) The partial eliminants are formed in the same way as before, and I shall, therefore, only write down the resultant.

The original equations being expanded in powers of θ, we put A, B, C, a_1, β_1, for the coefficients of $\cos^2\theta$, $\cos\theta\sin\theta$, $\sin^2\theta$, $\cos\theta$, and $\sin\theta$ in the first equation; and C, $-B$, A, a_2, β_2, for the coefficients of like powers in the second equation: (a_1,β_2) stands for $(a_1\beta_2-a_2\beta_1)$ according to the usual determinant notation, and the resultant is,

$$
=\left|\begin{array}{cccc}
(\beta_1,a_2)+2B\gamma\ , & (C-A)\gamma\ , & \left.\begin{array}{r}\beta_1(C-y^2)-\beta_2(A-x^2)\\ +B(a_1+a_2),\end{array}\right\} & a_1+a_2\\
(C-A)\gamma\ , & (\beta_1,a_2)-2B\gamma\ , & \left.\begin{array}{r}a_2(C-x^2)-a_1(A-y^2)\\ -B(\beta_1+\beta_2),\end{array}\right\} & \beta_1+\beta_2\\
\left.\begin{array}{r}\beta_1(C-y^2)-\beta_2(A-x^2)\\ +B(a_1+a_2),\end{array}\right\} & \left.\begin{array}{r}a_2(C-x^2)-a_1(A-y^2)\\ -B(\beta_1+\beta_2),\end{array}\right\} & (\beta_1,a_2)\ , & \gamma\\
a_1+a_2\ , & \beta_1+\beta_2\ , & \gamma\ , & .
\end{array}\right|
$$

This expression only differs from Δ_1 in the values of the coefficients, and in the circumstance that the first powers of $\sin\theta$ and $\cos\theta$ occur in each of the two original equations, and come out in duplicate in the resultant.

As this resultant can be formed from one of the original quadratic equations and the linear equation of addition, and as these two equations contain nine distinct coefficients, it follows that in the elimination of sines and cosines between any two equations of the 1st and 2nd degrees respectively the resultant is a bordered symmetrical determinant of the 4th order.

The Action of Persulphates on Iodine.

By Hugh Marshall, D.Sc.

(Read December 19, 1898.)

When solutions of potassium persulphate and potassium iodide are mixed, iodine is liberated and potassium sulphate formed. The reaction takes place somewhat slowly in the cold, much more rapidly on warming. It has generally been supposed that the equation

$$2KI + K_2S_2O_8 = I_2 + 2K_2SO_4$$

completely represents the action which takes place, and although I have had occasion to try the reaction very many times since I first noted it as a reaction for persulphates, it was only about a year ago that I observed a further change.

A solution of potassium iodide was being boiled with ammonium persulphate (which is much more soluble than the potassium salt) in order to decompose it and drive off the iodine by sublimation. It was observed that the free iodine in the liquid seemed to disappear very rapidly, more so than corresponded to the amount volatilised, and that the final disappearance of the brown colour took place with unexpected suddenness. This pointed to the possible conversion of the iodine into some compound, probably by the oxidising action of the persulphate. This proved to be the case, because, on the cautious addition of sulphurous acid to the colourless solution, iodine again appeared; on standing, the liquid again became decolourised, and was again turned brown by sulphurous acid. These operations could be repeated so long as persulphate remained in the solution. The action takes place slowly in the cold, and at high temperatures there is considerable volatilisation of iodine; when a solution of potassium iodide was corked up with ammonium persulphate in a test tube at the ordinary temperature, the iodine which separated in the solid form gradually dissolved on further standing, and had entirely disappeared in the course of several days. The oxidation product was not further investigated at the time; it was assumed to be iodic acid.

Some months ago, my attention was again called to the subject by seeing an abstract of a paper on the determination of chlorine, bromine, and iodine in organic compounds by distilling the substance with a mixture of sulphuric acid and persulphate. The possible oxidation of iodine under these conditions was apparently overlooked, and the method may therefore not be so accurate for that substance as in the case of chlorine or bromine; it was therefore important to have the nature of the action definitely settled by isolating some of the product. For this purpose, four or five grams of iodine were digested in an Erlenmeyer flask with a strong solution of ammonium persulphate, the quantity taken being more than sufficient to convert the iodine into periodate. The loosely-corked flask was placed on an ordinary heating coil, so that its temperature varied from 10°–15° during the night to 30°–35° during the day. In the course of fully a week the whole of the iodine had dissolved and oxidised, and in its place there was a moderate quantity of a white crystalline solid. This was filtered off, washed with a little cold water, and recrystallised from solution in hot water. Its solution was strongly acid, free from sulphate, and gave all the ordinary reactions of an iodate and of an ammonium salt; it was evidently ammonium hydrogen iodate. The original mother liquor and washings, containing chiefly ammonium hydrogen sulphate, were neutralised with ammonia solution, whereupon a considerable quantity of white crystalline precipitate formed, probably normal ammonium iodate. In order to prove conclusively that it was iodate and not periodate, its oxidising power was determined by means of potassium iodide and sulphuric acid, the liberated iodine being titrated with an empirical solution of sodium thiosulphate. The titre of the latter was found by comparison with pure ammonium iodate prepared by other methods. It was thus found that 0·1540 grams of substance corresponded to a volume of thiosulphate solution equivalent to 0·1534 grams of ammonium iodate. There is therefore no doubt that persulphate oxidises iodine to iodic acid.

Since the above investigation was carried out there has appeared a paper on the action of potassium persulphate on potassium iodide, by T. S. Price.* In it the author gives the result of his investigations on the rate of the reaction at different concentrations, and on the effects which other substances have when present in the solution. He finds that the reaction is one of the second degree, not of the third, as was anticipated. It therefore agrees with the equation

$$KI + KSO_4 = I + K_2SO_4$$

rather than

$$2KI + K_2S_2O_8 = I_2 + 2K_2SO_4$$

although all molecular weight determinations lead to the doubled formula for persulphuric acid. The solutions dealt with in Price's investigation were dilute, and it is impossible to say what effect the liberated iodine would have on the rate of decomposition; there may be a certain amount of "catalytic action," the iodine being alternately oxidised by the persulphate and reduced by hydriodic acid. The reaction investigated may not be the comparatively simple one represented by the equation

$$S_2O_8'' + 2I' = I_2 + 2SO_4''$$

but may really be the much more complex one, as represented by the *additional* equations

$$I_2 + 5S_2O_8'' + 6H_2O = 2IO_3' + 10SO_4'' + 12H^{\cdot}$$
$$2IO_3' + 10I' + 12H^{\cdot} = 6I_2 + 6H_2O$$

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* *Zeits. für phys. Chem.*, xxvi. 474 (1898).

On some Oceanographic Problems. By Vice-Admiral S. Makaroff, Imperial Russian Navy. (Plates I.–XII.)

(Read February 9, 1899.)

I am very glad to embrace the opportunity of addressing the Fellows of the Royal Society of Edinburgh, many of whose members have contributed much to our knowledge of Oceanography. For instance, your President, Lord Kelvin, besides his researches on the tides, is well known to practical seamen from his excellent compasses and sounding machines; your Secretary, Prof. Tait, is well known from his researches on the pressure errors of deep-sea thermometers; Dr Alexander Buchan has a world-wide reputation in the department of oceanic meteorology; the late Prof. Dittmar was a great authority on the chemistry of sea water. It is enough to say that Sir Wyville Thomson, Mr J. Y. Buchanan, and Sir John Murray, were members of the "Challenger" Expedition, which has given the world such valuable information about the depths of the sea.

Of course, it is not with the intention of giving to such scientific authorities a lesson that I address the Society, but if you represent scientists, I represent the seamen, and it is useful from time to time to have a talk between these two classes of men. Every scientific study should be started by the scientist, but the sooner they can associate ordinary practical men with the work the better it will be. We practical seamen are more numerous than scientists; we constantly navigate the sea, and we have more opportunities of making contributions to science than they have. Certainly, they can make their observations in a more exact way than we can, but the laws of nature—particularly those concerning Oceanography—are so imperfectly known, that there is very much to be done even by the rough hands of the ordinary seamen.

There are three principal elements to be studied at sea with regard to Oceanography:—the temperature of the water, the specific gravity of the same, and the currents. I need not say how easy it is to make observations on the temperature of the surface water; there is only one precaution, that is, not to forget that the correction of the thermometer should be known, and of course water should be brought to the thermometer in such a way as not to be subject to a change of temperature in the meanwhile.

The determination of the specific gravity of water was for a long time considered as a delicate piece of work, which could be done only by the hands of the scientist. It was very brave of me to dare to take in hand the hydrometer, but I became in a short time master of this ordinary sort of work. After taking charge of this operation in 1886, I, being a captain then, handed it to all my officers on board, and was really glad to see that every one of them was as particular about the exactness of the observations as a man who is accustomed to do this kind of work. It may be interesting to mention that out of ten instruments given to the officers of the watch only two were broken after six thousand observations had been made during the three years' voyage uninterruptedly in good weather and bad. The association of all the officers in this work gave me very great assistance. When the work is done by a scientist, observations of specific gravity may be taken once or several times a day, but, as every officer of the watch could do it on board my ship, I sometimes had observations made every five minutes. It is not necessary in ordinary circumstances to determine the specific gravity of the water every five minutes, but if you cross a certain current and wish to know the limits of it, you have to increase the number of observations.

I wish to draw your attention to one fact, which should not be neglected. The hydrometer changes its position, and consequently its reading, when you place the thermometer in the same vessel. I found out this fact and published it in my book *Le Vitiaz et l'Océan pacifique* (paragraph 58). Dr Krümmel, of Kiel, afterwards studied this subject particularly. I mention this fact because it is better not to have the thermometer in a vessel in which the hydrometer is floating. In any case, observations of the specific gravity should be done in the same way in which the errors of the hydrometer were determined.

I will now show you the results of my work done at different times, from 1881 to 1889. My books and papers are published chiefly in the Russian language, and for this reason they are not very well known in this country. I cannot, in the course of my address, make you familiar with all my works, and wish only to draw your attention to the interesting phenomena of double currents in the Straits of Bosphorus, Gibraltar, Bab-el-Mandeb, Formosa, and La Pérouse.

The Strait of Bosphorus joins the Black Sea and the Marmora Sea. The Black Sea water has in it—roughly speaking—half the quantity of salt found in the water of the Mediterranean. The water of the lower strata of the Marmora Sea has the same composition as the water of the Mediterranean. The upper strata, say from ten fathoms upward, contain water of intermediate salinity between the water of the Mediterranean and the water of the Black Sea (see Plate I., fig. 1). This difference in the salinity of the water is the chief reason of the enormous double current of the Bosphorus. Let us imagine that at a certain given moment the level of both seas is at the same height. The pressure of the column of water in the Marmora Sea will be greater than that in the Black Sea; the difference would increase with the depth and it would disappear at the surface. For this reason the water in the lower strata of the Marmora Sea rushes into the Black Sea, keeping close to the bottom. That rush of water after a certain time will raise the level of the Black Sea, producing a difference in the level of the two seas, which causes a superficial current to flow out of the Black Sea in the opposite direction to the under current. Here we see distinctly that the principal reason for the double current is the difference in the salinity of the water, and should that difference in salinity cease the double current would be discontinued. The fact is that in the Black Sea evaporation does not exceed the quantity of water supplied by rains and streams, and this excess of fresh water maintains the difference of salinity in the waters of the Black Sea and Mediterranean.

The existence of double currents in the Bosphorus was known long ago, and Marsilli in 1681, in his letter to Queen Christina of Sweden, has described them. Later, they were somehow forgotten, and some interesting papers have been published, in which the authors try to prove that the double current was legendary. Rear-Admiral Sir W. J. L. Wharton, of your Navy (who is now at the head of the Hydrographic Office), was the first to show by direct observations that a double current existed in the Bosphorus. I was there a few years after him, commanding the stationary steamer

"Taman." I began to take observations of the specific gravity of the water at different depths, and I found out that the water forming the lower strata contained twice as much salt as the water of the upper strata; after this a double current was quite evident to me.

In order to measure the velocity of both currents, I invented the instrument which is shown on Plate II. It consists of a propeller revolving on a horizontal spindle. A bell is attached to the propeller, the tongue of which is arranged to move on an axle in one direction; at every revolution of the propeller it strikes twice, and, as water is a very good conductor of sound, the number of revolutions could be counted through the bottom of the ship (provided the ship is not sheathed with wood) at all depths to which the instrument was lowered (40 fathoms). To this instrument I gave the name of fluctometer, and I used it during the whole year, lowering it on a wire rope with ballast of 80 lbs. of lead attached below.

Plate I. fig. 2 shows the velocity of both currents in feet per second, and the specific gravity of the water in each stratum. Observations were made every two hours, and the specific gravity of water from every depth was taken at the same time when the velocity of the current was observed.

I do not wish to keep you long with the different interesting details, and will only point out to you that the lower current is similar in many details to an ordinary river, while, on the contrary, the upper current differs much from an ordinary river, probably from the reason that, while the surface of it is falling gradually down, the bottom rises constantly (see Plate I. fig. 1).

The difference of level of the Black Sea and Marmora Sea calculated from the difference in the specific gravity of the water I found for the month of July 1882 to be 1·396 feet.

In the Strait of Gibraltar I had only five stations, and made my observations one day only. I had no opportunity of measuring the velocity of the current, but the phenomenon is very similar to what I found in the Bosphorus. Plate III. shows a section of the Gibraltar Strait and the Mediterranean as far as the Syrian coast. Of course, the Mediterranean is given in a different longitudinal scale from that of Gibraltar Strait. As shown by the diagram, the water of the Atlantic rushes into the Mediterranean, the difference between the surface levels being, according to my calculations, 0·54 feet.

The evaporation of water from the Mediterranean is greater than the quantity supplied by rivers and rains. For this reason, the water becomes more dense, settles down, and goes back to the Atlantic by the under current.

I wish to point out here that the temperature of the lower strata of the Mediterranean coincides with the mean winter temperature of the air in the eastern part of the sea. This is quite evident, because in winter the temperature of the water to a great depth corresponds to the temperature of the air. In summer, the surface water is much warmer, but this high temperature cannot penetrate to a great depth. I am sorry that I have not time to discuss more fully this question, but in the Straits of Bab-el-Mandeb we have the same phenomena as in the Gibraltar Strait and Mediterranean. Here again—by my observations—the temperature of the lower water strata coincides with the winter temperature of the air at the place where the water settles down.

In the three straits already mentioned, we have a double current: superficial and bottom current. In the Straits of Formosa and La Pérouse there are also two currents, but both are superficial.

I ought to mention that the influence of the rotation of the earth on the direction and velocity of the currents cannot be overestimated. I shall not discuss this question fully, but the fact that in every salt inland sea there is a circular rotation of the water in a direction opposite to the apparent movement of the sun, shows that the rotation of the earth has very much to do with the direction of the currents (see paragraph 222 of *Le Vitiaz et l'Océan pacifique*). In the vicinity of islands, for the same reason, the water follows a direction coinciding with the apparent movement of the sun. It is for this reason also that the water alongside the Chinese coast flows to the south during the north-easterly monsoon as well as during the south-westerly monsoon. The Kuro-Siwo current going to the north and north-east cannot touch the Chinese coast because there is brackish water flowing to the south-west.

Plates IV., V., VI., and VII. show that in the Strait of Formosa the specific gravity and temperature of the water at the Chinese coast are quite different to what is observed off the coast of Formosa. This difference in the temperature and specific gravity may give to a sailor a good guide for a fair passage through the Strait. As you see by Plate V. the temperature of the water, say, in the month of February at the Chinese coast is 11° C., while at the coast of Formosa it is 20°. If the captain will try during the month of February to follow the line of the temperature of 15° he will pass at a good distance from the dangers of both coasts. Plate VII. shows the distribution of the specific gravity. At the Chinese coast in winter it is possible to find water at less than 1·0240 $\left(S\frac{17\cdot5}{17\cdot5}\right)$, while at the coast of Formosa it is seldom less than 1·0265.

Every sailor knows how difficult the passage through the Strait of Formosa is. During the north-easterly monsoon the weather is very thick, and the depth of the sea cannot in these places be regarded as giving a good means for determining the position of the ship. It may happen that after a ship leaves say Nagasaki the captain never knows his position until he runs on the Chinese coast and wrecks his ship. My opinion is, that a regular temperature service should be arranged from Turnabout Lighthouse; every day a pilot boat should put to sea, taking temperatures both going out and returning, and the temperature of the water should be wired to all Chinese and Japanese ports for the information of the captains. By these means many ships would be saved from danger.

The currents in the Straits of La Pérouse are very complicated. Plate VIII. shows the distribution of the temperature of the surface water. There is a very narrow and long strip of cold water, which lies in the direction from N.W. to S.E.; a vessel crossing that strip in July may have temperatures of 18° C., then 5°, and again 16° or 18°. It would take me too long to explain the source from whence the cold water comes, and why it is constantly there; it is the cause of fogs which render navigation in that place very difficult. I may briefly say that the Kuro-Siwo current partly enters the Sea of Japan, and the excess of water escapes partly through the Strait of La Pérouse into the Okotsk Sea. Due to the rotation of the earth the current turns to the south-east and flows alongside the Island of Yezo. This water is warm and dense, having much salt in it. The water of the Okotsk Sea—particularly in the vicinity of the Island of Saghalien—is in summer also pretty warm, but it is much lighter than the water of the Kuro-Siwo, and thus while the denser water sinks down, the lighter water tries to rise on the top of it. The difference of level which is produced hereby brings to the surface the cold water of the lower strata (for particulars see paragraph 261 of my book *Le Vitiaz et l'Océan pacifique*).

I studied this Strait in 1887 and 1888, and published the results of my study, but when I came to the Pacific again in 1895, as the admiral commanding the squadron, I was very anxious to go to the Strait of La Pérouse to re-investigate the currents, and now I am in possession of very valuable material on this subject, which is almost ready for publication.

I do not propose to take up more of your time at present with particulars of these five straits. Those who desire further information may refer to my book *Le Vitiaz et l'Océan pacifique*, published in the Russian and French languages, and also my book on the *Exchange of Waters between the Black Sea and Mediterranean.* I only wish to remind you what important information the thermometer and hydrometer can give in the study of the different parts of our so little-known planet. You know better than I, that studies in that direction ought to be continued, and no nation in the world has been so liberal as England, which found means to send out for four years the "Challenger," with a scientific staff to explore the deep sea. But it is not always possible to find such means, and it is advantageous to associate ordinary seamen with that kind of work. Scientific societies might contribute very much in discussing the data collected by the ordinary seamen, and I wish now to refer to the manner of collecting data regarding the temperature of the superficial water.

Up to the present time it has been the custom to publish only charts of the temperature; I find that this system is not quite efficient; charts of temperature are very useful, but when the number of observations increases and one wishes to publish a new chart one does not know what value to attach to the previous chart, and has to do the same work over again.

In my book *Le Vitiaz et l'Océan pacifique* I have made a particular study in this respect of the Northern Pacific Ocean, and I collected from the log books of Russian vessels all the information that I could find. I divided the sea into 1° squares and gave to each ship one little column; the following table is an example of the method adopted :—

Tables showing the Temperature (Centigrade) of the Surface Water in the North Pacific arranged in One Degree Squares. Latitude 44° to 45° N.

		135—136 E.									136—137 E.				
1	Year	78	85	88	88	78	87	*m.*	75	78	59	78	75	*m.*	78
2	Month	VI	VII	VIII	VIII	VIII	VIII	VIII	X	XI	V	V	V	V	VI
3	Date	5	7	7	17	20	27	17.7	23	12	4	17	28	16.3	5
4	Name of Ship	Vostok	Tungus	Rynda	Rynda	Vostok	Rynda	—	Japonez	Tungus	Voev. int.	Tung.	Japonez		Vostok
5	No. of Observations	—	—	—	—	—	—	—	—	—		3	3	—	4
6	Minimum	5.6	11.6	—	—	—	—	—	—	—	—	5.8	5.0	—	5.0
7	Maximum	6.5	11.9	—	—	—	—	—	—	—	—	6.4	5.5	—	5.9
8	Mean	6.0	11.8	15.6	15.0	20.4	17.5	17.1	12.5	3.5	3.6	6.1	5.2	5.0	5.4
9	,, for the Month	7.9	12.9	—	—	—	—	17.1	13.2	3.1	—	—	—	4.9	—

Continuation.

1	87	65	87	*m.*	85	87	66	85	*m.*	88	88	61	88	78	87	*m.*
2	VI	VI	VI	VI	VII	VII	VII	VII	VII	VIII	VIII	VIII	VIII	VIII	VIII	VIII
3	6	17	30	17.3	7	17	19	24	16.7	7–8	15–7	18	18	20	28	21.0
4	Vladiv.	Variag	Nayez.	—	Tungus	Vladiv.	Ask.	Tung.	—	Rynda	Rynda	Gaid.	Vitiaz	Vostok	Rynda	—
5	—	10	—	—	5	—	—	4	—	6	—	4	7	4	—	—
6	4.4	7.7	—	—	12.8	—	—	14.4	—	12.9	14.7	18.3	15.2	18.7	—	—
7	4.4	8.7	—	—	13.9	—	—	16.0	—	15.0	15.6	20.0	17.0	20.6	—	—
8	4.4*	8.2	10.9	8.2	13.2	15.0	14.4	15.0	14.4	14.0[1])	15.1[1])	18.9	16.0	19.6	16.3	17.5
9	—	—	—	8.0	—	—	—	—	14.3	—	—	—	—	—	—	17.5

Continuation.

1	87	89	80	89	*m.*	90	88	75	88	87	75	*m.*	86	58	78	87	*m.*	87
2	IX	IX	IX	IX	IX	X	X	X	X	X	X	X	XI	XI	XI	XI	XI	XII
3	6	13	21	25	16.2	2	7	9	21	22	23	14	2	5	12	19	9.5	17
4	Rynda	Razb.	Djig.	Vlad.	—	Krs.	Vitiaz	Vostok	Alent	Vlad.	Japon,	—	Vlad.	Voev.	Tung.	Vitiaz	—	Alent int.
5	—	—	—	3	—	—	7	—	—	—	—	—	—	—	—	—	—	
6	—	—	17.9	15.6	—	—	11.4	12.3	—	10.6	—	—	8.7	—	2.3	5.6	—	—
7	—	—	18.4	16.9	—	—	11.9	12.8	—	11.2	—	—	8.7	—	4.2	6.2	—	—
8	17.8	18.7	18.1	16.1	17.7	16.9	11.7	12.3	8.1	10.9	12.2	13.7	8.7	6.6	3.2	5.9	6.1	2.4
9	—	—	—	—	17.7	—	—	—	—	—	—	13.1	—	—	—	—	5.2	2.5

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[1] Close to the Coast.

		137—138 E.													
1	Year	87	59	77	75	89	75	*m.*	87	78	87	65	87	67	87
2	Month	V	V	V	V	V	V	V	VI	VI	VI	VI	VI	VI	VI
3	Date	3	5	18	23	24	29	17	4—6	6	6	18	18	25	30
4	Name of Ship	Vlad.	Voev.	Btr.	Jap.	Vlad.	Jap.	—	Nay.	Vostok	Vlad.	Var.	Nay.	Mor.	Nay.
5	No. of Observations	3	—	—	4	3	3	—	4	—	—	—	—	—	—
6	Minimum	1.2	—	—	4.5	5.0	5.0	—	6.9	6.4	4.4	—	—	—	10.6
7	Maximum	2.2	—	—	5.5	5.9	6.7	—	8.4	7.0	5·0	—	—	—	11.5
8	Mean	1.5	3.7	7.2	4.9	5.4	5.8	4.8	7.4	6.7	4.7	8.7	10.3	12.5	11.0
9	,, for the Month	—	—	—	—	—	—	4.6	—	—	—	—	—	—	—

Continuation.

1	*m.*	85	87	66	77	60	85	87	*m.*	77	67	61	78	88	87	*m.*	87	89	80	80
2	VI	VII	VII	VII	VII	VII	VII	VII	VII	VIII	VIII	VIII	VIII	VIII	VIII	VIII	IX	IX	IX	IX
3	15.4	7	16	19	19	20	24	30	19.3	7	8	18	19	26	28	17.7	6	13	18	21
4	—	Tung.	Vlad.	Ask.	Btr.	Grid.	Tung.	Vlad.	—	Btr.	Gorn.	Gaid.	Vos.	Nay.	Ryn.	—	Ryn.	Razb.	Dj.	Dj.
5	—	6	—	3	—	*m.*	8	3	—	—	—	4	—	—	—	—	—	—	—	—
6	—	13.9	14.4	13.7	—	—	13.9	15.6	—	—	—	19.4	—	18.7	—	—	17.5	18.0	19.0	17.8
7	—	14.1	15.0	14.0	—	—	15.0	16.2	—	—	—	20.0	—	19.1	—	—	18.0	19.0	19.2	17.9
8	8.8	14.0	14.7	14.9	16.6	14.7	14.3	15.8	14.9	20.5	15.0	19.5	16.2	18.9	12.5	17.1	17.7	18.5	19.1	17.8
9	8.8	—	—	—	—	—	—	—	14.3	—	—	—	—	—	—	16.9	—	—	—	—

The first line gives the year of observation; the second, the month; the third, the date; the fourth, the name of the ship; the fifth gives the number of observations of temperature in a given square; the sixth line gives the minimum temperature, the seventh gives the maximum, and the eighth the mean. The columns of each square I put in chronological order with regard to the month and the day, but neglecting the year. After every month I have an extra column for the monthly mean, and as the mean of the date may not correspond to the middle of the month, I give in the ninth line the mean temperature reduced to the middle of the month.

Now, suppose somebody wishes to add new data upon the temperature observed on some other ships, he will have all my information at his disposal, and certainly his conclusions will be better than mine, because he will get the advantage of more extensive information. Suppose some one wishes to study the variations of temperature of water by certain periods of time, one year, five years, seven years, or whatever it may be, he will also find the necessary information in my tables.

Sir John Murray, whom I am glad to see present, wished to study the amplitude of the variation in the temperature of the surface waters, which has doubtless a great influence upon the organic life in the ocean; he found in my tables very necessary information, which assisted him in arriving at some very useful conclusions, published not long ago in the *Geographical Journal.*

I will be very glad if my tables of temperature can be discussed here, or in a separate Commission, and that the Oceanographers would come to certain definite opinions with regard to the mode of collecting the information about the temperature of the surface water. It would be a great advantage to knowledge to divide the study of the sea with regard to the temperature. Suppose Russia should take Okotsk Sea, Behring Sea, or Sea of Japan, Black Sea, White Sea, Kara Sea, and the Finnish Gulf, England takes the Atlantic, United States takes Northern Pacific, Germany takes Indian Ocean, France takes South Pacific, Sweden and Norway take North Sea, Baltic Sea and the Arctic Sea. Every nation should extract the information in regard to the temperature from ship's log books, put it in tables of approved description, and send it to the corresponding nation; this will give means to collect enormous information. The observations of every ship in a certain square ought to be placed on a separate card. Boxes containing these cards, say for the North Pacific, will not occupy more space than can be found in a good-sized bookcase.

When a new journal of a ship is received, temperatures of sea water observed on board that ship should be placed on the cards, and the cards put in their corresponding place. In this way we shall, each year, become richer in the knowledge of the temperature of the surface water, and no observation will be lost. Every observation will increase our knowledge of the temperature of sea water. It will be a real pleasure to see that progress of knowledge, and if ever this system or any other system be accepted, it will help us to study many details, which, up to the present time, are unknown. From time to time, each nation would publish its information for the use of scientists and seamen of all nations.

I wish to draw your attention to one instrument of my design, which will be of great service in the study of the temperature of the sea. Direct observations of the temperature even when taken every hour do not show the exact limits of the different currents. Ships nowadays steam so quickly that in one hour they cover sometimes as much as 20 miles. The direct observations only show that the limit of the current was somewhere between the two places of observation, but they do not show whether the dividing line is sharp or not. In the case of the Strait of La Pérouse a ship might pass the cold region without noticing it at all. In one of the log books from which I extracted the temperature of the sea, I found recorded temperatures of about 15°, then 5°, and then again 15°, evidently at that time the reading of 5° was taken at the very place of the strip of cold water. Somebody examining the journal afterwards added 10 to the 5 making it 15, probably thinking that the men who made the observations were in error, while he (the corrector) really made the mistake himself.

The self-recording instruments are very important for studying the temperature of the sea, and on Plate IX. I show you the arrangement I proposed for that purpose. When the ship is under way, water enters into one part of the cock, washes round to the thermometer, and returns to the sea, thus keeping the thermometer constantly at the temperature of the sea; the moment a change of the temperature takes place, the pencil of the thermograph will move showing the exact time of the change of the temperature. The ice-breaker "Ermack" will be fitted with one of these instruments.

The part of the earth least known is the Arctic Sea, and information about the temperature and specific gravity of the water in that part of the world would be very interesting. It is long ago since I had a great desire to penetrate into this region with the hydrometer and thermometer in my hand, but means which I think are the best adapted for the purpose are very expensive. To collect the necessary funds is sometimes almost impossible. There ought to be a pretext that some influential persons should approve, and without good pretext it is impossible to find money. When Dr Nansen — whose personal acquaintance I made after his journey across Greenland—conceived the idea of penetrating into the Arctic with his "Fram," I wrote him a letter in which I stated that I was entirely of his opinion, that he would be carried by the currents somewhere in the direction he imagined, and advised him that help should be sent for him to Franz Josef Land; my letter to him and his answer were duly published in the Russian newspapers and the Geographical Proceedings.

I thought it quite possible that he would not fulfil his voyage in three years; I also thought that if in four years nothing was heard of him, people would be anxious to send help, and that would be a good pretext for collecting necessary money. In my opinion the best way to penetrate into the Arctic region is by means of a large ice-breaker. Certainly I did not wish to mention in my letter to Dr Nansen that I would go and help him, because being on a Government Service I could not dispose of myself, but I asked him in my letter, if he had any intention to leave some trace of his voyage; he then answered me that he intended to put on every island that he might discover, a pole with a small Norwegian flag on it, and under that pole a letter with information about the voyage of his ship.

Fortunately for Dr Nansen the current carried him on very well, and on my return from the Pacific Station, I was happy to learn that he and his "Fram" had safely returned home. Of course, that deprived me of my pretext for collecting the necessary money for building a large ice-breaker, but I found another motive—this time purely commercial.

You know that the Gulf of Finland of the Baltic Sea is covered with ice during the whole winter, and that the navigation and transport of cargo to such an important commercial port as St Petersburg is interrupted for five months in the year. In a severe winter ice may be found at a distance of 200 miles from St Petersburg. The ice of the Finnish Gulf is very strong because the water of the Baltic has very little salt in it. The thickness of plain ice does not exceed two feet in deep places, and three feet in shallow water, but the winds break the ice and pile it up, and large fields of packed ice 12 feet deep may often be found. An ice-breaker ought to be strong enough to go through ice of this description.

The first ice-breaker was built by a Russian merchant, Britneff, who took one of his steam tugs and cut the fore part into such a shape that it would run on the top of the ice and break it with its own weight. The trial took place in 1864 and proved very successful, and then the authorities of the city of Hamburg and some important ports of the Baltic constructed ice-breakers for themselves on the same principle, which prevails up to this day.

A great improvement in ice-breakers was made in America by introducing a fore propeller. I am sorry to say that I do not know the name of the inventor of the fore propeller, but it adds considerably to the efficiency of the ice-breaker. The fore propeller sucks the water from under the ice, and washes the skin of the ice-breaker, delivering it from the pieces of ice which otherwise would stop the progress of the ship. Anyhow, if it happens that the ice "torros" is very powerful and stops the ice-breaker, then the fore propeller is reversed, and the water pushes forward the pieces of ice that compose that "torros." Plate X. shows how this is done. One minute of such working of the propeller is usually enough to shake the "torros"; after this the engine is again reversed to speed ahead (stern engine always working ahead), and then the ship proceeds. The ice-breakers in America are used on ice, but this would not obstruct the steamer following. With regard to this, a large ice-breaker is preferable to a small one, as the large ice-breaker leaves a larger channel and destroys the ice more efficiently on its passage.

Holding the above-mentioned views, I opened my ice-breaking campaign by addressing the Geographical Society of St Petersburg in the month of March 1887. I proposed to build an ice-breaker of 10,000 horse-power with four engines of equal size, three of them being placed at the stern and the fourth at the bow of the ship. The lecture interested many people in the subject. Professor Mendeleeff, who is well known in England, was first to join me. The Minister of Finance of my country, Mr Witte, took a great interest in my proposition, and said to me that if my ideas were correct, and if really the ice-breakers could deal with Finnish Gulf and Kara Sea ice, then they would be of great service to the commerce of Russia. On his suggestion I made a voyage to the Kara Sea, and on my return the question was studied by the Commission, and it was on the representation of the Minister of Finance that His Majesty the Emperor of Russia authorised one experimental ice-breaker to be constructed.

Several shipbuilding firms were invited to tender, and that of Messrs Sir W. G. Armstrong, Whitworth & Company, Limited, which promised better conditions, was accepted. It is a striking fact that the first steam tug that Britneff turned into an ice-breaker was built at the same yard some forty years ago. Lately, the same firm was engaged in building several ice-breakers for the River Volga, the Port of Hango, and the Lake of Baikal, and I may state that they have done their best to fulfil the very extraordinary conditions that I have put upon them, which were necessary in building a polar ice-breaker.

I will not take up much of your time in the technical details of the ship; the model here exposed (Plate XII.) shows the exterior. You will notice that the submerged part has no vertical lines, even in the midship section there is an angle of 20° from the vertical line, and her bow and stern lines are inclined 70° and 65° respectively from the vertical line.

The ship has a double bottom and wing passages throughout the whole length. Transversely, the ship is divided by eight principal bulkheads, and the total number of independent compartments is forty-eight.

The sub-division of the ship is not a novelty in the science of naval architecture. The novelty is that each one of her compartments was filled with water to the upper deck; the compartments which do not rise so high were tested with a pressure of water up to the upper deck. After the engines, boilers, doors, pipes, etc., were fixed in their places, the largest compartment of the vessel was again filled with water to the upper deck, so as to ensure that the attaching of these things did not compromise the water-tightness of the bulkheads. The last trial was as successful as the previous ones, and if such a severe test were prescribed in the case of every iron ship, the number of casualties would greatly decrease. It is a shameful thing that the large liners carrying thousands of passengers are built in such a way that a little hole in them is sufficient to settle them to the bottom. The loss of the "Elbe," "Utopia," and "La Bourgogne," with a great many passengers, is a sufficient testimony that I am perfectly right on insisting that ships' bulkheads ought to be properly tried before the ships are allowed to go to sea.

However, this is not the main topic of my address. I may here briefly state that the ribs of the ice-breaker and her skin are calculated to bear the greatest pressure that ice can force upon it. The ice-breaker can charge at any floating ice (icebergs excluded) at any speed going ahead or astern.

There are trimming and heeling tanks of great capacity, and there is a pump in the centre of the ship which is able to force the water from one extreme end of the ship to the other with a velocity of 10 to 13 tons per minute. The hot water from the condenser may be utilized for warming the fore part of the ship in order to make the surface of the ice-breaker more slippery; snow at a low temperature washed with water at the freezing-point is very sticky, and the warm skin of the ship, melting the very surface which touches it, will produce a sort of lubrication.

In order to have means for navigating economically every main engine can be disconnected from its propeller, and each propeller may be turned by special auxiliary engines provided for this purpose. The ship is fitted with a mast and lifting crow's nest, which allows of always keeping a look-out from a height of 100 feet in order to find the best way through the ice.

The ice-breaker is now almost ready to go to the Baltic and to commence her work.

Doubtless you wish to question me as to whether with this ship I propose to proceed to the North Pole. To this question I may answer, that in my first proposition I imagined it would be necessary to have two such ships to go into the middle of the Arctic Ocean; we have now built one, and the trial of it will show how much the ice-breaker can do in the polar ice, which I shall try to negotiate if circumstances permit.

The Arctic Sea in summer is certainly full of ice, but this ice is in the shape of islands, divided by the canals which are mostly filled with broken ice. The islands are of different sizes, some being as much as five miles in diameter; the others are smaller, and the great majority of them do not exceed hundreds of feet. Sometimes these islands are pressed against each other, and there may be days during which it is difficult to proceed, but with a change of weather and current the ice islands may become separated from each other so as to render a passage possible. I do not think that going with the ice-breaker into the Polar Region it would be necessary to keep a straight course and cut the ice; I believe that the ship should go in a "zig-zag" line, shaping her course between ice-floes. In some cases it will be necessary to apply the full power, but in other places the ship will proceed easily.

I spoke on this subject with Captain Sverdrup of the "Fram" and Dr Nansen. Captain Sverdrup is entirely of my opinion, but Dr Nansen did not wish to express his views; he only said that he wished me success, and he would be the first to congratulate me upon it.

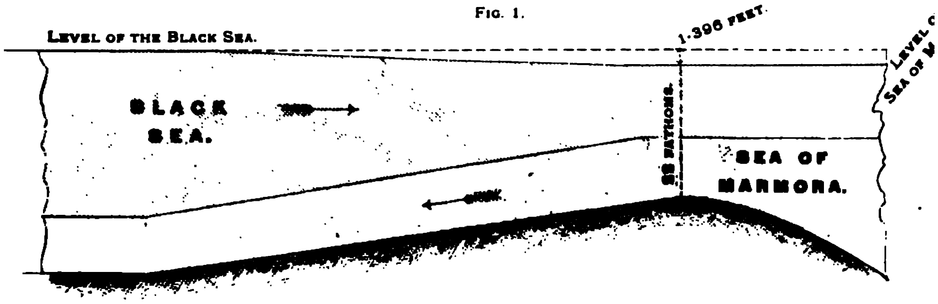
The near future will show whether my proposition and calculations are sound or not.

ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

PLATE I.

LONGITUDINAL SECTION OF THE BOSPHORUS.

FIG. 1.

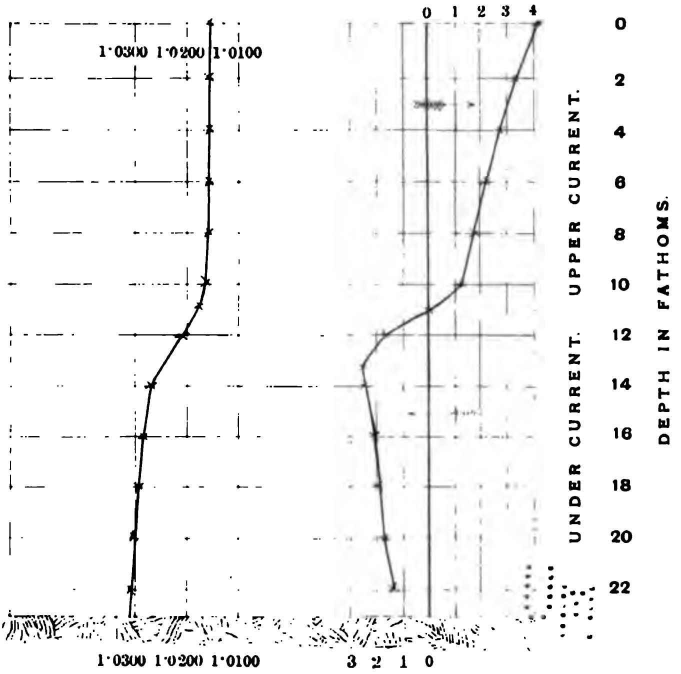


BOSPHORUS.

Mean specific gravity $(S\frac{17.5}{17.5})$ **of water for two hours observation during one day.**

Mean velocity of double current for two hours observation during one day.

FIG. 2.



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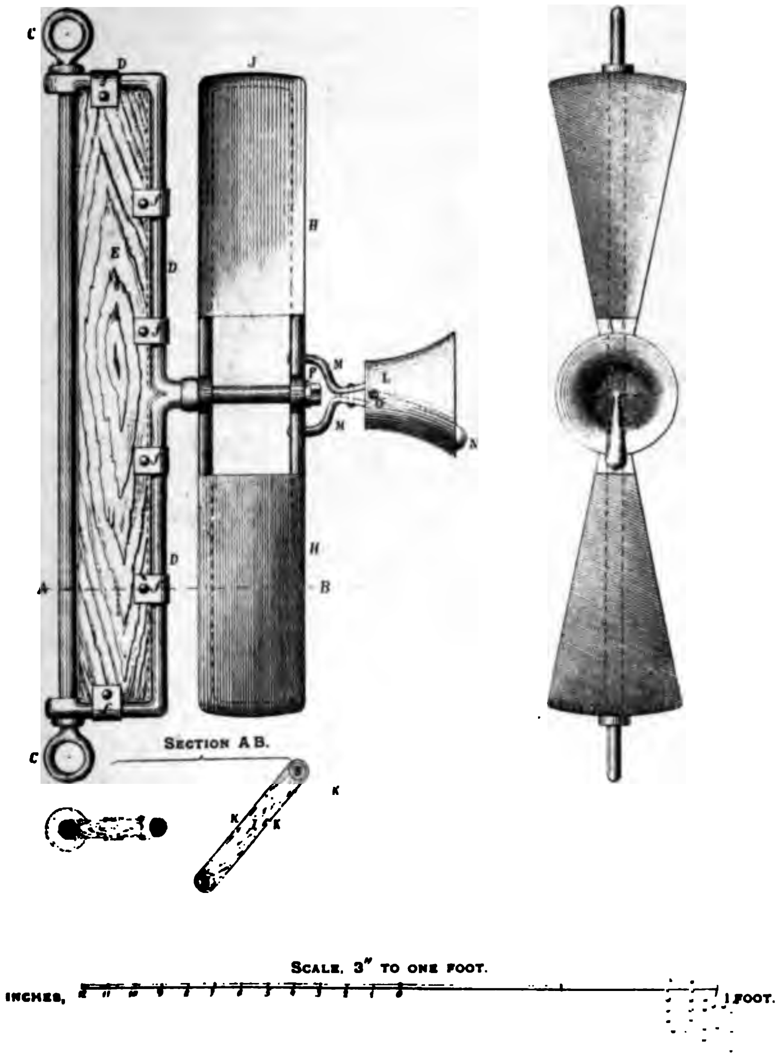
ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

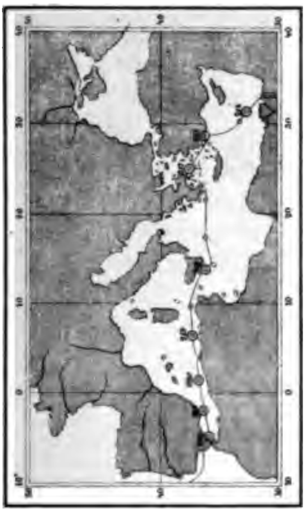
PLATE II.

FLUCTOMETER.

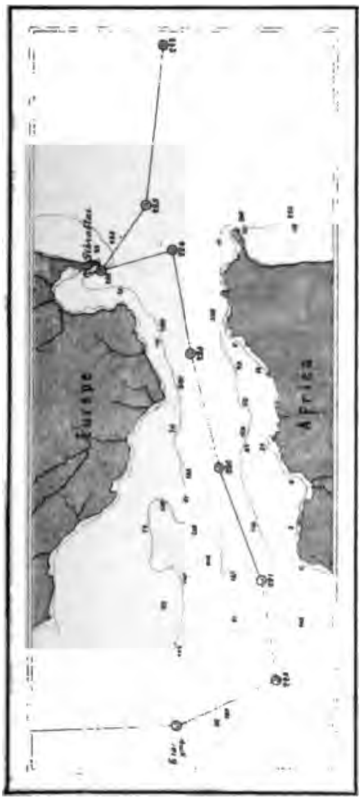


ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

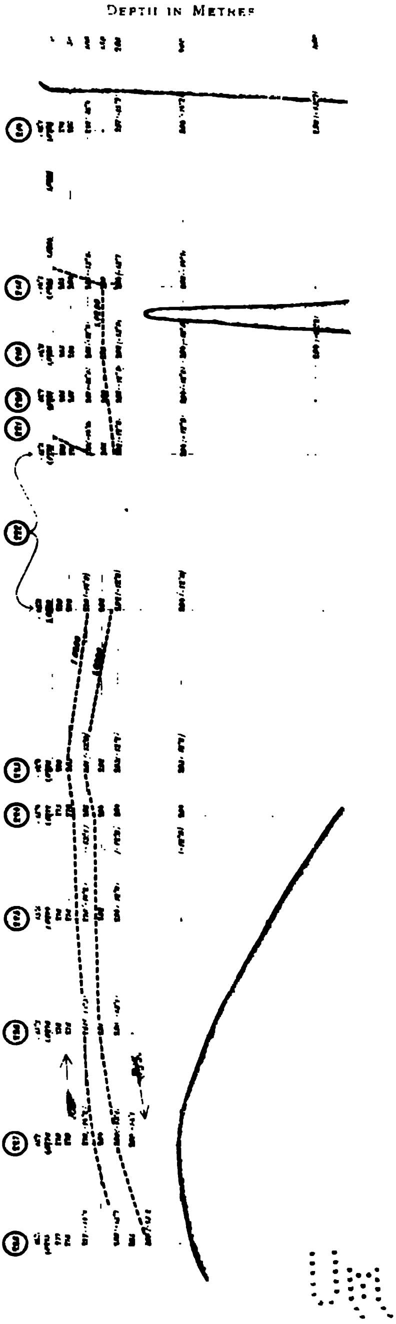
PLATE III.



MEDITERRANEAN



GIBRALTAR STRAIT.



Temperature (Centigrade) and specific gravity of the water $(S\frac{17.5}{17.5})$

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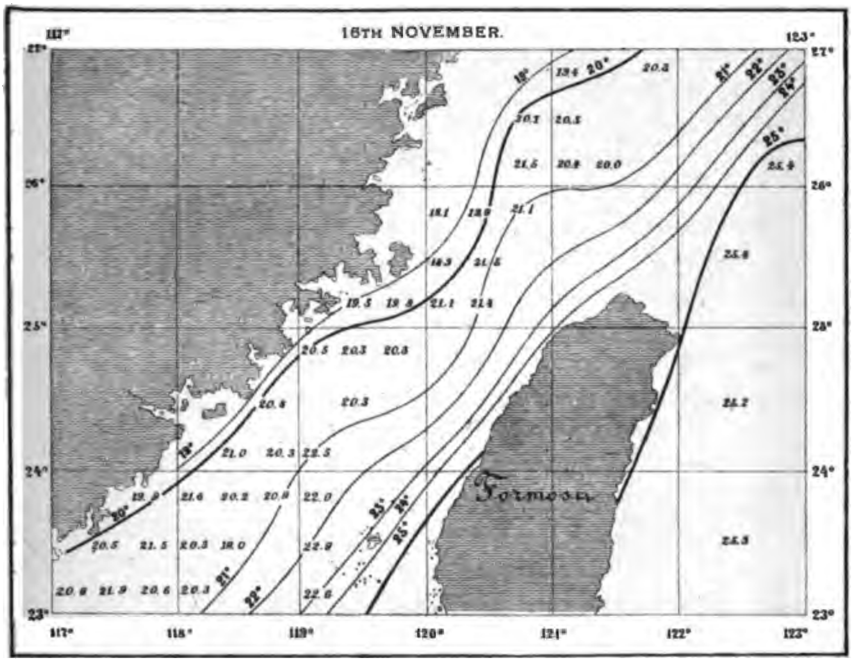
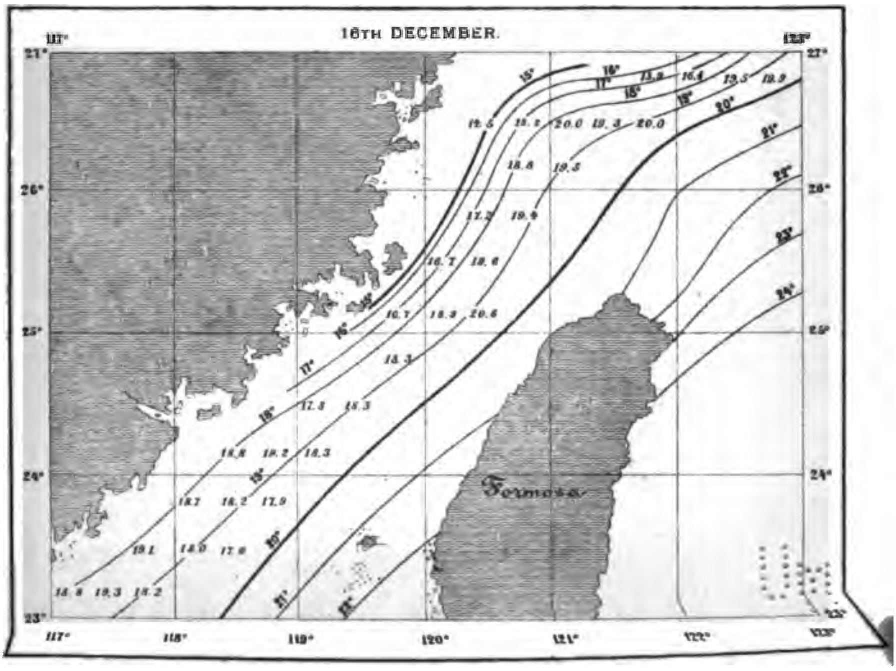
ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

PLATE IV.

ISOTHERMS IN CENTIGRADE OF THE SURFACE WATER OF THE STRAIT OF FORMOSA.


16TH NOVEMBER.


16TH DECEMBER.

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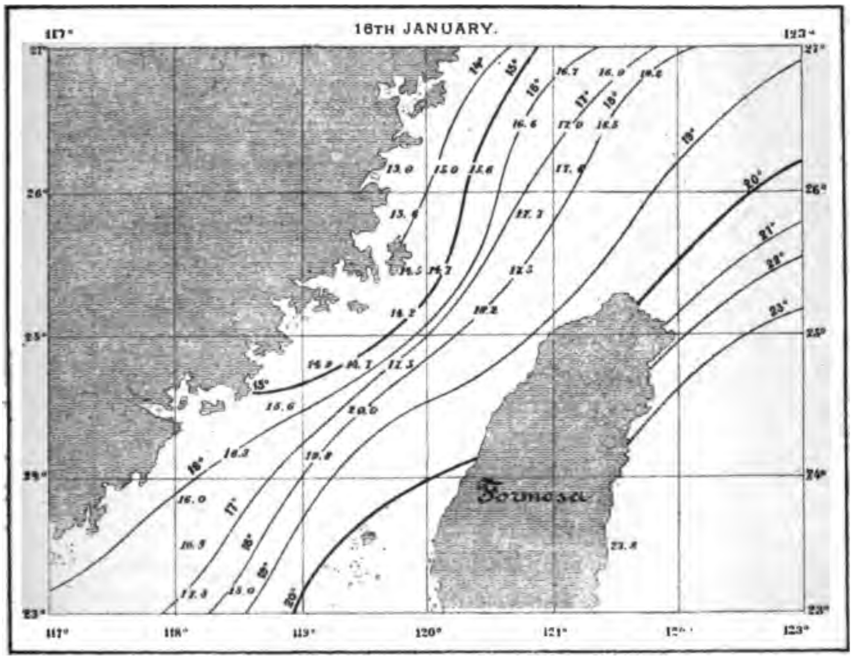
ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

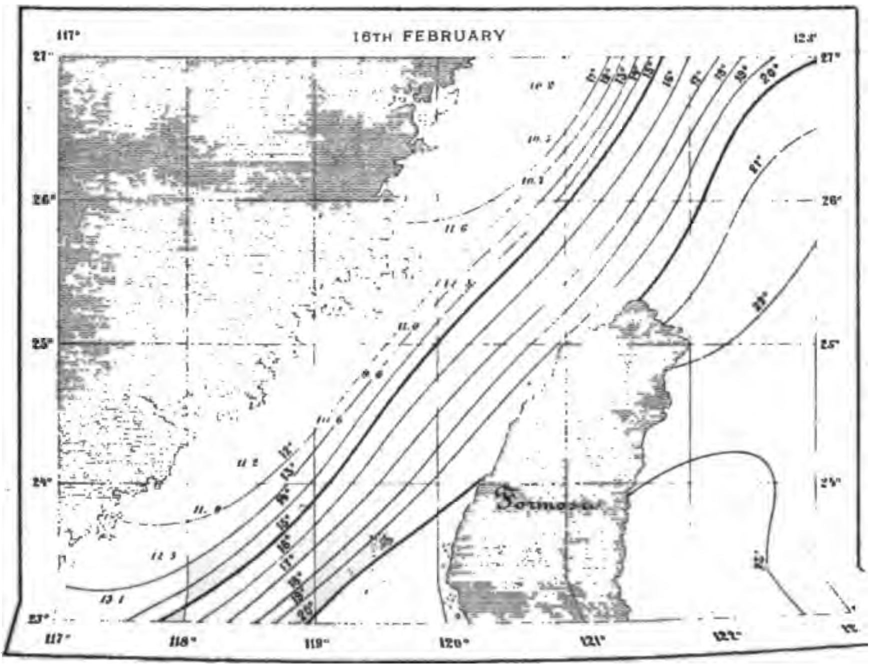
PLATE V.

ISOTHERMS IN CENTIGRADE OF THE SURFACE WATER OF THE STRAIT OF FORMOSA.

16TH JANUARY.



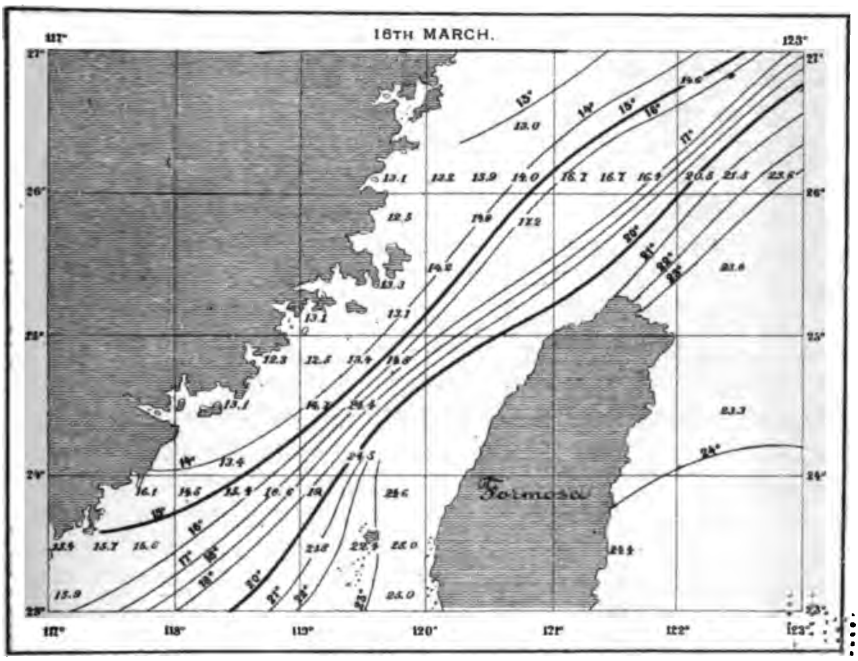
16TH FEBRUARY.



ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

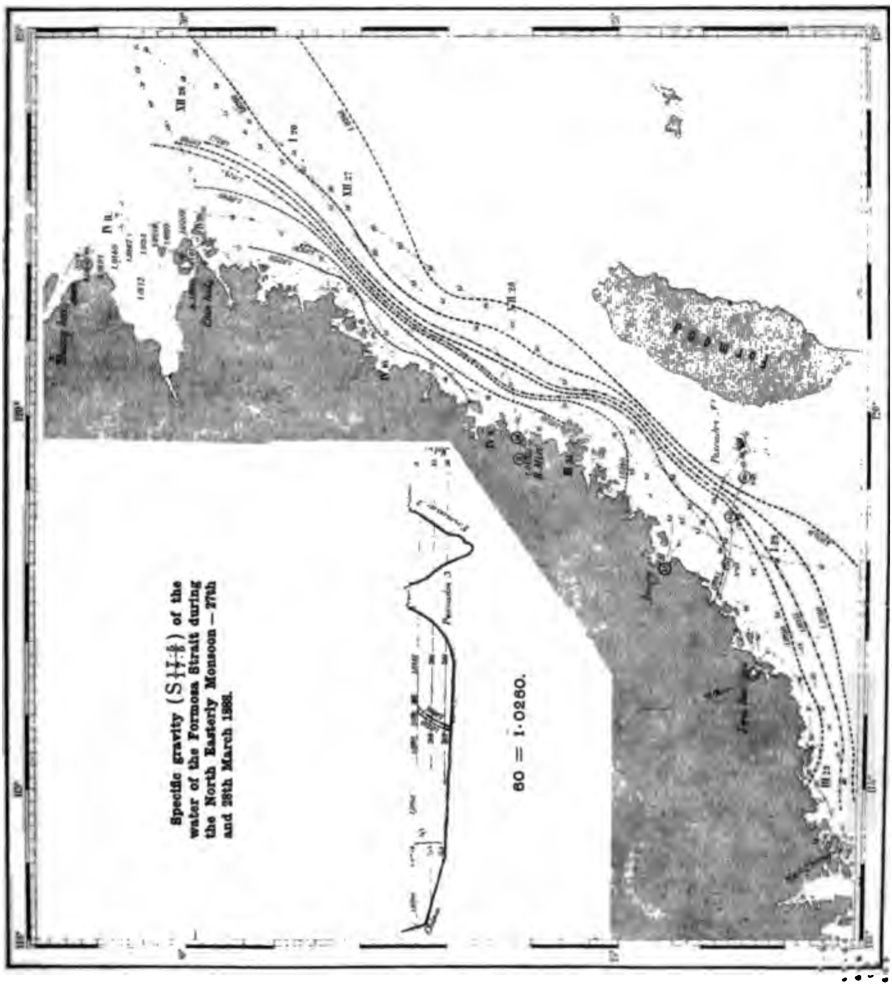
PLATE VI.

ISOTHERMS IN CENTIGRADE OF THE SURFACE WATER OF THE STRAIT OF FORMOSA.



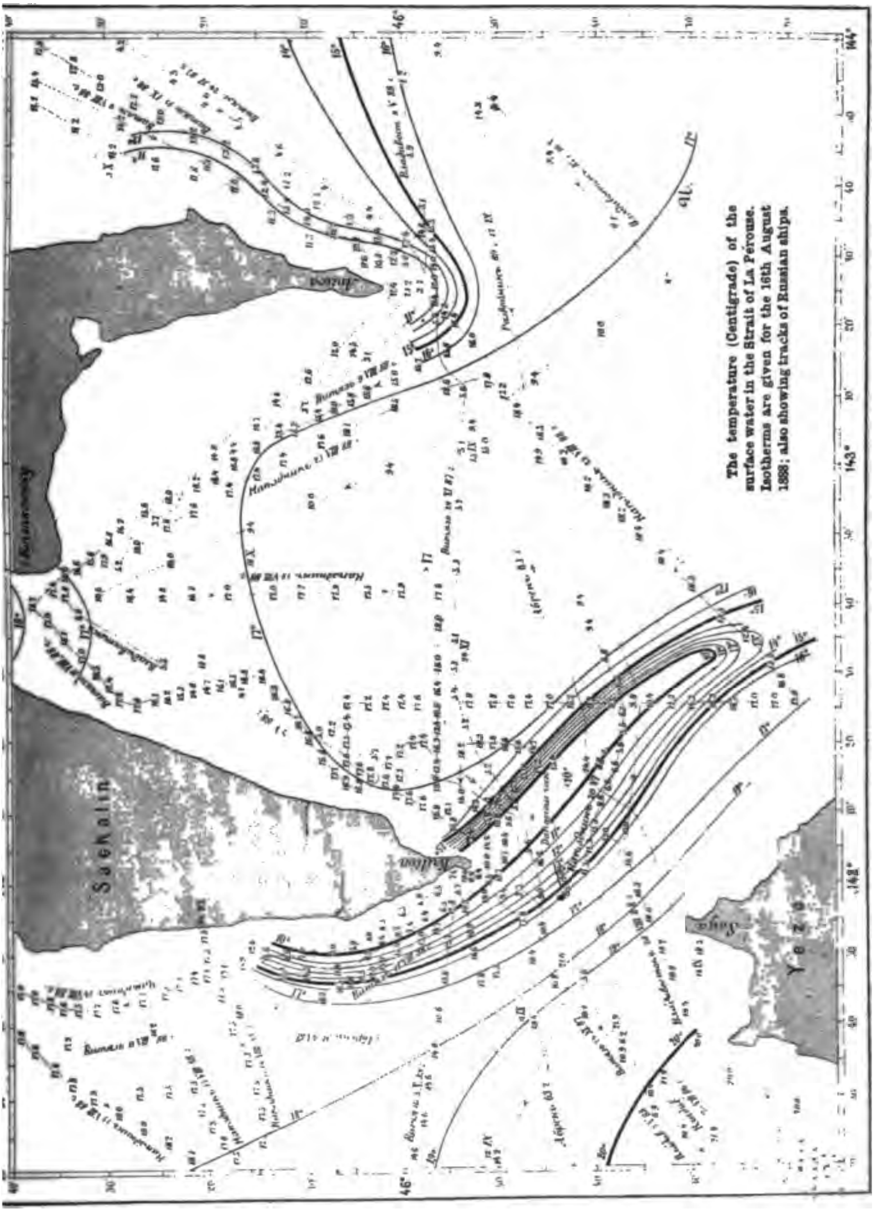
ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

PLATE VII.


Specific gravity ($S\frac{17.5}{17.5}$) of the water of the Formosa Strait during the North Easterly Monsoon — 27th and 28th March 1888.
60 = 1·0260.
FORMOSA
Pescadores
XII 27
XII 28

ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

PLATE VIII.

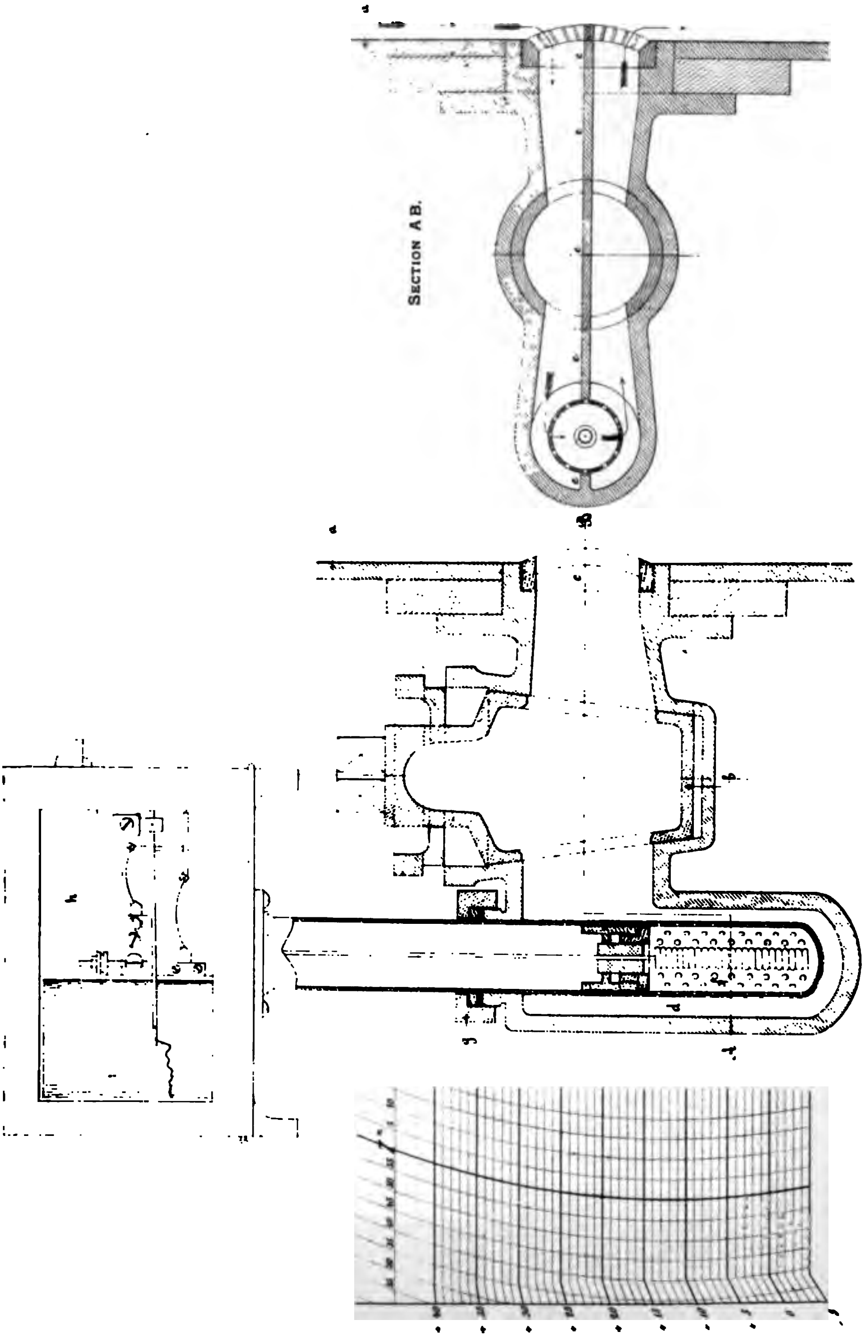


The temperature (Centigrade) of the surface water in the Strait of La Pérouse. Isotherms are given for the 16th August 1888; also showing tracks of Russian ships.

ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

PLATE IX.

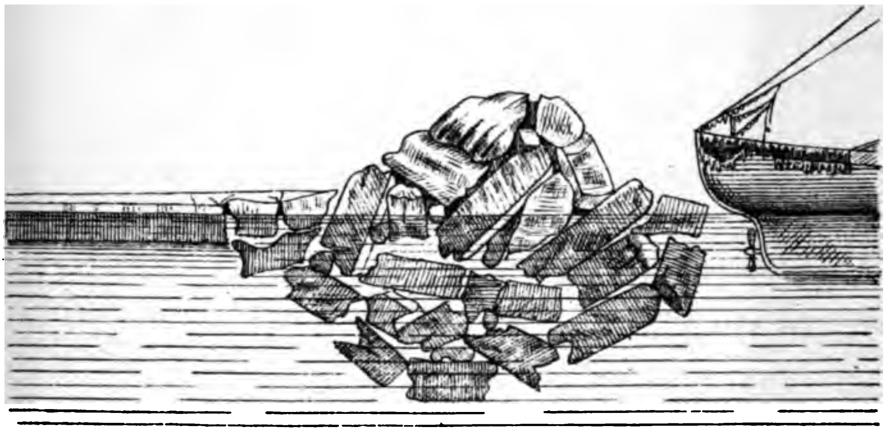
ARRANGEMENT OF THE RICHARD'S THERMOGRAPH
FOR REGISTERING THE TEMPERATURE OF THE SEA WATER.



ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.

PLATE X.

ICE TORROS.



REMOVING THE ICE OF THE TORROS BY THE STREAM OF WATER FROM THE FORE PROPELLER.

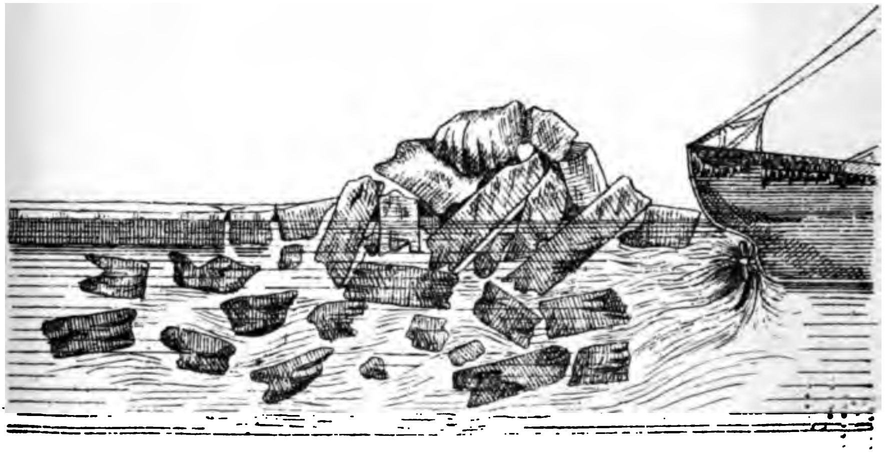


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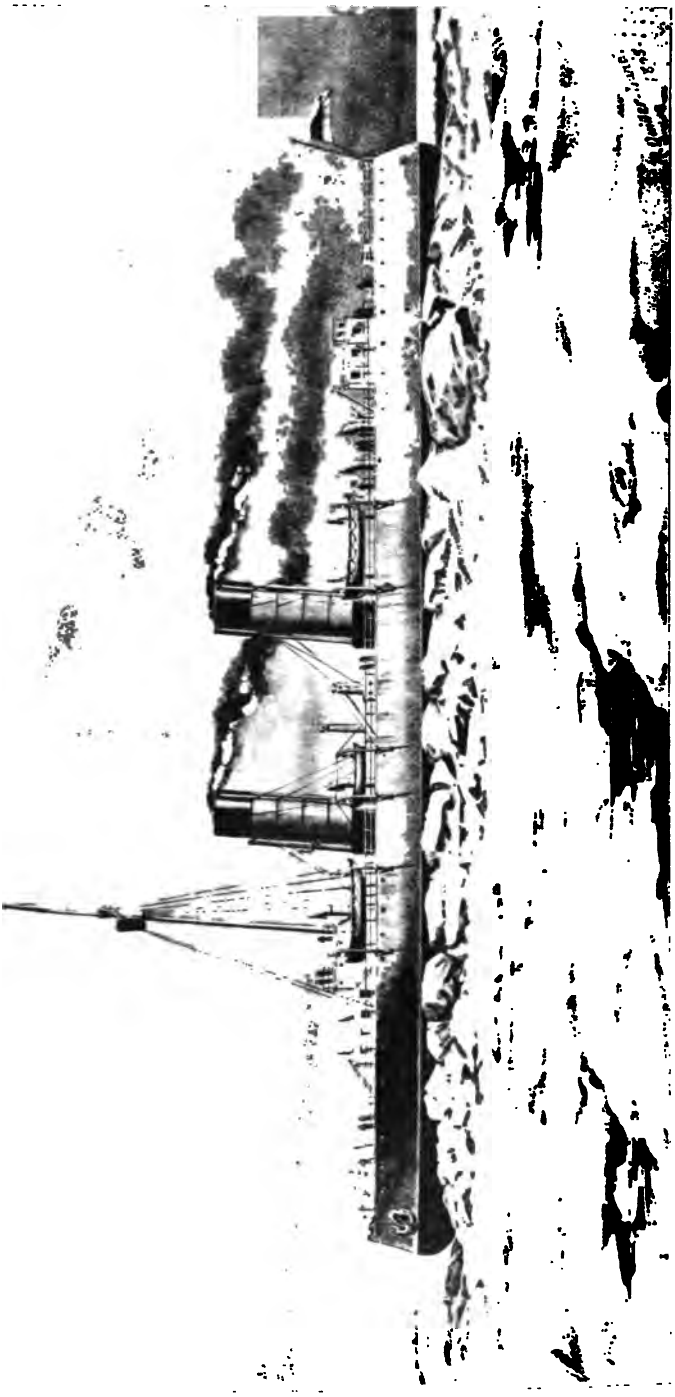
PLATE XII.

ADMIRAL MAKAROFF ON OCEANOGRAPHIC PROBLEMS.



THE RUSSIAN ICE-BREAKING STEAMER ERMACK.

Messrs Sir W. G. Armstrong, Whitworth, and Co., Ltd., Newcastle-on-Tyne, Builders.

On the Survey by the S.S. 'Britannia' of the Cable Route between Bermuda, Turk's Islands, and Jamaica. By R. E. Peake, M. Inst. C.E. With Descriptions by Sir JOHN MURRAY, K.C.B., F.R.S., of the Marine Deposits brought home. (*Communicated by* Sir JOHN MURRAY.) (With a Map.)

(Read July 18, 1898.)

I. NOTES ON THE SOUNDING CRUISE OF S.S. 'BRITANNIA,' by R. E. PEAKE.

The following is a brief account of the principal points of interest resulting from the survey lately made to find a suitable route for a cable to be laid from Bermuda to Turk's Islands and from thence to Jamaica.

As engineers to the Direct West India Cable Co., my firm, Messrs Clark, Forde, & Taylor, strongly advised that a more than usually complete set of soundings should be taken along the route over which it was proposed that the cable should be laid, our principal reason being that, as the Bermuda group has been raised almost vertically from the bed of the Atlantic by volcanic action, it was impossible, with the limited information available, to say whether similar action might not have taken place elsewhere on the proposed route for the new cable.

The task of carrying out this survey was undertaken by the Telegraph Construction and Maintenance Co. with their s.s. 'Britannia,' which was fitted up with all necessary apparatus, and was also well adapted for the work.

Sounding operations were commenced off Grand Turk Island on the 4th November 1897, and completed on the 2nd December following at Bermuda, during which time 182 soundings were taken, apart from the profile soundings taken off Grand Turk. The laying of the cable was commenced at Bermuda on the 5th January 1898, and completed on the 18th of the same month.

As a result of this work the following suggest themselves as points worthy of notice:—

(1) The even slope of the ground from the neighbourhood of Bermuda to the deepest water found in Lat. 24° 50′ N., no indication being discovered of any upheaval similar to that which has produced the Bermudas with their adjoining shoals.

(2) The large extent of water with depths of over 3000 fathoms.

(3) Sounding No. 91. The undoubted evidence of a hard bottom or hard object in such a great depth as 2994 fathoms; in this case the instrument for obtaining a specimen of the bottom, known as a "snapper," was brought up dented and bent, with a small portion of calcareous substance enclosed in the jaws.

(4) The formation of the coral bank on the west side of Grand Turk Island, as shown by the profile soundings taken off that island.

(5) That the south rocks near Sand Cay in the Turk's Island Passage are placed about ½ mile too far south on Chart 1441.

(6) That at its southern end the Turk's Island Passage is blocked by a bank which joins the Turk's Islands to the Caicos, having a comparatively narrow ridge, with between 200 and 300 fathoms of water on it. This bank, especially on its south side, shows a very steep gradient into deep water.

(7) That the cable between Bermuda and Grand Turk Island lies in by far the deepest water of any cable laid up to the present time, not only in the maximum depth attained, but also in the average depth of the whole section. The deepest water obtained on the route of the cable was 3113 fathoms, and the mean depth from Cable House to Cable House 2732 fathoms. In one continuous run of twenty-four hours, during which 172 nautical miles of cable were paid out, the average depth was 3044 fathoms.

A further point of interest is the comparison of the mean bottom temperature derived from thermometer observations with that obtained by electrical tests of the laid cable. Along the route followed by the cable over a distance of 750 miles, 102 soundings were taken, at a considerable number of which the bottom temperature was observed. It was possible, therefore, to determine, with considerable accuracy, the mean depth and mean temperature by thermometer.

The following table gives the manner in which these results were arrived at:—

BERMUDAS—TURK'S ISLANDS CABLE.

Statement showing Mean Depths and Bottom Temperatures for different portions of Cable as laid.

Landing Place.	Cable.	Depth.	Observed Temp. by Therm.
	Nautical Miles.	Fathoms.	F. °
Bermudas Cable House,	1·0	3	75·0
" "	1·5	8	70·7
" "	0·5	47	66·0
" "	3·0	325	62·0
" "	6·0	1000	38·5
" "	6·0	1400	38·0
" "	6·0	1800	36·6
" "	6·0	2000	36·4
" "	25·0	2350	36·3
" "	101·0	2600	36·2
" "	167·0	2865	36·2
" "	168·0	2904	36·2
" "	172·0	3044	36·2
" "	133·0	2697	36·2
" "	4·5	875	39·6
" "	8·0	749	39·9
" "	1·0	700	40·2
" "	2·5	500	44·0
" "	0·3	132	66·0
Grand Turk Island Cable House,	0·2	5	80·0
	812·5	Mean 2732	Mean 36·57 say 36·5

Note.—To obtain the mean thermometer temperature, the lengths in the first column were multiplied by the corresponding temperatures in the third column, and the sum of the products divided by the total length.

It will be seen from the previous statement that the short length of shallow water at each end of the line is only sufficient to raise the mean temperature one-third of a degree above the minimum obtained, viz., 36°·2 Fahrenheit, and as this temperature appears to be practically the same for all depths exceeding, say, 2300 fathoms, the conditions are exceptionally favourable for comparing the mean temperature given by thermometer with that obtained from electrical observations on the laid cable.

Owing to the small and steady difference of earth potential between Bermuda and Turk's Islands it was possible to determine the copper resistance of the laid cable with great accuracy, and the results so obtained were further checked by comparison with standard coils.

The copper resistance at 75° of the different coils of core used in the manufacture of the cable was separately determined by the contractors and by ourselves with different instruments, and as very little difference was found between these results there is every reason to believe that the electrical values are accurate.

The application of Matthiessen's co-efficient for the resistance of copper at different temperatures, to the observed resistance of the laid cable, gave a mean temperature of 33°·3 Fahrenheit, while the mean observed temperature by thermometer was 36°·5 Fahrenheit. It appeared to us at first that this discrepancy must be due to an error in the thermometer values, but after reading Professor Tait's Report on the Pressure Errors of the 'Challenger' thermometers,* and having had the instruments used on the 'Britannia' verified at Kew, this did not seem possible. The only other explanation which occurred to us was that the temperature co-efficient of the copper used in the conductor was different to that hitherto employed in calculation, and it was accordingly decided to investigate this point experimentally.

A determination of the co-efficient was carried out with great care by Mr H. A. Taylor, who, through the courtesy of Mr W. Shuter, the Managing Director of the Telegraph Construction and Maintenance Co., was enabled to test samples of wire supplied by five of the leading manufacturers. The results obtained from these tests, which gave singularly even values for the different samples, showed that the temperature co-efficient for this class of copper was sensibly different from that obtained by Matthiessen, and the new co-efficient applied to the observed resistance of the laid cable gave a mean temperature of 36°·26 against 36°·57 by thermometer. These results are so close as to practically reconcile the two methods and confirm the accuracy of both.

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* *Narr. Chall. Exp.*, vol. ii., Appendix A, 1882.

The alteration in this co-efficient has certainly occurred only within the last few years, and is probably due to changes in the methods employed in the manufacture. All conductivity copper is now electrically deposited, and owing to improved processes the density of the metal is appreciably higher than that of the laboratory specimens on which Dr Matthiessen's observations were made. Increased density is accompanied by higher conductivity, and Matthiessen himself pointed out that under these circumstances an increased temperature co-efficient was to be expected.*

II. NOTES ON THE DEPOSIT-SAMPLES OBTAINED BY S.S. 'BRITANNIA,'

by Sir JOHN MURRAY, K.C.B.

In November and December 1897, the s.s. 'Britannia,' belonging to the Telegraph Construction and Maintenance Co., took an excellent series of soundings between Bermuda and Turk's Islands and thence through the Windward Passage to Jamaica.† Through the kindness of Mr R. E. Peake, M. Inst. C.E., who supervised the carrying out of the work on behalf of the Direct West India Cable Co., we have been permitted to examine the deposits brought home, numbering in all 116 samples, ranging in depth from 224 to 3150 fathoms. These samples may be arranged in zones of 500 fathoms as follows:—

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* See Temperature Co-efficients of 'Conductivity' Copper, by Clark, Forde, and Taylor, London, 1899 (published privately).

† For complete list of the soundings, see *Admiralty Blue-book*: List of Oceanic Depths and Serial Temperature Observations received at the Admiralty during the year 1897, from H.M. Surveying Ships, Indian Marine Survey, and British Submarine Telegraph Companies; Hydrographic Department, Admiralty, London, February 1898, pp. 52–56. See also the accompanying map, on which nearly all the 'Britannia' soundings are laid down in a distinctive type, as well as all other known deep-water soundings in the same region.

17	samples	come from depths	less than 500 fathoms.
22	,,	,,	between 500 and 1000 fathoms.
4	,,	,,	,, 1000 ,, 1500 ,,
2	,,	,,	,, 1500 ,, 2000 ,,
7	,,	,,	,, 2000 ,, 2500 ,,
45	,,	,,	,, 2500 ,, 3000 ,,
19	,,	,	greater than 3000 fathoms.
116			

In the majority of cases the samples are small, especially from very deep water, where, however, the available material shows that the bottom is covered by Red Clay extremely uniform in character and composition; some of the samples weighed from a half to one ounce, and in a few cases even more. All the samples, large and small, were carefully examined, and in nearly every case the type of deposit was determined. In the case of large samples the percentage of carbonate of lime was determined by chemical analysis, and in other cases it was estimated by inspection. Typical examples, where there was sufficient material, have been fully described, and the descriptions will be found in the sequel.

Before proceeding to consider the general character of the bottom, we may point out that this admirable series of soundings has considerably increased our knowledge of the depth of the sea in this part of the world. The line of deep soundings has altered the outlines and increased the extent of the area over 3000 fathoms (called Nares Deep on the bathymetrical charts published in the 'Summary of Results,' forming the final two volumes of the *Challenger Reports*, 1895), which now approaches much closer to the coast of Turk's Islands. The area over 2000 fathoms is also extended towards the northern entrance of the Windward Channel, between Caicos Bank and Haïti. The present state of our knowledge of the depths and deposits in this region of the ocean is shown on the accompanying map.

The deposit samples show that in shallow water close to the shore of Jamaica the bottom is covered by Blue Mud, containing 20 or 25 per cent. of carbonate of lime. Off the eastern point of Jamaica the deposit passes into Pteropod Ooze, containing 60 to 80 per cent. of carbonate of lime, which continues through the Windward Passage in depths down to 1800 fathoms, but is replaced again in deep water (over 2000 fathoms), between the northern coast of Haïti and Caicos Bank, by Blue Mud, containing 5 or 10 per cent. of carbonate of lime. A very peculiar sample comes from this locality, containing an extraordinary proportion of volcanic mineral particles, and therefore called a Volcanic Mud (see description of No. 49, 2293 fathoms). On approaching the southern entrance of Turk's Island Passage, in the neighbourhood of a reported danger called Fawn Shoal, which was unsuccessfully searched for in 1880 by H.M.S. 'Fantome,' Pteropod Ooze, with about 90 per cent. of carbonate of lime, again covers the bottom in depths between 250 and 300 fathoms. The four samples are peculiar in that they contain internal casts of Foraminifera, Pteropods, and other organisms, these casts being composed of carbonate of lime, and not of a silicate as is usually the case; similar casts were observed in the deposit collected by the 'Challenger' off Raine Island in Torres Strait (see detailed description further on). Off Turk's Islands the deposit is Coral Mud or Sand, with 80 to 90 per cent. of carbonate of lime, in depths less than 1000 fathoms. From the steep slope to the north of Turk's Island, in depths between 1000 and 2000 fathoms, no deposit-samples were brought home; but in depths slightly over 2000 fathoms Globigerina Ooze, with 60 to 70 per cent. of carbonate of lime, was met with, succeeded by Red Clay containing about 3 to 15 per cent. of carbonate of lime in depths approaching and exceeding 3000 fathoms, which continues until the bottom gradually shoals on approaching Bermuda, the percentage of carbonate of lime increasing to 20 and 25 per cent., the deposit insensibly passing into Globigerina Ooze with from 30 to 70 per cent. of carbonate of lime in depths between 2162 and 2562 fathoms, with Pteropod Ooze, containing 80 or 85 per cent. of carbonate of lime, nearer the shores of Bermuda in 1000 and 1395 fathoms.

This series of soundings modifies somewhat the distribution of deep-sea deposits as laid down in this region on the chart accompanying the Challenger Report on Deep-Sea Deposits, published in 1891. Thus:—

(1) Blue Mud is introduced off the coasts of Jamaica and Haïti.

(2) Pteropod Ooze is introduced northward from Jamaica through the Windward Channel, again off the southern entrance of Turk's Island Passage, and again near the island of Bermuda at the northern end of the series.

(3) Coral deposits are introduced off Turk's Islands; and

(4) The deep-water Red Clay area is extended and approaches closer to the coast of Turk's Islands than shown on the map referred to.

In the following table we have collected together the information regarding this series of deposits, giving the number, date, position, depth, type of deposit, estimated percentage of carbonate of lime, with a few remarks on the varying character of the bottom. This table is followed by detailed descriptions of a few of the typical samples representing the different kinds of deposits, and by a general description of the map which accompanies this paper.

[TABLE.

No.	Date.		Latitude, N.			Longitude, W.			Depth, Fathoms.	Deposit.	Estimated Percentage of Calcium Carbonate.*	Remarks.
	1897.											
3	Nov.	5	21°	26′	43″	71°	11′	8″	598	Coral Mud, ...	[86·04]	
5	,,	,,	21	25	12	71	10	47	544	,, ,, ...	80	
7	,,	,,	21	23	36	71	13	20	784	,, ,, ...	80	
9	,,	,,	21	22	6	71	14	12	796	,, ,, ...	80	
10	,,	,,	21	21	7	71	14	0	761	,, ,, ...	80	
11	,,	,,	21	20	18	71	14	47	800	,, ,, ...	80	
12	,,	,,	21	18	12	71	15	15	800	,, ,, ...	80	
13	,,	6	21	16	42	71	16	32	860	,, ,, ...	80	
15	,,	,,	21	14	38	71	17	20	880	,, ,, ...	80	
16	,,	,,	21	14	6	71	18	5	912	,, ,, ...	80	
17	,,	,,	21	13	0	71	18	6	903	,, ,, ...	80	
18	,,	,,	21	11	49	71	17	34	836	Fragments of Corals, Foraminifera, and Molluscs,	...	
19	,,	,,	21	11	13	71	18	38	872	Coral Mud, ...	80	
20	,,	,,	21	9	48	71	19	13	830	,, ,, ...	80	
23	,,	,,	21	6	6	71	20	35	681	,, ,, ...	80	
25	,,	7	21	2	35	71	21	44	345	,, ,, ...	85	Pteropods numerous.
26	,,	,,	21	2	36	71	22	55	349	,, ,, ...	85	,, ,,
30	,,	,,	21	1	55	71	26	38	349	,, ,, ...	85	Pteropods and Pelagic Foraminifera becoming more numerous.
31	,,	,,	21	1	50	71	25	5	439	,, ,, ...	85	Many Pteropods and Foraminifera.

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* The few cases in which the percentage of calcium carbonate has been determined by chemical analysis are distinguished by having the figures placed in brackets.

No.	Date.	Latitude, N.			Longitude, W.			Depth, Fathoms.	Deposit.	Estimated Percentage of Calcium Carbonate.*	Remarks.
		°	′	″	°	′	″				
32	1897. Nov. 7	21	2	53	71	20	9	224	Fragments of Coral, Foraminifera, and Molluscs,	...	
33	,, ,,	21	2	11	71	22	46	341	Coral Mud, . . .	85	Many Pteropods and Foraminifera.
37	,, ,,	20	55	16	71	27	34	254	Pteropod Ooze, . .	90	Calcareous casts of organisms.
38	,, ,,	20	52	55	71	30	7	260	,, ,, . .	90	,, ,, ,,
39	,, ,,	20	53	7	71	34	12	258	,, ,, . .	90	,, ,, ,,
40	,, ,,	20	50	36	71	36	35	290	,, ,, . .	90	,, ,, ,,
44	,, ,,	20	43	18	71	58	24	2388	Blue Mud, . . .	10	
45	,, 8	20	38	12	72	9	3	2308	,, ,, . . .	[11·34]	
46	,, ,,	20	33	0	72	12	39	2298	,, ,, . . .	10	
49	,, ,,	20	27	12	72	40	39	2293	Volcanic Mud, . .	[19·70]	Peculiar deposit (see description).
51	,, ,,	20	22	42	72	53	15	1745	Pteropod Ooze, . .	70	Not typical; much clayey matter.
58	,, 9	19	50	6	73	54	17	1589	,, ,, . .	70	Not typical; much clayey matter.
59	,, ,,	19	37	24	74	3	47	1485	,, ,, . .	60	Very many mineral particles (quartz, &c.).
63	,, ,,	19	20	42	74	21	17	998	,, ,, . .	70	
65	,, ,,	19	11	36	74	32	59	1109	,, ,, . .	70	
66	,, ,,	18	59	48	74	43	11	962	,, ,, . .	75	
68	,, 10	18	51	0	74	53	24	940	,, ,, . .	75	
72	,, ,,	18	13	42	75	31	18	865	,, ,, . .	80	

74	,,	,,	17	57	30	75	49	30	653	Coral with Manganese, .	...	
75	,,	,,	17	45	0	76	4	30	492	Pteropod Ooze, . .	70	
76	,,	,,	17	44	10	76	9	22	469	,, ,, . .	70	
77	,,	,,	17	45	0	76	14	20	450	,, ,, . .	65	Fewer Pteropods; more muddy.
78	,,	,,	17	46	47	76	17	25	420	,, ,, . .	60	Fewer Pteropods; coherent.
79	,,	14	17	54	37	76	41	54	365	Blue Mud, . .	20	
81	,,	,,	17	49	37	76	39	37	672	,, ,, . . .	20	
82	,,	,,	17	48	18	76	29	52	319	,, ,, . . .	25	
83	,,	16	21	32	9	71	10	38	431	Coral Sand, . . .	90	
86	,,	,,	21	29	48	71	12	0	793	Coral Mud, . . .	90	
89	,,	17	21	46	24	71	3	54	2037	Globigerina Ooze, . .	[63·43]	
90	,,	,,	21	55	42	70	57	36	2060	,, ,, . .	70	
91	,,	,,	22	8	18	71	1	54	2994	Small calcareous fragments,	...	
92	,,	,,	22	12	48	70	53	6	2993	Red Clay, . . .	3	
93	,,	,,	22	16	48	70	39	18	3052	,, ,, ...	3	
97	,,	18	22	44	54	70	24	0	2959	,, ,, ...	5	
98	,,	,,	22	52	0	70	27	0	2975	,, ,, ...	[11·50]	
99	,,	,,	23	1	30	70	31	18	2881	,, ,, ...	5	
100	,,	,,	23	4	0	70	20	6	3009	,, ,, ...	5	
102	,,	19	23	15	18	70	12	0	3023	,, ,, ...	3	
104	,,	,,	23	25	30	70	5	42	3036	,, ,, ...	3	
106	,,	20	23	36	12	69	59	12	3075	,, ,, ...	5	
108	,,	,,	23	47	30	69	52	30	3000	,, ,, ...	5	
109	,,	,,	23	53	54	69	39	30	2984	,, ,, ...	5	
110	,,	,,	24	3	36	69	44	6	2990	,, ,, ...	10	
112	,,	21	24	18	36	69	34	24	3035	,, ,, ...	3	
113	,,	,,	24	20	18	69	22	48	3020	,, ,, ...	3	
116	,,	,,	24	51	0	69	13	42	3084	,, ,, ...	3	
117	,,	,,	24	50	0	69	5	30	3150	,, ,, ...	3	
118	,,	22	25	1	30	69	11	48	3113	,, ,, ...	3	

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* The few cases in which the percentage of calcium carbonate has been determined by chemical analysis are distinguished by having the figures placed in brackets.

No.	Date.	Latitude, N.	Longitude, W.	Depth, Fathoms.	Deposit.	Estimated Percentage of Calcium Carbonate.*	Remarks.
119	1897. Nov. 22	25° 13′ 0″	69° 18′ 12″	2981	Red Clay, . . .	3	
120	,, ,,	25 16 24	69 7 54	3053	,, ,, . . .	3	
121	,, ,,	25 17 0	68 51 54	3055	,, ,, . . .	3	
122	,, ,,	25 29 36	68 54 48	3021	,, ,, . . .	3	
123	,, ,,	25 42 18	68 59 24	3049	,, ,, . . .	3	
124	,, ,,	25 42 12	68 47 48	3050	,, ,, . . .	5	
125	,, 23	25 43 0	68 33 18	3040	,, ,, . . .	8	
126	,, ,,	25 53 12	68 39 12	3014	,, ,, . . .	8	
127	,, ,,	26 3 30	68 45 6	2990	,, ,, . . .	10	
128	,, ,,	26 7 0	68 32 12	2944	,, ,, . . .	10	
129	,, ,,	26 10 0	68 17 48	2992	,, ,, . . .	10	
130	,, ,,	26 21 24	68 21 12	2865	,, ,, . . .	12	
131	,, ,,	26 32 36	68 24 42	2990	,, ,, . . .	10	
132	,, ,,	26 34 48	68 12 12	2968	,, ,, . . .	5	
133	,, 24	26 37 6	67 59 36	3029	,, ,, . . .	5	
134	,, ,,	26 48 6	68 4 18	2990	,, ,, . . .	5	
135	,, ,,	26 58 54	68 8 36	2910	,, ,, . . .	5	
138	,, ,,	27 10 0	67 46 0	2876	,, ,, . . .	5	
140	,, 25	27 26 0	67 37 30	2810	,, ,, . . .	8	
141	,, ,,	27 30 30	67 25 0	2778	,, ,, . . .	12	
144	,, 27	27 52 12	67 20 54	2720	,, ,, . . .	12	
145	,, ,,	27 54 54	67 6 30	2765	,, ,, . . .	12	
146	,, ,,	28 5 12	67 14 18	2819	,, ,, . . .	15	
147	,, ,,	28 15 24	67 19 18	2885	,, ,, . . .	12	
148	,, ,,	28 18 12	67 6 6	2918	,, ,, . . .	10	

149	,,	,,	28	21	0	66	52	18	2820	,, ,,	8	
150	,,	,,	28	31	6	66	59	54	2930	,, ,,	5	
151	,,	28	28	41	18	67	7	30	2910	,, ,,	5	
152	,,	,,	28	44	12	66	54	6	2910	,, ,,	5	
153	,,	,,	28	47	6	66	40	54	2912	,, ,,	5	
154	,,	,,	28	57	0	66	45	30	2898	,, ,,	5	
155	,,	,,	29	7	36	66	47	48	2850	,, ,,	8	
158	,,	29	29	23	24	66	33	12	2930	,, ,,	10	
159	,,	,,	29	33	30	66	40	30	2921	,, ,,	12	
162	,,	,,	29	50	30	66	16	0	2850	,, ,,	15	
163	,,	,,	30	1	54	66	15	42	2769	,, ,,	20	
164	,,	30	30	5	6	66	4	12	2766	,, ,,	20	
166	,,	,,	30	19	48	65	59	12	2686	,, ,,	20	
168	,,	,,	30	30	0	65	44	0	2719	,, ,,	25	
170	,,	,,	30	45	42	65	39	30	2672	,, ,,	25	Lighter colour, approaching Globigerina Ooze.
171	,,	,,	30	55	6	65	41	54	2671	Globigerina Ooze,	30	
172	Dec.	1	30	58	30	65	29	54	2644	,, ,,	40	
174	,,	,,	31	13	0	65	22	36	2548	,, ,,	40	
175	,,	,,	31	21	42	65	23	42	2562	,, ,,	40	
176	,,	,,	31	24	0	65	8	54	2516	,, ,,	50	
177	,,	,,	31	33	12	65	14	6	2515	,, ,,	50	
179	,,	,,	31	56	30	64	51	30	2162	,, ,,	70	
181	,,	2	32	10	14	64	42	13	1395	Pteropod Ooze,	85	Might be called Globigerina Ooze.
182	,,	,,	32	14	26	64	39	45	1000	,, ,,	[79·59]	

* The few cases in which the percentage of calcium carbonate has been determined by chemical analysis are distinguished by having the figures placed in brackets.

S.S. 'Britannia,' No. 3, 5th November 1897. Lat. 21° 26′ 52″ N., long. 71° 10′ 22″ W., 598 fathoms.

CORAL MUD, white, chalky.

Calcium Carbonate [86·04%], small particles of Corals and calcareous Algæ, with bottom-living and pelagic Foraminifera (including *Polytrema* and *Globigerina rubra*), fragments of Pteropods, Heteropods, and other pelagic Molluscs, Alcyonarian and Tunicate spicules, Ostracodes, Echini spines, Cephalopod beaks, coccoliths.

Residue [13·96%],

Minerals (1%), glassy particles, quartz.

Siliceous Organisms (3%), Sponge spicules, Diatoms, Radiolarians, and imperfect casts of calcareous organisms.

Fine Washings (9·96%), amorphous clayey matter, with very minute particles of minerals and siliceous organisms.

S.S. 'Britannia,' No. 18, 6th November 1897. Lat. 21° 11′ 49″ N., long. 71° 17′ 34″ W., 836 fathoms.

The material consists of fragments of Coral or calcareous Algæ (or Polyzoa?), fragment of a large Foraminifer (*Orbitolites* or *Carpenteria*), and fragment of a Gasteropod shell.

S.S. 'Britannia,' No. 32, 7th November 1897. Lat. 21° 2′ 53″ N., long. 71° 20′ 9″ W., 224 fathoms.

The material is similar to No. 18, consisting of fragments of Coral or calcareous Algæ, of Foraminifera (*Carpenteria*), and of Mollusc shells.

S.S. 'Britannia,' No. 38, 7th November 1897. Lat. 20° 52′ 55″ N., long. 71° 30′ 7″ W., 260 fathoms.

PTEROPOD OOZE, cream colour.

Calcium Carbonate (90%), Pteropods, Heteropods, and other pelagic Molluscs and their fragments, pelagic and bottom-living Foraminifera, Echini spines, Alcyonarian and Tunicate spicules, coccoliths, rhabdoliths. Many of the organisms (or rather casts of organisms) are of a dull brownish or greyish colour, and are composed of crystalline carbonate of lime (see note below).

RESIDUE (10%),

Minerals (1%), a few small particles of augite and glass.

Siliceous Organisms (1%), Sponge spicules, imperfect casts in a clayey matter.

Fine Washings (8%), amorphous clayey matter, etc.

In this and the three neighbouring deposits (254 to 290 fathoms) casts of the calcareous organisms in carbonate of lime occurred. Such a deposition was observed in the deposit obtained by the 'Challenger' off Raine Island, Torres Strait (155 fathoms), and is thus described in the Challenger Report on Deep-Sea Deposits (pp. 169–170) :—

"If some of the Foraminifera be treated with dilute acid, the action stopped after it has continued for some time, and the substance dried and examined by reflected light, a number of casts of the organisms are seen in carbonate of lime looking quite like milky quartz. If, however, the action be continued, it is seen that they are composed of carbonate of lime as they entirely disappear, leaving a small residue of a reddish colour, or very areolar casts of the shells in the same red substance. Examined in thin sections, it is observed that the shells are filled with a red, yellowish, or greenish matter, frequently extending into the foramina. The shell is at once distinguished from the cast by its structure, transparency, and optical properties. It is sometimes observed that two or three shells or fragments are cemented by the same red substance forming the casts. This substance, when sufficiently transparent, appears of a yellowish red colour, and gives sometimes aggregate polarisation, but is never extinguished between crossed nicols. Often the casts inclose small mineral particles. With very high powers it is seen that the structure of the grey carbonate of lime casts is granular, and between crossed nicols it is evident that the grains are crystalline. This is one of the few instances in which it has been possible to point out the deposition of carbonate of lime in the shells forming deposits, and it evidently took place in the deeper layers."

S.S. 'Britannia,' No. 45, 8th November 1897. Lat. 20° 38′ 12″ N., long. 72° 9′ 3″ W., 2308 fathoms.

BLUE MUD, of a green-grey colour.

CALCIUM CARBONATE [11·34%], a few fragments of pelagic Foraminifera (one perfect specimen of *Globigerina conglobata* observed) and Pteropods, coccoliths, rhabdoliths.

RESIDUE [88·66%],

Minerals (60%), m.di. 0·1 mm., augite, quartz, mica, and glassy particles.

Siliceous Organisms (1%), Sponge spicules.

Fine Washings (27·66%), amorphous clayey matter with many minute mineral particles.

S.S. 'Britannia,' No. 49, 8th November 1897. Lat. 20° 27′ 12″ N., long. 72° 40′ 39″ W., 2293 fathoms.

This deposit is very peculiar, consisting of two distinct kinds of material: (1) a fine yellow mud or ooze with many pelagic Foraminifera, and (2) a green volcanic sandy mud with many mica flakes.

VOLCANIC MUD, yellow and greenish, partly granular, partly coherent.

CALCIUM CARBONATE [19·70%), pelagic Foraminifera (*Pulvinulina menardii, tumida,* and *micheliniana, Candeina nitida, Globigerina rubra, bulloides, conglobata, dubia,* and *æquilateralis*), bottom-living Foraminifera, Pteropod fragments, Tunicate spicules, coccoliths, rhabdoliths.

RESIDUE [80·30%],

Minerals (50%), angular, m.di. 0·08 mm., volcanic particles (green olivine being most conspicuous), mica, felspar, quartz, glassy particles.

Siliceous Organisms (1%), Sponge spicules.

Fine Washings (29·3%), amorphous clayey matter with minute mineral particles.

There was also a little black vegetable matter.

From an inspection of the two samples sent us by Mr Peake (one wet, one dry), it is impossible to say whether there were two layers at the bottom, and, if so, whether the yellow ooze or the green sandy material formed the upper layer; the yellow material occurs in small patches surrounded by the sandy material. It is probable that the bottom was originally covered by Globigerina Ooze, and that subsequently a submarine eruption took place, the volcanic mineral matter resulting therefrom being brought up mixed with the ooze in such abundance that the deposit must be called a Volcanic Mud. The volcanic debris is apparently not widespread, for the neighbouring deposits are Blue Muds in similar depths to the north-east, and Pteropod Oozes in shallower depths to the south-east.

S.S. 'Britannia,' No. 74, 10th November 1897. Lat. 17° 57′ 30″ N., long. 75° 49′ 30″ W., 653 fathoms.

Fragment of branching Coral (1 cm. in diameter), much decomposed and coated and impregnated by peroxide of manganese. This fragment is similar to what was obtained by H.M.S. 'Challenger' at two stations in the north-east Atlantic, between the Canary and Cape Verde Islands (Stations 3 and 85, 1525 and 1125 fathoms),* except that in this case the decomposition of the internal parts of the coral is much further advanced, so that the structure is almost entirely lost. A few fragments of a similar *Corallium*, coated with manganese, were obtained by the 'Blake' off the north coast of Haïti, in 772 fathoms.†

S.S. 'Britannia,' No. 89, 17th November 1897. Lat. 21° 46′ 24″ N., long. 71° 3′ 54″ W., 2037 fathoms.

GLOBIGERINA OOZE, of a grey colour.

CALCIUM CARBONATE [63·43%], pelagic Foraminifera (including *Hastigerina* and many *Candeina nitida*), bottom-living Foraminifera, Pteropod and Heteropod fragments, Ostracodes, Echini spines, otoliths of fishes, coccoliths, rhabdoliths.

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* See Murray and Renard, *Deep-Sea Deposits Chall. Exp.*, pp. 41, 61, 149, and 153.

† See Murray, *Bull. Mus. Comp. Zoöl.*, vol. xii. p. 46, 1885.

RESIDUE [36·57%],

Minerals (2%), small glassy particles and mica (one rounded crystal of quartz 0·4 mm. in diameter was observed).

Siliceous Organisms (2%), Sponge spicules, Radiolaria.

Fine Washings (32·57%), amorphous clayey matter, etc.

S.S. 'Britannia,' No. 91, 17th November 1897. Lat. 22° 8′ 18″ N., long. 71° 1′ 54″ W., 2994 fathoms.

The following extract is quoted from the Sounding Report by Mr Peake to draw attention to undoubted evidence of a hard substance on the bottom at this great depth:—"New snapper came up dented and bent, so that it would not close properly; small specimen of rocky substance in neck of snapper."

The material consists of one or two small calcareous fragments, too minute to determine accurately, though they appear to be of organic origin. The fragments were submitted by Mr Peake to an expert in the British Museum, who writes: "Small granules composed of calcite; it is scarcely possible to determine the source of such small particles,—whether of organic or purely mineral origin."

S.S. 'Britannia,' No. 98, 18th November 1897. Lat. 22° 52′ N., long. 70° 27′ W., 2975 fathoms.

RED CLAY, of a brown colour.

CALCIUM CARBONATE [11·50%], a few pelagic Foraminifera and coccoliths.

RESIDUE [88·50%], brown.

Minerals (2%), m.di. 0·06 mm., glassy particles, etc.

Siliceous Organisms (1%), Sponge spicules.

Fine Washings (85·5%), amorphous clayey matter and minute mineral particles.

S.S. 'Britannia,' No. 112, 21st November 1897. Lat. 24° 18′ 36″ N., long. 69° 34′ 24″ W., 3035 fathoms.

RED CLAY, brown or dark grey in colour.

CALCIUM CARBONATE (3%), one or two small bottom-living Foraminifera.

Residue (97%).

Minerals (1%), small particles of glass and mica.

Siliceous Organisms (1%), Sponge spicules.

Fine Washings (95%), amorphous clayey matter, with very minute mineral particles.

S.S. 'Britannia,' No. 182, 2nd December 1897. Lat. 32° 14′ 26″ N., long. 64° 39′ 45″ W., 1000 fathoms.

PTEROPOD OOZE, pure white when dry.

Calcium Carbonate [79·59%], pelagic Foraminifera, Pteropods, Ostracodes, Alcyonarian spicules, Echini spines, Fishes teeth, a few bottom-living Foraminifera, Tunicate spicules, coccoliths, rhabdoliths. Some of the shells are discoloured black by manganese.

Residue (20·41%).

Minerals (1%), small glassy and green particles.

Siliceous Organisms (3%), Sponge spicules, Radiolaria, arenaceous Foraminifera.

Fine Washings (16·41%), amorphous clayey matter.

DESCRIPTION OF THE MAP.

The map accompanying this paper represents the present state of our knowledge concerning the marine deposits in the south-western portion of the North Atlantic. The various kinds of deposits are shown in different colours, but it must be remembered that each variety of deposit slowly passes into adjacent varieties without any marked line of separation such as the colours on the map tend to suggest. It is often difficult to say whether a given sample should be called a Red Clay or a Globigerina Ooze, a Pteropod Ooze or a Coral Mud, a Blue Mud, a Green Mud, or a Volcanic Mud, although typical samples of each of these varieties are quite distinct from one another.

The Bermudas, which are situated towards the north-east corner of the map, consist of a number of coral islets which crown the summit of a huge cone rising abruptly from the bed of the Atlantic, the islands being surrounded at no great distance by depths of over 2500 fathoms. This cone, on which the Bermudas are situated, was originally—there is little doubt—a volcano. No trace of volcanic rock has as yet, however, been dredged from its slopes; it is now completely covered by a mantle of carbonate of lime—the broken shells and skeletons of marine organisms. It will be noticed that four colours surround the Bermudas. The inner colour (yellow) shows the coral reefs and Coral Sands and Muds. Outside these, the Coral Mud passes into a Pteropod Ooze (pale green colour) with increasing depth and distance from the islands. Though only shown in one small patch to the east of Bermuda, future investigations will probably show that Pteropod Ooze occurs all round the cone in depths of about 800 to 1400 fathoms. Beyond the depth of about 1400 fathoms, the Pteropod Ooze passes into a Globigerina Ooze (pink colour), and, with increase of depth, the quantity of carbonate of lime shells and skeletons becomes less and less till the Globigerina Ooze passes, at depths between 2700 and 3000 fathoms, into a Red Clay (brown colour), in which deposits in the greater depths there may be scarcely a trace of the carbonate of lime shells.

It will be observed that the Red Clay occupies all the deeper parts of the bed of the ocean in this region, and covers the greater part of the area represented by the map. In passing towards the shallower depths, in the direction of the coasts of the United States and the West Indies, it will be noticed that a band of Globigerina Ooze (pink colour) occupies the bed of the ocean from a depth of about 2100 fathoms up to 500 or 600 fathoms, the deposit varying much in composition according to depth and position.

The deposits close to land, and generally within the 500 or 600 fathoms-line, show great variety dependent on position and the nature of the adjacent land. Off the coast of Florida, just beyond the 100 fathoms-line, there is a deposit of glauconitic sand and mud (dark green colour), containing generally about 50 per cent. of carbonate of lime. In this deposit some remarkable phosphatic concretions have been found,* and similar phosphatic

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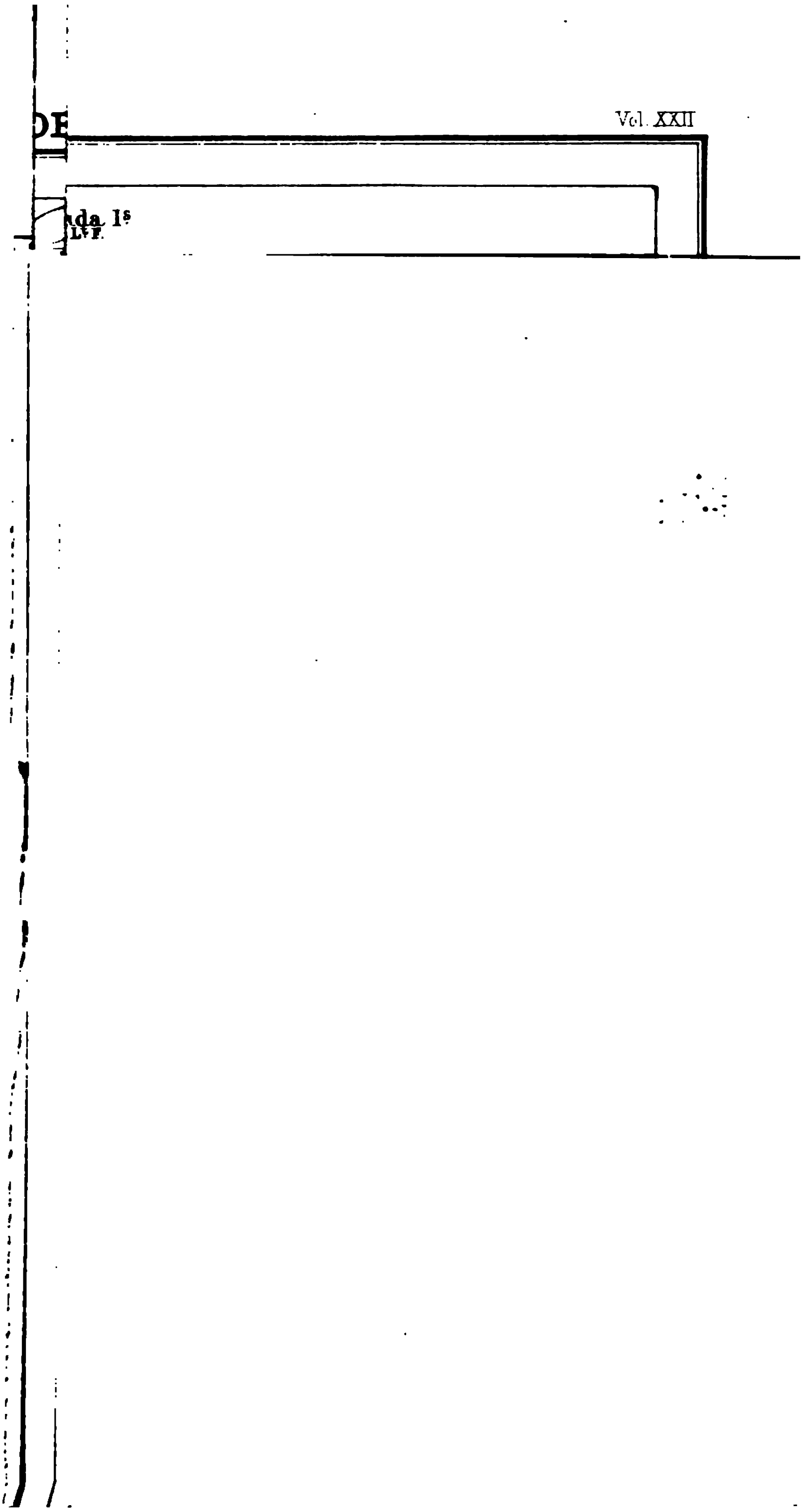
* See Murray, *Bull. Mus. Comp. Zoöl.*, vol. xii. p. 42, 1885; also Agassiz, *Three Cruises of the 'Blake,'* vol. i. p. 276, 1888.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concretions occur all through the Straits of Florida, some of them being fragments of Manatee bones.

Around all coral reefs the deposit consists of Coral Sands and Muds (yellow colour). At some distance from the reefs, and at greater depths, the coral deposits pass into a Pteropod Ooze (pale green colour), and in some cases it is impossible to say whether the sample should be called a Coral Mud or a Pteropod Ooze. Where rivers enter the sea, and generally off coasts where rocks belonging to ancient formations are exposed, the deposit is usually a Blue Mud (blue colour) principally made up of land detritus. In some places the quantity of volcanic detritus is so abundant as to give the deposit the character of Volcanic Mud, as in one sample obtained by the ‘Britannia’ (bright red colour). In the northern part of the Gulf of Mexico, owing to the influence of the Mississippi and other rivers, Blue Mud is found at a great distance from land and in very deep water. In the deepest water of the Caribbean Sea and the Gulf of Mexico, far removed from land, it will be observed that the deposit again passes into Globigerina Ooze.

It should be remarked that the limits of many of these deposits, as laid down on the map, especially in shallow water, are to a large extent hypothetical, owing to the small number of deposits which have been carefully examined up to the present time. Future investigations will no doubt extend the limits of some of the deposits as laid down on the map, especially as regards Globigerina Ooze, Pteropod Ooze, Coral deposits, Blue Mud, and, in fact, all the shallower water deposits.

The Energy of the Röntgen Rays. By Rev. Alexander Moffat, M.A., B.Sc. (*Communicated* by Dr C. G. Knott.)

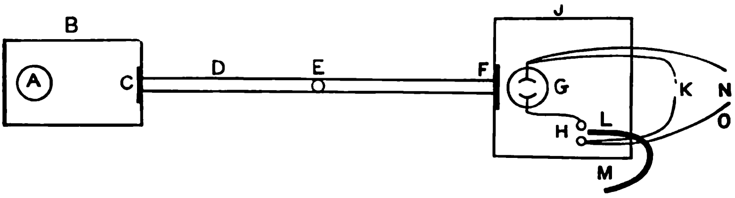
(Read January 9, 1899.)

The number of experiments that have been made on the Röntgen rays is already very large, and much has been learned from them regarding the properties of these rays. There has been, however, as yet, but little done to determine their energy. Indeed, the only accurate work which has been carried out in this direction is that of E. Dorn (*Wied. Ann.*, Bd. 63, p. 160), who measured the heat produced by Röntgen rays falling on a metal plate. This investigation is, however, not complete, as Dorn did not determine the number per second or the duration of the discharges of the Röntgen rays.

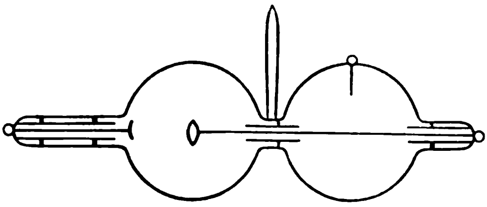
On the suggestion of Professor E. Wiedemann (Erlangen), in whose laboratory I have been working, I have tried to investigate this subject; and, although I have not had time to complete my experiments, as I now find I have to return to India much earlier than I expected, I shall describe the methods adopted and the results obtained, in the hope that they may not be without interest.

I first sought to determine in absolute measure the quantity of light produced on a screen of barium platino-cyanide by Röntgen rays. For this purpose I used an optical bank with the barium platino-cyanide screen and the Röntgen lamp at one end, the source of light which I used as a standard of comparison at the other end, and between them a movable photometer. The arrangement of the apparatus is shown in the following diagram.

The Röntgen lamp which I used was one supplied by the Voltohm-Elektricitäts-Gesellschaft, A. G., Munich. It was as shown in the second diagram, the anti-cathode being a copper hemisphere with a platinum disc for the plane face, set at an angle of 45° to the axis of the lamp, and the vacuum being one suitable for a short spark. The radius of the bulb was 4·2 cm. The current was obtained from a twenty-plate Töpler machine, which was driven by a hydraulic motor made by A. Schmid of Zürich. I regulated the water supply so that the motor ran at the rate of two revolutions per second. In order to obtain as strong Röntgen rays as possible, I introduced an air-spark into the circuit. It has been shown by Dr A. Wehnelt and others that a Röntgen lamp is the more efficient the more abruptly the discharge takes place, and I found this to be the case in my experiments. The discharge in the air-spark was made as disruptive as possible by blowing a strong current of air through it. This drives away the particles of dust in the air, and consequently makes the discharge more sudden.



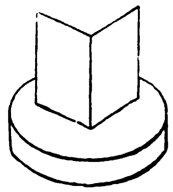
A = Amyl acetate lamp; B = Box to screen off light; C = Aperture in B; D = Optical bank; E = Photometer; F = Barium platino-cyanide screen; G = Röntgen lamp; H = Spark interrupter; J = Box to screen off light; K = Shunt; L = Tube from bellows; M = Bellows; N = Influence machine; O = Motor to drive it.



The Röntgen lamp, with its air-spark, was enclosed in a box with a wooden frame and sides of thick cardboard. The lamp was fixed near one side of the box, and at the place where the rays from the anti-cathode were to be transmitted a hole 6 cm. in diameter was bored and covered with thin black paper. On the other side of this thin black paper the barium platino-cyanide screen was fastened. In front of the screen was fixed a piece of cardboard with a small hole in it, and through this the light-emitting surface of the screen appeared. In order to stop the discharge in the Röntgen lamp when I wished to do so without stopping the motor, I introduced a shunt, consisting of an air-spark, parallel to the lamp.

A Hefner lamp was used as the standard of comparison. As preliminary experiments showed that the light proceeding from the barium platino-cyanide screen was very faint, the light coming from the Hefner lamp had to be weakened considerably. This was done by putting the lamp inside a wooden box, provided with tubes for the entrance of fresh air and the escape of the combustion products. A hole was made in front of the box, and covered with translucent paper of as uniform texture as could be procured. In front of this, again, was put a piece of green glass, and as this did not weaken the light enough, another piece of somewhat darker smoked glass was put in front of it. The Hefner lamp was placed sufficiently far behind the paper to secure that it was practically uniformly illuminated, and as in the case of the barium platino-cyanide screen, a piece of cardboard with a small hole in it was set in front to give the light-emitting surface a convenient magnitude.

In the choice of the photometer to be employed, I was restricted by the very small luminosity to be measured. I could not, therefore, adopt either Joly's paraffin photometer or the photometer of Lummer and Brodhun. I used simply two mirrors set at right angles to one another, and in this way lost very little of the light. At the barium platino-cyanide screen and the Hefner lamp, I used apertures of different sizes, varying from 5 cm. to 1·5 cm. in diameter, in order that the images seen in the photometer might appear about the same size in whatever position on the optical bank it might happen to be.



To determine the quantity of light emitted by the translucent paper, which served as the source of light when illuminated by the Hefner lamp, I put at one end of the optical bank a Hefner lamp, and, at the other end, the box with the translucent paper thus illuminated. The photometer I now used was that of E. W. Lehmann, described in *Wied. Ann.*, Bd. 49, p. 672, as it gives two uniformly illuminated surfaces, which can be readily compared when the sources of light are strong enough. I found that the light emitted by the paper was one-twentieth of that of the Hefner lamp. I then removed the paper, and determined the extinction coefficient of the green glass and the darker glass by measuring the intensity of the light transmitted by them in comparison with that of the lamp alone. I used four different pieces of dark glass (A, B, C, D) in my experiments, and found that the intensity of the light coming from the translucent paper, when weakened by transmission through the green glass, and one of these pieces of dark glass was,

When A was used, $\frac{1}{5000}$ Hefner lamp.
" B " $\frac{1}{11.000}$ " "
" C " $\frac{1}{16.000}$ " "
" D " $\frac{1}{50.000}$ " "

When we seek to compare the energy of the Röntgen rays with that of light rays, we must determine how much energy is supplied by them in equal times. Now, the light of the Hefner lamp is emitted during the whole time it is burning, but the Röntgen rays only during very short intervals of time. I therefore tried to get at least an upper limit to the duration of these intervals. This I did by putting in front of the barium platino-cyanide screen an opaque screen with a small vertical slit, and viewing the image of the slit in a rotating mirror. The mirror was driven by a centrifugal machine, and the image viewed directly by the eye. Now, since the barium platino-cyanide screen does not show any appreciable phosphorescence, it follows that if the emission of the Röntgen rays continues for a certain time, we should see a broadening of the image in the mirror.* Had there been a broadening of one-tenth of the image, it could have been detected; but no such broadening was observed. Now, since the slit was 3 mm. broad, and the mirror exactly opposite it at a distance of 27·5 cm., and revolving at the rate of about ten revolutions per second, it follows that, if the image were broadened one-tenth part, the duration of the radiation must have been about

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* It is clear that, if the emission of light from the screen continued after the Röntgen rays had ceased to fall upon it, we could not use the above-mentioned method, as there would be a broadening of the image, owing to this phosphorescence. This would be the case if barium platino-cyanide phosphoresced, *e.g.*, like the uranium salts or Balmain's luminescent paint.

$$\frac{1}{2} \times \frac{1}{10} \times \frac{1}{10} \times \frac{3}{273} \times \frac{7}{44} = \frac{1}{120.000} \text{ second.}$$

We may, therefore, conclude that the duration of the Röntgen rays produced by each discharge cannot be greater than $\frac{1}{100.000}$ second, and shall use this as an upper limit.

This number is less than any given by those who have previously determined the duration of the Röntgen rays.

A. Roiti (*Rendiconti della R. Acc. dei Lincei*, vol. v. p. 243), who seems to have been the first to attempt to do this accurately, used a Ruhmkorff coil with a rotating interrupter to make and break the current in the primary circuit. On the interrupter he mounted a photographic plate, and in front of it had a screen of lead with a slit in it. The Röntgen rays passing through the slit produced a photograph of it on the plate, which appeared broadened into a small sector of a circle. From the amount of broadening, Roiti concluded that the duration of a discharge was about $\frac{1}{600}$ second.

Dr E. Trouton (*Report of the British Association, Liverpool*, 1896) adopted the method of rotating a zinc-toothed wheel between the Röntgen lamp and the photographic plate. A photograph of the moving teeth was obtained by making one interruption of the primary current in the induction coil, and so allowing one discharge to pass. The departure from sharpness of outline of the photograph of the teeth indicated the time that the radiation lasted. The results obtained in this way varied from $\frac{1}{800}$ to $\frac{1}{10.000}$ second.

M. Colardeau (*Éclairage électrique*, vol. viii. p. 112), using a similar method, found the duration to be about $\frac{1}{1000}$ second.

H. Morize (*Comptes rendus*, vol. cxxvii. p. 546), like Roiti, makes the photographic plate revolve behind a slit in a metal screen. The plate is fixed to one end of the axle of an electromotor, and at the other end is a toothed wheel which interrupts an electric current, and makes a contact which registers itself as well as the mean seconds of an electric chronometer on the band of a Breguet chronograph. He is thus able to determine the speed of the motor at any moment. From the broadening of the image of the slit he deduces the duration of the emission of the Röntgen rays, and gives as the mean result 0·00109 second.

Now, with reference to all these experiments, it has to be observed that they were made with induction coils. A great difficulty in the way of using an induction coil for such an investigation is that, owing to the nature of the apparatus, each discharge given by a coil lasts for a certain time. This is not the case, or, at least, is so to a very much less extent with influence machines. The discharges from these machines being, because of the small capacity, practically instantaneous, experiments conducted with them give much clearer results.

To get the whole duration of the discharge of the Röntgen rays in one second, we must multiply the duration of each discharge by the number of discharges. The method adopted for determining the number of discharges was that given long ago by Professor E. Wiedemann (*Wied. Ann.*, 10, p. 210). In the circuit containing the Röntgen lamp and its air-spark, the length of the spark being 14 mm., a Geissler tube (H) was set upright outside the box. Parallel to it was fixed another Geissler tube (N) excited by an induction coil. The interruptions of the primary current in the coil were made by a vibrating tuning-fork which made 100 vibrations per second. By viewing the images of the Geissler tubes in a rotating mirror, one could easily determine the relation between the number of discharges in the tube H and those in the tube N.* The number so obtained was about 90. The whole duration of the emission of the Röntgen rays is, therefore, not more than $90 \times \frac{1}{100,000}$, *i.e.*, $\frac{1}{1000}$ of the whole time which elapses.

My experiments gave the following results:—

Let a = length of air-spark in millimetres.
b = the extent to which the light from the Hefner lamp was weakened.
c = distance in centimetres of the photometer from the paper in front of the Hefner lamp.

* Between the bright images of the tube H, due to the discharges which gave rise to the Röntgen rays in the Röntgen lamp, I observed about half-a-dozen faint images. These became fewer in number as the length of the air-spark was decreased, and were probably due to discharges from the walls of the Röntgen lamp.

d = distance in centimetres of the photometer from the barium platino-cyanide screen.

e = Luminosity of the screen as compared with the Hefner lamp deduced from b, c, and d.

a	b	c	d	e
14	A	80	118	4×10^{-4}
13	A	97	101	2 ,,
12	A	105	93	1·6 ,,
11	A	120	78	0·8 ,,
10	B	105	93	0·7 ,,
9	B	116	82	0·5 ,,
8	C	109	89	0·4 ,,
7	C	117	81	0·3 ,,
6	D	140	58	0·03 ,,
5	...	...	...	very faint.

The luminosity of the screen is thus about 4×10^{-4} Hefner lamp when the air-spark is 14 mm., and diminishes rapidly as the air-spark is shortened.

In these experiments I have measured the intensity of the rays emitted by only one screen. I shall take it to correspond to the whole energy of the Röntgen rays, *i.e.*, as if the whole of the Röntgen rays were absorbed by the barium platino-cyanide screen. I shall do this because I found that when I put a second screen before the first, its brightness was only one-eighth of that of the first. The correction to be made for the Röntgen rays not transformed into light by the first screen would thus be only 10 per cent., a number which we may here neglect, as being of the order of the experimental errors.

In order to compare directly the intensity of the Röntgen radiation with that of the Hefner lamp, we must bear in mind that the Röntgen rays come from the anti-cathode, and produce a luminescence that proceeds in a diffused way in all directions from the luminescent screen. If, therefore, the distance of the anti-cathode from the screen is r, each part of the screen gets an intensity proportional to $\frac{1}{r^2}$ of the intensity of the whole radiation. Now the radius of the bulb of the Röntgen lamp was 4·2 cm., and its distance from the screen 2 cm., so that the distance of the anti-cathode from the screen was 6·2 cm. If, then, the screen had been at a distance of 1 cm. from the anti-cathode, the intensity of the radiation would have been $(6 \cdot 2)^2$, *i.e.*, 38·4 times the intensity stated above, viz., $38 \cdot 4\ (4 \times 10^{-4}) = 154 \times 10^{-4}$ Hefner lamp.

If we admit that the whole energy of the Röntgen rays is transformed into light energy, the intensity of the Röntgen rays would be 154×10^{-4} Hefner lamp. Now, the energy of the light emitted by the Hefner lamp in one second is 0·189 watt. Therefore the part of the energy of the Röntgen rays transformed every second into luminescence is 0·002911 watt. Now, we have found that the luminescence lasts for only $\frac{1}{1000}$ of the whole time which elapses. If, then, the energy were emitted at the same rate during the whole time, we would obtain for the energy of the luminescence 2·911 watts. From the experiments of Professor E. Wiedemann we know that the transformation coefficient of radiant energy into luminescence is not more than about 4 per cent. (*Wied. Annal.*, Bd. 37, p. 233; *Sitzungsber. d. phys. med. Gesellsch. zu Erlangen*, 1888). Assuming the same proportion to hold good for the transformation of the energy of Röntgen radiation into luminescence, we find that the Röntgen rays producing the luminescence supply energy at the rate of 73 watts. This energy corresponds to that of 18 gram-calories per sec. Now the energy of the sun's rays falling upon one square centimetre is equal to that of 0·035 g. Cal. per sec., so that if the Röntgen rays were to fall perpendicularly on a square centimetre, they would produce $18 \div 0 \cdot 035$, *i.e.*, 500 times the effect of the sun. Something like the same conclusion is reached if we take the result given by E. Dorn. He finds, for the total radiation during one second, 1·51 mg. Cal. (*Wied. Ann.*, Bd. 63, p. 175). Supposing, then, that the radiation here also lasts for $\frac{1}{1000}$ second, we would get from his determination the value 1·51 g. Cal. per second, which nearly corresponds to that obtained by me. The energy of the cathode rays is much greater than this, being, according to the determination of E. Wiedemann and H. Ebert (*Sitzungsber. d. phys. med. Societät zu Erlangen*, 1891), $1 \cdot 4 \times 10^5$ times that of the sun. The coefficient of transformation of cathode rays into Röntgen rays must, therefore, be very small.

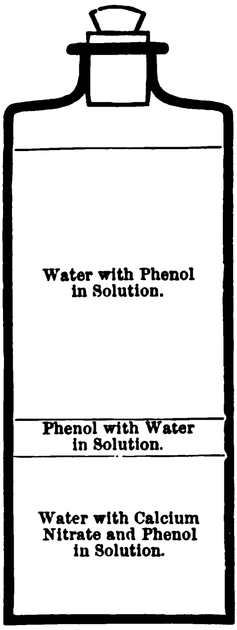
In drawing a conclusion from this investigation, one must bear in mind that no account has been taken of the energy lost by the Röntgen radiation in passing through the walls of the Röntgen lamp, and that a particular coefficient of transformation of energy into luminescence has been assumed. It is evident, however, from the results obtained, that the true average power of the Röntgen rays, *i.e.*, their energy divided by their duration as such, is much greater than generally supposed, and that about ten gram-calories per second may be taken as a lower limit to its magnitude.

To Professor E. Wiedemann and his assistant Dr E. Müller my best thanks are due for the help they so kindly gave me in carrying out this investigation.

On Nernst's "Osmotic Experiment" and a Definition of Osmotic Pressure. By Prof. Crum Brown.

(Read February 6, 1899.)

In vol. vi. of the *Zeitschrift für Physikalische Chemie* (1890), pp. 16–36, Nernst demonstrates the relation between the osmotic pressure of a given solution of N in A and the difference of concentration of two solutions of A in B, the one made by shaking up B with A and the other by shaking up B with the solution of N in A; where A and B are two liquids miscible with each other, but not in all proportions, as, for instance, water and ether, and N a substance soluble in A but not in B. Immediately after this paper, Nernst describes (*l.c.*, pp. 37–40) an osmotic experiment in which the "semipermeable membrane" is a layer of the liquid B held in its place by capillarity. Through this layer no N can pass, because N is insoluble in B, but A will pass from what we may call the A side, on account of the concentration gradient, the layer of B containing more A dissolved in it on the A side than on the solution side. At the same time a pressure is developed on the solution side equal to the osmotic pressure of the solution of N.


Water with Phenol in Solution.
Phenol with Water in Solution.
Water with Calcium Nitrate and Phenol in Solution.

So far as the diffusion of A through the layer of B from the A side to the solution side is concerned, Nernst's experiment can be shown without fixing the layer of B. In the form exhibited to the Society, A is water, B phenol, and N calcium nitrate. The solution was taken of such concentration that its density is considerably greater than that of phenol. It will be seen that the layer of phenol gradually rises in the cylinder.

There is one point not explicitly referred to by Nernst, which is, however, of interest. It is the cause of the stoppage of the diffusion through the layer of B when the difference of pressure on the two sides becomes equal to the osmotic pressure. Increase of pressure increases the solubility of A in B, and when this increase of solubility just balances the diminution of solubility due to the presence of N, the layer of B will become homogeneous, and the gradient of concentration, the cause of the passage of A from the A side, having disappeared, there will be equilibrium.

We may therefore say, if A is shaken up with B under pressure p, and a given solution of N in A is shaken up with B under pressure p', the solutions of A in B formed in the two cases will have the same concentration if $p'-p$ is equal to the osmotic pressure. As it is possible, by optical means, to ascertain when the two solutions have the same concentration, a practical method of determining osmotic pressure might be founded on this relation.

Determination of the Sign of a Single Term of a Determinant. By Thomas Muir, LL.D.

(Read January 9, 1899.)

(1) As is well known, the first rule given for ascertaining the sign of a single term of a determinant was made known by Cramer in his *Introduction à l'Analyse des Lignes Courbes algèbriques*, published at Geneva in 1750. On page 658 he says—

> "On donne à ces termes les signes + ou −, selon la Règle suivante. Quand un exposant est suivi dans le même terme, médiatement ou immédiatement, d'un exposant plus petit que lui, j'apellerai cela un *dérangement*. Qu'on compte, pour chaque terme, le nombre des dérangements: s'il est pair ou nul, le terme aura le signe +; s'il est impair, le terme aura le signe −. Par ex. dans le terme $Z^1Y^2V^3$ il n'y a aucun dérangement: ce terme aura donc le signe +. Le terme $Z^3Y^1X^2$ a aussi le signe +, parce qu'il a deux dérangements, 3 avant 1 & 3 avant 2. Mais le terme $Z^3Y^2X^1$ qui a trois dérangements, 3 avant 2, 3 avant 1, & 2 avant 1, aura le signe −."

According to Cramer, therefore, *If* δ *be the number of* derangements *in the permutation corresponding to any term, the sign of the term is* $(-)^\delta$.

Instead of the word "dérangement," Gergonne * in 1813 used "inversion" in speaking of Cramer's rule: Cauchy † did the same in 1841: and, consequently, the latter term or "inversion of order" has come into pretty general use. "Inverted-pair" is probably a still better expression.‡

(2) The next rule originated with Rothe, having been foreshadowed by him in the second volume of Hindenburg's *Sammlung combinatorisch-analytischer Abhandlungen*, published at Leipzig in

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* *Annales de Math.*, iv. pp. 148–155.

† *Exercices d'analyse et de phys. math.*, ii. pp. 145–150.

‡ *Proc. Roy. Soc. Edin.*, xvi. p. 449.

1800—that is to say, half a century after the appearance of Cramer's.

In § 5, p. 266, he considers a pair of permutations which are so related that the one is got from the other by the interchange of two of the elements of the latter—"zwo Permutationen, welche sich nur dadurch unterscheiden, dass zwey Elemente in ihnen versetzt sind"—and he succeeds in proving that two permutations having this relationship must, according to Cramer's rule, differ in sign. As a corollary to this he asserts that we can easily determine whether two permutations have like or unlike signs by counting the number of interchanges necessary to transform the one into the other. His words are—

> "Da man durch successive Vertauschung zweyer Elemente, von jeder Permutation auf jede kommen kann, so wird man zu bestimmen im Stande seyn, ob zwey gegebene Permutationen einerley oder verschiedene Zeichen haben, nachdem eine gerade oder ungerade Anzahl Vertauschungen dazu erfordert wird. Gesetzt man sollte untersuchen, ob die beyden Permutationen für $r = 10$
>
> $$\begin{array}{ll} & 2,\ 10,\ 9,\ 3,\ 1,\ 7,\ 4,\ 5,\ 8,\ 6 \\ \text{und} & 6,\ 2,\ 7,\ 10,\ 5,\ 3,\ 9,\ 8,\ 1,\ 4 \end{array}$$
>
> einerley oder verschiedene Zeichen haben, so leite man die letztere aus der erstern, durch successive Vertauschung zweyer Elemente (die allemal hier mit Puncten bezeichnet werden mögen) also ab:
>
> $$\begin{array}{l} \dot{2},\ 10,\ 9,\ 3,\ 1,\ 7,\ 4,\ 5,\ 8,\ \dot{6} \\ 6,\ 1\dot{0},\ 9,\ 3,\ 1,\ 7,\ 4,\ 5,\ 8,\ \dot{2} \\ 6,\ 2,\ \dot{9},\ 3,\ 1,\ \dot{7},\ 4,\ 5,\ 8,\ 10 \\ 6,\ 2,\ 7,\ \dot{3},\ 1,\ 9,\ 4,\ 5,\ 8,\ 1\dot{0} \\ 6,\ 2,\ 7,\ 10,\ \dot{1},\ 9,\ 4,\ \dot{5},\ 8,\ 3 \\ 6,\ 2,\ 7,\ 10,\ 5,\ \dot{9},\ 4,\ 1,\ 8,\ \dot{3} \\ 6,\ 2,\ 7,\ 10,\ 5,\ 3,\ \dot{4},\ 1,\ 8,\ \dot{9} \\ 6,\ 2,\ 7,\ 10,\ 5,\ 3,\ 9,\ \dot{1},\ \dot{8},\ 4 \\ 6,\ 2,\ 7,\ 10,\ 5,\ 3,\ 9,\ 8,\ 1,\ 4 \end{array}$$
>
> Da man also durch 8 Vertauschungen aus der ersten die zweyte ableiten kann, so haben beyde einerlei Zeichen, nehmlich — "

Another little step and he might have laid down explicitly the rule :—

If v be the number of interchanges *necessary to transform a given permutation into the standard permutation* (i.e., *the permutation in which the elements occur in their natural order*) *the sign of the given permutation is* $(-)^{v}$.

This step he did not take, although he begins another corollary with the words "da die Permutation 1, 2, 3, . . . , r allemal das Zeichen + hat." Notwithstanding the omission, however, we shall be justified in associating the rule just formulated with Rothe's name.

(3) The third rule appeared twelve years after Rothe's. It is due to Cauchy, and was published in his great "Mémoire sur les fonctions qui ne peuvent obtenir que deux valeurs égales et de signes contraires par suite des transpositions opérées entre les variables qu'elles renferment."[*]

We are now required to count the number of *circular substitutions* which are necessary for the transformation of the given permutation into the standard permutation. Cauchy's words are (p. 56)—

> "Soit
>
> $$a_{\alpha 1}\ a_{\beta 2}\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ a_{\zeta n}$$
>
> le produit symétrique dont il s'agit, et désignons par g le nombre des substitutions circulaires équivalentes à la substitution
>
> $$\begin{pmatrix} 1 & 2 & 3 & \ldots\ldots & n \\ \alpha & \beta & \gamma & \ldots\ldots & \zeta \end{pmatrix}.$$
>
> Ce produit devra être affecté du signe +, si $n-g$ est un nombre pair, et du signe − dans le cas contraire."

For example, if the permutation whose sign is wanted be 6 8 3 1 9 2 5 4 7, we write above[†] it the standard permutation 1 2 3 4 5 6 7 8 9, thus obtaining the substitution

$$\begin{pmatrix} {\nearrow}1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 6 & 8 & 3 & 1 & 9 & 2 & 5 & 4 & 7 \end{pmatrix};$$

---

[*] *Journ. de l'Ec. Polyt.*, x. (*Cah.* xvii.) pp. 29–112.

[†] Or *below* it. The arrow-head is useful in this connection.

decompose this into circular substitutions

$$\begin{pmatrix}1 & 4 & 8 & 2 & 6\\ 6 & 1 & 4 & 8 & 2\end{pmatrix}, \quad \begin{pmatrix}3\\ 3\end{pmatrix}, \quad \begin{pmatrix}5 & 7 & 9\\ 9 & 5 & 7\end{pmatrix};$$

and the number of the latter being 3, and the total number of elements in the permutation being 9, we take for our sign-factor $(-1)^{9-3}$.

Cauchy's rule may thus be formulated:—*If* κ *be the number of* circular substitutions *necessary to transform the given permutation into the standard permutation, and* n *be the number of elements in either permutation, the sign of the given permutation is* $(-)^{n-\kappa}$.

(4) The fourth rule appeared in 1831, its author being J. E. Drinkwater. In his paper "On Simple Elimination" in the *Philos. Mag.*, x. pp. 24–28, he says:—

> "Any permutation may be derived from the first by considering a requisite number of figures to move from left to right by a certain number of single steps or descents of a single place. If the whole number of such single steps necessary to derive any permutation from the first be even, that permutation has a positive sign prefixed to it: the others are negative. For instance, 4 2 1 3 . . . . n may be derived from 1 2 3 4 . . . . n by first causing the 3 to descend below the 4, requiring one single step: then the 2 below the new place of the 4, another single step: lastly, the 1 below the new place of the 2, requiring two more steps, making in all 4. Therefore this permutation requires the positive sign."

This amounts to saying, that, *If* μ *be the number of* moves *necessary to transform the given permutation into the standard permutation, the sign of the given permutation is* $(-)^{\mu}$.

To a certain extent Rothe, to whom the rule of interchanges has been attributed, may be considered a co-discoverer with Drinkwater, one of Rothe's corollaries being (p. 272):—

> "Ensteht eine Permutation dergestalt aus einer andern, dass ein einziges Element aus seiner Stelle genommen, und in eine andere Stelle gesetzt wird, so haben beyde Permutationen einerley Zeichen, wenn der Unterschied der Stellen gerade,

> und verschiedene, wenn er ungerade ist, weil man durch so viel Vertauschungen, als der Unterschied der Stellen beträgt, aus der einen auf die andere kommen kann. So haben also die beyden Permutationen
>
> 4 7 8 10 1 9 5 2 6 3
>
> und
>
> 4 8 10 1 9 5 2 7 6 3
>
> einerley Zeichen, weil blos das Element 7 bey der einen in der zweyten, bey der andern in der achten Stelle steht, denn man kann durch sechs Vertauschungen aus der ersten Permutation die zweyte ableiten."

The question of priority, however, is of very little moment.

(5) The fifth, and perhaps the only other rule, is of quite recent date, viz., 1895, having been given by Mr Morgan Jenkins in the *Messenger of Mathematics*, xxv. pp. 60–68. As in Cauchy's case it is *circular substitutions* that are counted. It may be stated as follows:—*If κ_e be the number of* even circular substitutions *necessary to transform the given permutation into the standard permutation, the sign of the given permutation is* $(-)^{\kappa_e}$. For example, the given permutation being 1 7 3 6 5 4 2 we decompose the substitution

$$\begin{pmatrix}1&7&3&6&5&4&2\\ 1&2&3&4&5&6&7\end{pmatrix}$$

into circular substitutions, neglecting all those which contain an odd number of elements, viz.,

$$\begin{pmatrix}1\\1\end{pmatrix},\quad \begin{pmatrix}3\\3\end{pmatrix},\quad \begin{pmatrix}4&5&6\\5&6&4\end{pmatrix},$$

and keeping count of those which contain an even number of elements; and there being only 1 of the latter, viz.,

$$\begin{pmatrix}2&7\\7&2\end{pmatrix},$$

the sign is $(-)^1$.

(6) The main points connected with the five rules may be summed up as follows:—

Year.	Author.	Things counted.	Symbol for No.
1750	Cramer, .	inverted-pairs (dérangements)	δ
1800	Rothe, . .	interchanges (Vertauschungen)	υ
1812	Cauchy, .	circular substitutions	κ
1831	Drinkwater, .	moves	μ
1895	Jenkins, .	even circular substitutions	κ_e

In the case of the third rule we have to remember that the sign-factor is not $(-)^{\kappa}$ but $(-)^{n-\kappa}$.

Our object is now to give some investigations regarding the five entities specified in the third column of this table. The "Index of Contents" at the end will show the main features of the work.

INVERTED-PAIRS.

(7) As regards "inverted-pairs," it is manifest at the outset that the fewest possible number of them in any case is 0, which happens when the n elements are in their natural order; and that the greatest possible number is $(n-1)+(n-2)+\ldots+2+1$, *i.e.*, $\frac{1}{2}n(n-1)$, which occurs when the natural order is reversed.

Consequently, if $T_{n,r}$ be used to stand for the sum of the terms which have r inverted-pairs, the final expansion of the determinant is

$$T_{n,0} - T_{n,1} + T_{n,2} - \ldots\ldots + (-)_{\frac{1}{2}n(n-1)} T_{n,\frac{1}{2}n(n-1)}.$$

Also, if $V_{n,\delta}$ be used to stand for the *number* of terms which have δ inverted-pairs, we have

$$1.2.3.\ldots.n = V_{n,0} + V_{n,1} + V_{n,2} + \ldots + V_{n,\frac{1}{2}n(n-1)};$$

or, more definitely

$$\tfrac{1}{2}(n!) = V_{n,0} + V_{n,2} + V_{n,4} + \ldots\ldots$$

$$\text{and} \quad \tfrac{1}{2}(n!) = V_{n,1} + V_{n,3} + V_{n,5} + \ldots\ldots$$

(8) The full details of the number of inverted-pairs of a permutation—that is to say, the items which go to constitute δ—may be specified in different orderly ways. Thus, the number of inverted-pairs of the permutation

$$5\ 2\ 4\ 1\ 3$$

may be given as

$$4+1+2+0+0\,,$$

where 4 is the number of inverted-pairs in which the first element 5 of the given permutation comes first, 1 the number in which the second element 2 of the given permutation comes first, and so on; or, it may be given as

$$3+1+2+1+0\,,$$

where 3 is the number of inverted-pairs in which 1 comes second, 1 the number in which 2 comes second, 2 the number in which 3 comes second, and so on.

The former of these two ways is that generally followed; and, clearly, when this is done, the last item is necessarily 0, the second item from the end 1 or 0, the third item from the end 2 or 1 or 0, and so on.

A modification of the former mode is got by arranging the items differently, viz.,

$$4+2+0+1+0\,,$$

where 4 is the number of inverted-pairs in which the highest element 5 comes first, 2 the number in which the next highest element 4 comes first, and so on. If the inverted-pairs be written in rows according to this arrangement, thus

$$\begin{array}{cccc} 51 & 52 & 53 & 54 \\ 41 & . & 43 & . \\ . & . & . & . \\ 21 & . & . & . \end{array}$$

the columns so formed will give the items obtained by the second mode, viz.,

$$3+1+2+1+0\,.$$

(9) If with a view to finding all the possible forms of δ we try to fill n places subject to the conditions that the last place is to be filled with 0, the second from the end by 1 or 0, the third from the end by 2 or 1 or 0, and so on, we see that the filling of the places can be done in $1.2.3.\ldots.\,n$ different ways. And, as this is exactly the number of different permutations of n things, it follows that the determination of the permutation from the detailed specification of the number of its inverted-pairs must be always possible.

The mode of dealing with this converse problem will be understood from an example, say the case where

$$\delta = 2 + 3 + 0 + 1 + 0 .$$

We look to the first item 2 and ask ourselves which of the five elements 1, 2, 3, 4, 5 must be put first so that there may be only two elements less than it to follow after, and the answer is clearly the element 3. Then we turn to the second item of δ, viz., 3, and ask which of the remaining elements 1, 2, 4, 5 must be put next so that there may be exactly three elements less than it to follow after, and the answer is the element 5. Again, we turn to the third item of δ, viz., 0, and ask which of the remaining elements 1, 2, 4 must be put next so that there may be no element less than it to follow after, and the answer is 1. Continuing this procedure we find the required permutation to be

$$3 \; 5 \; 1 \; 4 \; 2 .$$

(10) Taking the case of $n = 4$ and arranging the permutations in order, with the appropriate detailed value of δ opposite each, we have the following table:—

Ordinal No.	Permutation.	No. of Inverted-Pairs.	
1	1234	0+0+0+0 *i.e.,*	0
2	1243	0+0+1+0	1
3	1324	0+1+0+0	1
4	1342	0+1+1+0	2
5	1423	0+2+0+0	2
6	1432	0+2+1+0	3
7	2134	1+0+0+0	1
8	2143	1+0+1+0	2
9	2314	1+1+0+0	2
10	2341	1+1+1+0	3
11	2413	1+2+0+0	3
12	2431	1+2+1+0	4
13	3124	2+0+0+0	2
14	3142	2+0+1+0	3
15	3214	2+1+0+0	3
16	3241	2+1+1+0	4
17	3412	2+2+0+0	4
18	3421	2+2+1+0	5
19	4123	3+0+0+0	3
20	4132	3+0+1+0	4
21	4213	3+1+0+0	4
22	4231	3+1+1+0	5
23	4312	3+2+0+0	5
24	4321	3+2+1+0	6

(11) A glance at the details under the heading "No. of Inverted-Pairs" shows that the first column of items consists of 6 (*i.e.*, 3!) zeros, 6 ones, 6 twos, 6 threes; that the second column consists of 2 (*i.e.*, 2!) zeros, 2 ones, 2 twos, followed repeatedly by the same; and that the third column consists of 1 (*i.e.*, 1!) zero, 1 one, followed repeatedly by the same. The explanation of this is that if in the case of the first, second, third, or fourth set of six permutations we strike off the first column of items of the inverted-pairs we obtain the reduced column

$$\begin{array}{c} 0+0+0 \\ 0+1+0 \\ 1+0+0 \\ 1+1+0 \\ 2+0+0 \\ 2+1+0, \end{array}$$

which are the numbers of inverted-pairs for the permutations of 234, or 134, or 124, or 123; and that if we treat this reduced column in the same way we obtain

$$\begin{array}{c} 0+0 \\ 1+0, \end{array}$$

which are the numbers of inverted-pairs for the permutations of 12.

(12) It follows from this that each of the integers from 1 up to $n!$ has corresponding to it a special form of δ, that therefore when any integer is given we ought to be able to determine the corresponding value of δ in full detail, and conversely, that when the details of δ are given the number of the corresponding permutation should be obtainable.

The theorem which effects the latter determination is—

If for the N^{th} *permutation of* n *elements the number of inverted-pairs be*

$$a_1 + a_2 + a_3 + \ldots\ldots + a_n$$

then

$$\mathrm{N} - 1 = a_1.(n-1)! + a_2.(n-2)! + \ldots\ldots + a_{n-1}.1! + a_n.$$

Let all the $n!$ permutations be arranged in order in a column, and opposite each permutation in another column the corresponding value of δ in detail, as has been done in § 10 for the case of $n=4$. Then it will be seen that the first permutation whose detailed value of δ begins with the item a_1 is preceded by a_1 sets of permutations, each set containing $(n-1)!$ permutations, so that there must precede the permutation in question $a_1.(n-1)!$ permutations at least. Following up the matter, however, we shall find for the same reason that the existence of the item a_2 implies that $a_2.(n-2)!$ more permutations precede the said permutation, and similarly in the case of a_3, a_4, . . . Consequently the number of this permutation is 1 more than

$$a.(n-1)! + a_2.(n-2)! + \ldots + a_{n-1}.1! + a_n$$

as was to be proved.

For example, the number of inverted-pairs being

$$2+3+0+1+0$$

as in § 9, we multiply 2 by 4!, 3 by 3!, and 1 by 1!, take the sum of the products, and thus find the permutation to be the 67th.

From this it is evident that if, on the other hand, the ordinal number of the permutation be known, we have only to take the integer next less than it, divide this integer by 2 and note the remainder, then divide the integral part of the quotient by 3 and note the remainder, then divide the integral part of the new quotient by 4 and note the remainder, and so on. This being done, the remainders are, when reversed in order, the items which constitute δ for the permutation in question. For example, if it were the 54th permutation, the process would be

$$\begin{array}{r|l} 2 & \underline{53} \\ 3 & \underline{26},\ 1 \\ 4 & \underline{\ 8},\ 2 \\ 5 & \underline{\ 2},\ 0 \\ & \ 0,\ 2 \end{array}$$

and $2+0+2+1+0$ would be the required number of inverted-pairs in detailed form.

(13) Looking again to the table of § 10 we see that for any one of the last six permutations of 1 2 3 4 the number of inverted-

pairs is 1 more than for the corresponding one of the third set of six permutations, 2 more than for the corresponding one of the second set of six permutations, and 3 more than for the corresponding one of the first set, which first set consists simply of the permutations of 1 2 3. Further, for either of the last two permutations of 1 2 3, the number of inverted-pairs is 1 more than for the corresponding one of the second set of two permutations, and 2 more than for the corresponding one of the first set, which first set consists simply of the permutations of 1 2.

It follows, therefore, that from the numbers of inverted-pairs for the case of the permutations of 1 2, viz.,

0, 1,

we can write the numbers of inverted-pairs for the case of the permutations of 1 2 3, viz.,

0, 1,
1, 2,
2, 3;

and that from these again we can write the numbers of inverted-pairs for the case of the permutations of 1 2 3 4, viz.,

0, 1, 1, 2, 2, 3, 3, 4,
1, 2, 2, 3, 3, 4, 4, 5,
2, 3, 3, 4, 4, 5, 5, 6.

Consequently, if we denote by

$$V_{n,\delta}$$

the number of permutations of 1 2 3 . . . n which have δ inverted-pairs, we have

$$V_{2,0}=1,\quad V_{2,1}=1,$$

$$V_{3,0}=1,\quad V_{3,1}=2,\quad V_{3,2}=2,\quad V_{3,3}=1,$$

$$V_{4,0}=1,\quad V_{4,1}=3,\quad V_{4,2}=5,\quad V_{4,3}=6,\quad V_{4,4}=5,\quad V_{4,5}=3,\quad V_{4,6}=1.$$

(14) Since in the case of the standard permutation 1 2 3 . . . n

none of the $\frac{1}{2}n(n-1)$ pairs of elements is inverted, and in the case of $n, n-1, \ldots, 2, 1$ all of them are, it follows that, if we transform $1\ 2\ 3 \ldots n$ so as to obtain a permutation with δ inverted-pairs, the same transformation made upon $n, n-1, \ldots, 2, 1$ will produce a permutation with δ uninverted-pairs, and therefore with $\frac{1}{2}n(n-1)-\delta$ inverted-pairs. This implies that for every permutation of $1\ 2\ 3 \ldots n$ with δ inverted-pairs, there must thus be a permutation of $n n-1 \ldots 2\ 1$ with $\frac{1}{2}n(n-1)-\delta$ inverted-pairs. But the permutations of $1\ 2\ 3 \ldots n$ are exactly the permutations of $n n-1 \ldots 2\ 1$; hence

$$V_{n,\delta} = V_{n,\frac{1}{2}n(n-1)-\delta},$$

as is seen in the preceding § to be the case for $V_{2,\delta}$, $V_{3,\delta}$, $V_{4,\delta}$.

(15) Having got in § 10 the numbers of inverted-pairs for the 24 permutations of 1 2 3 4, viz., 0, 1, 1, 2, 2, 3, . . ., we can immediately write the numbers for the 120 permutations of 1 2 3 4 5. All that is necessary is to copy out in one column the 24 numbers referred to, and then make four adjacent columns with each component number of any of the four greater by 1 than the corresponding number in the immediately preceding column: thus—

0	1	2	3	4
1	2	3	4	5
1	2	3	4	5
2	3	4	5	6
2	3	4	5	6
3	4	5	6	7
1	2	3	4	5
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

It follows, of course, from this mode of constructing the five columns, that if we wish to know how often any number, say 7, occurs in the complete set of columns, we have only got to ascertain from previous work how often 7 occurs in the first column, and then recall the fact that 7 occurs in the second column exactly as often as 6 occurs in the first, that 7 occurs in the third column exactly as often as 5 occurs in the first, and so on. Consequently, denoting as before by $V_{5,7}$ the number of permutations of 1 2 3 4 5 in which there are 7 inverted-pairs, we have

$$\begin{aligned} V_{5,7} &= V_{4,7} + V_{4,6} + V_{4,5} + V_{4,4} + V_{4,3}\,, \\ &= 0 + 1 + 3 + 5 + 6\,, \\ &= 15\,; \end{aligned}$$

or, using the theorem of § 14,

$$\begin{aligned} V_{5,7} &= V_{5,3}\,, \\ &= V_{4,3} + V_{4,2} + V_{4,1} + V_{4,0}\,, \\ &= 6 + 5 + 3 + 1\,, \\ &= 15\,. \end{aligned}$$

Making now one continuous column of the above 120 numbers by putting the second column under the first, the third under the new position of the second, and so on, we should obtain the first 120 of the 720 numbers of inverted-pairs wanted for the case of the permutations of 1 2 3 4 5 6, the 600 others being got by forming five adjacent columns as before. It is clear, therefore, that generally we have

$$V_{n,\delta} = V_{n-1,\delta} + V_{n-1,\delta-1} + V_{n-1,\delta-2} + \ldots + V_{n-1,\delta-n+1}\,,$$

the number of terms on the right being n.

(16) With this difference-equation, and the knowledge that $V_{n,0} = 1$ and $V_{1,\delta} = 0$ (except when $\delta = 0$), the accompanying table is easily constructed.

[TABLE.

TABLE OF $V_{n,\delta}$, THE NUMBER OF PERMUTATIONS OF 1, 2, 3, . . ., n WHICH HAVE δ INVERTED-PAIRS.

$n=$	1	2	3	4	5	6	7	8	9	10	11
$\delta =$											
0	1	1	1	1	1	1	1	1	1	1	1
1		1	2	3	4	5	6	7	8	9	10
2			2	5	9	14	20	27	35	44	54
3			1	6	15	29	49	76	111	155	209
4				5	20	49	98	174	285	440	649
5				3	22	71	169	343	628	1068	1717
6				1	20	90	259	602	1230	2298	4015
7					15	101	359	961	2191	4489	8504
8					9	101	455	1415	3606	8095	16599
9					4	90	531	1940	5545	13640	30239
10					1	71	573	2493	8031	21670	51909
11						49	573	3017	11021	32683	84591
12						29	531	3450	14395	47043	131625
13						14	455	3736	17957	64889	&c.
14						5	359	3836	21540	&c.	
15						1	259	3736	24584		
16							169	3450	27073		
17							98	3017	28675		
18							49	2493	29228		
19							20	1940	28675		
20							6	1415	27073		
21							1	961	24584		
22								602	21540		
23								343	17957		
etc.								etc.	etc.		

The practical rule corresponding to the difference-equation will be found to be:—*To find any number in any column, say the column headed "5," add five consecutive numbers of the preceding column, beginning at the corresponding place in the latter column.* For example,

$$\begin{aligned} V_{5,8} &= 0+0+1+3+5\,, \\ &= 9. \end{aligned}$$

(17) Knowing that

$$V_{n,\delta} = V_{n-1,\delta} + V_{n-1,\delta-1} + V_{n-1,\delta-2} + \dots + V_{n-1,\delta-n+1}$$

we have, of course,

$$V_{n,\delta-1} = \qquad V_{n-1,\delta-1} + V_{n-1,\delta-2} + \dots\dots + V_{n-1,\delta-n},$$

and therefore by subtraction

$$V_{n,\delta} - V_{n,\delta-1} = V_{n-1,\delta} - V_{n-1,\delta-n}.$$

This means that the series of first differences of the numbers in the column headed "n" are the same as a certain other series of differences in the preceding column. This renders the construction of the table still easier.

(18) No reasonably simple solution of the difference equation

$$V_{n,\delta}=V_{n,\delta-1}+V_{n-1,\delta}-V_{n-1,\delta-n}$$

seems attainable.

The first form (§ 15) of the equation, however, and especially that form when put as a practical rule (§ 16), suggests another mode of dealing with the problem. For, when each member of a series of numbers is got from another series by adding a fixed number of consecutive members of the latter, the process is exactly similar to the multiplication of the sum of the members of the series by $1+1+1+\dots$ Beginning, therefore, with the only number in the first column, viz., 1, and multiplying by $1+1$, then multiplying the product by $1+1+1$, and so forth, we reproduce the columns of the table with ease. Thus—

$$\begin{array}{lll}
 & 1 & \text{1st column} \\
\text{Multiply by} & \underline{1+1} & \\
 & 1+1 & \text{2nd column} \\
\text{Multiply by} & \underline{1+1+1} & \\
 & 1+1 & \\
 & 1+1 & \\
 & \underline{1+1} & \\
 & 1+2+2+1 & \text{3rd column} \\
\text{Multiply by} & \underline{1+1+1+1} & \\
 & 1+2+2+1 & \\
 & 1+2+2+1 & \\
 & 1+2+2+1 & \\
 & \underline{1+2+2+1} & \\
 & 1+3+5+6+5+3+1 & \text{4th column.}
\end{array}$$

(19) On the face of this process there is witness to the truth of the theorem of § 7, viz., that

$$V_{n,0}+V_{n,1}+V_{n,2}+\dots+V_{n,\frac{1}{2}n(n-1)}=n\,!$$

for, the successive results being equal to

$$\begin{array}{l} 1\,(1+1)\,, \\ 1\,(1+1)(1+1+1)\,, \\ 1\,(1+1)(1+1+1)(1+1+1+1)\,, \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}$$

are equal to

$$2\,!\ ,\ 3\,!\ ,\ 4\,!\ ,\ \ldots .$$

(20) It is only a variant of the statement in § 18 to say that the numbers of the 2nd, 3rd, 4th, ... columns of the table are the coefficients in the expansions of

$$\begin{array}{l} 1+x \\ (1+x)(1+x+x^2) \\ (1+x)(1+x+x^2)(1+x+x^2+x^3) \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{array}$$

respectively. Consequently we have the result

$$V_{n,\delta} = \text{coefficient of } x^{\delta} \text{ in the expansion of}$$

$$(1+x)(1+x+x^2)(1+x+x^2+x^3)\ldots(1+x+x^2+\ \ldots\ +x^{n-1})$$

or of

$$(1-x)(1-x^2)(1-x^3)\ \ldots .\ (1-x^n)/(1-x)^n.$$

Taking the case of $n=4$ as an example, we have

$$\begin{array}{l} (1-x)(1-x^2)(1-x^3)(1-x^4)\cdot(1-x)^{-4} \\ =(1-x-x^2+2x^5-x^8-x^9+x^{10}) \\ \quad\cdot(1+C_{4.1}x+C_{5.2}x^2+C_{6.3}x^3+\cdots\quad) \end{array}$$

$$\begin{array}{llll} = & 1 & = & V_{4.0} \\ & +(C_{4.1}-1)x & & +V_{4.1}\cdot x \\ & +(C_{5.2}-C_{4.1}-1)x^2 & & +V_{4.2}\cdot x^2 \\ & +(C_{6.3}-C_{5.2}-C_{4.1})x^3 & & +V_{4.3}\cdot x^3 \\ & +(C_{7.4}-C_{6.3}-C_{5.2})x^4 & & +\ldots . \\ & +(C_{8.5}-C_{7.4}-C_{6.3}+2)x^5 & & \\ & +(C_{9.6}-C_{8.5}-C_{7.4}+2C_{4.1})x^6 & & \\ & +(C_{10.7}-C_{9.6}-C_{8.5}+2C_{5.2})x^7 & & \\ & +\ldots\ldots\ldots & & \end{array}$$

so that

$$V_{4.\delta}=C_{3+\delta,3}-C_{2+\delta.3}-C_{1+\delta.3}+2C_{\delta-2,3}.$$

And, since the coefficients of all powers of x higher than the 6th must vanish, we have also

$$0=C_{r+9,3}-C_{r+8,3}-C_{r+7,3}+2C_{r+4,3}-C_{r+1,3}-C_{r,3}+C_{r-1,3}$$

for all positive integral values of r: and consequently the identity

$$\begin{aligned}(r+9)(r+8)(r+7)-(r+8)(r+7)(r+6)-(r+7)(r+6)(r+5)\\+2(r+4)(r+3)(r+2)-(r+1)(r)(r-1)-r(r-1)(r-2)\\+(r-1)(r-2)(r-3)=0.\end{aligned}$$

For the case of $n=3$ the three results similar to these are

$$V_{3,\delta}=C_{\delta+2,2}-C_{\delta+1,2}-C_{\delta,2},$$

$$0=C_{r+7,2}-C_{r+6,2}-C_{r+5,2}+C_{r+3,2}+C_{r+2,2}-C_{r+1,2},$$

$$\begin{aligned}0=(r+7)(r+6)-(r+6)(r+5)-(r+5)(r+4)\\+(r+3)(r+2)+(r+2)(r+1)-(r+1)r.\end{aligned}$$

There is little hope, however, of generalisation, the difficulty lying in the fact that the expansions of

$$1-x,\ (1-x)(1-x^2),\ (1-x)(1-x^2)(1-x^3),\ \ldots\,.$$

do not proceed in accordance with a sufficiently simple law. Thus after

$$V_{3,\delta}=C_{\delta+2,2}-C_{\delta+1,2}-C_{\delta.2},$$

$$V_{4,\delta}=C_{\delta+3,3}-C_{\delta+2,3}-C_{\delta+1,3}+2C_{\delta-2,3},$$

we find

$$\begin{aligned}V_{5,\delta}=C_{\delta+4,4}-C_{\delta+3,4}-C_{\delta+2,4}+C_{\delta-1,4}+C_{\delta-2,4}\\+C_{\delta-3,4}-C_{\delta-4,4}-C_{\delta-5,4}-C_{\delta-6,4}.\end{aligned}$$

(21) The case where $\delta=n$ is interesting, as then the aggregate of the first three terms may be replaced by one term, and there is no variation in the expression as we proceed from case to case, except through the appearance of an additional term. Thus—

$$\begin{aligned}V_{3.3}&=C_{5.2}-C_{4.2}-C_{3.2}=C_{4.1}-C_{3.2}\\&=C_{3.3}\\V_{4.4}&=C_{5.4}\\V_{5.5}&=C_{7.5}+C_{4.4}\\V_{6.6}&=C_{9.6}+C_{6.5}\end{aligned}$$

$$\begin{aligned}
V_{7.7} &= C_{11.7} + C_{8.6} + C_{6.6} \\
V_{8.8} &= C_{13.8} + C_{10.7} + C_{8.7} \\
V_{9.9} &= C_{15.9} + C_{12.8} + C_{10.8} \\
V_{10.10} &= C_{17.10} + C_{14.9} + C_{12.9} \\
V_{11.11} &= C_{19.11} + C_{16.10} + C_{14.10} \,.
\end{aligned}$$

From a consideration of these it might possibly occur that the general expression for $V_{n,n}$ was

$$C_{2n-3,n} + C_{2n-6,n-1} + C_{2n-8,n-1} .$$

The very next case, however, would serve to disprove it, for

$$V_{12.12} = C_{21.12} + C_{18.11} + C_{16.11} - C_{11.11} ,$$

the expansion of $(1-x)(1-x^2) \ldots . (1-x^{12})$ being

$$1 - x - x^2 + x^5 + x^7 - x^{12} + \ldots .$$

and $\therefore$

$$V_{12,\delta} = C_{\delta+11,11} - C_{\delta+10,11} - C_{\delta+9,11} + C_{\delta+6,11} + C_{\delta+4,11} - C_{\delta-1,11} + \ldots$$

and $V_{12.12}$, as stated. The expression given for $V_{n,n}$ should therefore be reinforced by the term $-C_{2n-13,n-1}$ if it is to be correct for $n=12$. And no alteration in this is necessary, save by way of an addendum, if we proceed to make the expression suffice also for $n = 13, 14, \ldots .$ For, having got the expansion of

$$(1-x)(1-x^2)(1-x^3) \ldots . (1-x^{12})$$

to be, as far as is needed for this purpose,

$$1 - x - x^2 + x^5 + x^7 - x^{12} + \ldots .$$

we see that the multiplication of it by $1 - x^{13}$ will not affect the coefficient of x^{12}, the expansion of

$$(1-x)(1-x^2)(1-x^3) \ldots . (1-x^{13})$$

being in fact, as far as is necessary,

$$1 - x - x^2 + x^5 + x^7 - x^{12} + 0 \cdot x^{13} + \ldots$$

from which we have

$$V_{13,\delta} = C_{\delta+12,12} - C_{\delta+11,12} - C_{\delta+10,12} + C_{\delta+7,12} + C_{\delta+5,12} - C_{\delta,12} + 0 \cdot C_{\delta-1,12} + \ldots$$

and $\therefore$

$$\begin{aligned}V_{13.13} &= C_{25.12} - C_{24.12} - C_{23.12} + C_{20.12} + C_{18.12} - C_{13.12}\\ &= C_{23.13} + C_{20.12} + C_{18.12} - C_{13.12}.\end{aligned}$$

By proceeding as far as the factor $1 - x^{20}$ it is found that

$$V_{n,n} = C_{2n-3,n} + C_{2n-6,n-1} + C_{2n-8,n-1} - C_{2n-13,n-1} - C_{2n-16,n-1} + \dots\dots$$

(22) If δ be less than n, the last term $V_{n-1,\delta-n}$ of the difference-equation (§ 17) is of no moment, and the equation to be satisfied is

$$V_{n.\delta} = V_{n,\delta-1} + V_{n-1.\delta}.$$

Now it is easily shown that one form of solution for this is an aggregate of multiples of combinatorials, that is to say, we may have

$$V_{n,\delta} = A_1 \cdot C_{n+\delta+a_1,\delta+b_1} + A_2 \cdot C_{n+\delta+a_2,\delta+b_2} + \dots$$

where $A_1, A_2, \dots, a_1, a_2, \dots, b_1, b_2, \dots$ are constants with regard to n and δ. Thus, on substitution in the right hand side of the difference-equation, we have

$$\begin{aligned}V_{n,\delta-1} + V_{n-1,\delta} = {} & A_1 \cdot C_{n+\delta-1+a_1.\delta-1+b_1} + A_2 \cdot C_{n+\delta-1+a_2.\delta-1+b_2} + \dots\\ & + A_1 \cdot C_{n+\delta-1+a_1,\delta+b_1} \quad + A_2 \cdot C_{n+\delta-1+a_2,\delta+b_2} \quad + \dots\end{aligned}$$

and this by repeated applications of the theorem

$$C_{p,q} = C_{p-1,q} + C_{p-1,q-1}$$

is clearly

$$= A_1 \cdot C_{n+\delta+a_1,\delta+b_1} + A_2 \cdot C_{n+\delta+a_2,\delta+b_2} + \dots$$

and $\therefore$

$$= V_{n,\delta}.$$

It follows from this that if in the tables of values of $V_{n,\delta}$ the diagonal

$$0,\ 0,\ 1,\ 5,\ 22,\ 90, \dots\dots$$

which consists of the values of $V_{n,n}$ be expressible as an aggregate of combinatorials, all the values of $V_{n,\delta}$ on the upper side of this diagonal will be expressible in the same way. Now such an aggregate is

$$C_{n+\delta-2,\delta}-C_{n+\delta-3,\delta-2}+C_{n+\delta-6,\delta-5}+C_{n+\delta-8,\delta-7}- \;\ldots\,.$$

which when $\delta=n$ becomes

$$C_{2n-2,n}-C_{2n-3,n-2}+C_{2n-6,n-5}+C_{2n-8,n-7}-\ldots\,.\,.$$

or

$$C_{2n-3,n}+C_{2n,-6,n-1}+C_{2n-8,n-1}-\ldots\,.\,.$$

as it should be. Hence

$$C_{n+\delta-2,n-2}-C_{n+\delta-3,n-1}+C_{n+\delta-6,n-1}+C_{n+\delta-8,n-1}-C_{n+\delta-13,n-1}-\ldots\,.$$

is the correct expression of $V_{n,\delta}$ so long as $\delta \not> n$.

For example—

$$\begin{aligned} V_{11,8} &= C_{17,9}-C_{16,6}+C_{13,3}+C_{11,1}, \\ &= 24310-8008+286+11, \\ &= 24607-8008, \\ &= 16599. \end{aligned}$$

(23) The generating function of $V_{n,\delta}$ given in § 20 is already well known as the generating function of the numbers of combinations of $1+2+3+\ldots+(n-1)$ things of which one is unique, two are alike but different from the first, three are alike but different from both the first and second sets, and so on. We have therefore the following curious proposition:—

The number of permutations of 1 2 3 . . . n *which have* δ *inverted-pairs is equal to the number of* δ-*combinations of* $\frac{1}{2}$n (n − 1) *letters, one of which is* a, *two of which are* b's, *three of which are* c's, *and so forth.*

If we denote the number of such combinations by

$$C_{1+2+\ldots+(n-1),\delta}$$

the proposition is, in symbols,

$$V_{n,\delta}=C_{1+2+\ldots+(n-1),\delta}\,.$$

(24) The reason of this unlooked-for relationship becomes more apparent when we actually try to form in order the combinations referred to. For the first three cases the results may be tabulated as follows:—

	Things given for Combination.		
	a.	*a,bb.*	*a,bb,ccc.*
(0)	.	.	.
Sets of 1 letter,	*a.*	*a,b.*	*a,b,c.*
,, 2 letters,		*ab,bb.*	*ab,bb* : *ac,bc* : *cc.*
,, 3 letters,		*abb.*	*abb* : *abc,bbc* : *acc,bcc* : *ccc.*
,, 4 letters,			*abbc* : *abcc,bbcc* : *accc,bccc.*
,, 5 letters,			*abbcc* : *abccc,bbccc.*
,, 6 letters,			*abbccc.*

Now it will be found that the combinations in any column here are got from those of the preceding column in a manner closely resembling that specified in the practical rule for the construction of the table of § 16. For example, if we wish to form the combinations of four letters taken from *a,bb,ccc*, we go to the preceding column and take over without change the combinations there given (*i.e.*, none) of four letters, then take the combinations of three letters, viz., *abb*, and annex a *c*, then the combinations of two letters, viz. *ab,bb*, and annex two *c*'s to each, and finally the separate letters *a,b*, and annex three *c*'s to each. The corresponding operation in connection with the table of § 16 is

$$V_{4,4} = 0 + 1 + 2 + 2 = 5.$$

It is thus seen that if the problem be to find not the combinations themselves but the number of them, the difference-equation is exactly the same as before, and the initial conditions are also the same if we make the usual convention that in every case when we omit all the letters one combination is to be counted; that is to say,

$$C_{1+2+\,.\,.\,+(n-1),0} = 1.$$

With the same convention we have quite generally, as is well known,

$$C_{1+2+\ldots+(n-1),\delta}=C_{1+2+\ldots+(n-1),\frac{1}{2}n(n-1)-\delta},$$

and therefore the theorem of § 14

$$V_{n,\delta}=V_{n,\frac{1}{2}n(n-1)-\delta}.$$

(25) There is still another problem which has for its solution exactly the same double series of numbers as we have tabulated in § 16, viz., *the problem of finding the number of positive integral divisors of* $ab^2c^3d^4$ . . . *where* a,b,c,d, . . . *are integers prime to one another.* In fact, the related table in § 24 may be viewed either as a table of combinations, as was intended, or as a table of the said divisors. Of course, for the latter purpose, the insertion of the row of 1's referred to at the close of § 24 is a necessity, 1 being a divisor in every case--the co-factor, in fact, of the divisor of highest degree.

(26) To bring out still more clearly the connection between the three problems, let it be observed that the table of combinations in § 24 is also by implication a table of the permutations of 1 2 3 . . . arranged in order of the number of inverted-pairs which they contain. This fact is not so readily perceived as the identity of the combinations of § 24 with the divisors of § 25, but a little care suffices to establish it.

Taking the third column of combinations or factors, viz. :—

1,
a,b,c.
ab,bb : *ac,bc* : *cc.*
abb : *abc,bbc* : *acc,bcc* : *ccc.*
abbc : *abcc,bbcc* : *accc,bccc.*
abbcc : *abccc,bbccc.*
abbccc.

and viewing each *a*, each *b*, each *c* as representative of an inverted-pair of which 2, 3, 4 are respectively the first element, we translate the column into

1234.
2134,1324,1243.
2314,3124 : 2143,1342 : 1423.
. . . .
4321.

and this new column is seen to contain the 24 permutations of 1 2 3 4 separated into seven sets, viz., those which have *no* inverted-pairs, those which have one, those which have two, and so forth.

(27) A comparison of the number of inverted-pairs in one permutation with the number in the conjugate permutation* was originally made by Rothe in the paper above referred to. The following simple fundamental theorem, however, puts the matter in a fresh light:—

If in any permutation of 1, 2, 3, . . . , n *any one of the elements* λ *be in the* l^{th} *place and any other* μ *be in the* m^{th} *place, then according as* $\lambda\mu$ *is or is not an inverted-pair there will be in the conjugate permutation an inverted-pair* ml *or an uninverted-pair* lm.

To say that $\lambda\mu$ is an inverted-pair of the original permutation is the same as to say that $\lambda > \mu$ and precedes μ, that is, that

$$\lambda > \mu \text{ and } m > l.$$

Now, in the conjugate permutation the element l will be in the λ^{th} place and the element m in the μ^{th} place: consequently, since $\lambda > \mu$ the element m will in that permutation precede the element l: and therefore, since $m > l$, ml will be an inverted-pair. The reasoning is exactly similar when $\lambda\mu$ is an uninverted-pair.

Thus in the permutation

$$3\ 6\ 7\ 8\ 9\ 2\ 1\ 5\ 4$$

75 is an inverted-pair, and since the places of these two elements are respectively the 3rd and 8th, it follows that 83 must be an

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* "Two permutations of the numbers 1, 2, 3, . . ., n are called *conjugate* when each number and the number of the place which it occupies in the one permutation are interchanged in the case of the other permutation." See Muir, "History of Determinants," pp. 59, 60; Muir, "On Self-Conjugate Permutations."—*Proc. Roy. Soc. Edin.*, xvii. pp. 7–13.

inverted-pair of the conjugate permutation. Similarly the uninverted-pair 39 in the given permutation corresponds to the uninverted-pair 15 in the conjugate permutation.

From this it follows that conjugate permutations have the same number of inverted-pairs, and therefore have the same sign—which is Rothe's proposition.

Moves.

(28) The next subject which it is convenient to consider is that of "moves," and at the outset it is important to note that whereas the number of inverted-pairs in any given permutation is definite, the number of moves which may be used to transform a given permutation into the standard permutation is in general indefinite, because of the possibility of proceeding in divers ways in making the moves. If it be agreed, however, to make the moves in orderly fashion, viz., so as to put the elements in order into their standard places, the number is quite definite, and it is this number which μ is used to denote. Thus, for the permutation

$$2\ 4\ 1\ 5\ 3$$

we have $\mu=4$, because to attain the standard permutation it is necessary to put 1 into its standard place—which requires 2 moves, and then 3 into its standard place—which requires other 2.

Of course, we might put the elements *in reverse order* into their standard places, but, as will be seen immediately, the total number of moves would not then be different. Thus, for the permutation 2 4 1 5 3, we should have to put 5 into its standard place, then 4, and then 2, the number of necessary moves thus being $1+2+1$, *i.e.*, 4 as before.

(29) The fundamental proposition in regard to "moves" is the following:—

The number of orderly moves necessary to transform any given permutation of the first n *integers into the standard permutation is equal to the number of inverted-pairs in the former;* i.e., *in symbols,*

$$\mu=\delta\,.$$

In the given permutation let $\alpha,\beta,\gamma,\ldots$ be the numbers of "inverted-pairs" in which 1, 2, 3, . . . are respectively the second elements, so that the total number of inverted-pairs is $\alpha+\beta+\gamma+\ldots$ Now the number of "moves" necessary to transform the permutation into the standard permutation is the number necessary to put 1 into the first place, 2 into the second place, 3 into the third place, and so on. But the number necessary to put 1 into the first place must be α, because from consideration of the inverted-pairs we know that there are exactly α integers preceding 1. Again, the number of "moves" necessary to put 2 into the second place must be β, because at the outset there were β integers greater than 2 preceding 2, and these could not be affected by the movement of 1 into its standard place. Similarly the number of moves necessary to put 3 into the third place is seen to be γ, and so on; so that the total number of "moves" is $\alpha+\beta+\gamma+\ldots$, that is to say, is the same as the number of "inverted-pairs."

(30) If in the preceding proof we had arranged the inverted-pairs differently, viz., if we had begun with those in which the highest element, say 5, came first, then taken those in which 4 came first, and so on, the total number being thus partitioned into $\alpha'+\beta'+\gamma'+\ldots$ we could have proved in the same manner that the number of moves necessary to put 5, 4, 3, . . . into their respective places would have been $\alpha', \beta', \gamma', \ldots$

In this way we see that *the number of moves necessary to transform a given permutation into the standard permutation is the same whether we put the elements in order, or in reverse order, into their standard places.*

(31) Further, *the lowest possible number of moves is secured when the elements are put in order, or in reverse order, into their standard places.*

No move can free us of more than one inverted-pair; and, there being δ inverted-pairs, it follows that the least possible number of moves is δ. But δ, as we have just seen, is the number of moves made when we proceed in orderly fashion; the theorem is thus established.

INTERCHANGES.

(32) Coming now to the subject of "interchanges" we have again to observe that the number of them is indefinite unless they are made in accordance with a more or less orderly plan.

Interchanges may be divided into two kinds, *effective* and *ineffective*, an interchange being *effective* when it is practised upon two elements which are not in their standard places, and, as the result of it, one at least of them is brought into its standard place. An effective interchange may be *singly* or *doubly effective*, according as one or both elements are brought by it into their standard places. Thus, in the permutation

3 6 5 1 4 2 7

the interchange $6 \leftrightarrow 2$ would be doubly effective, $3 \leftrightarrow 1$ singly effective, $4 \leftrightarrow 2$ and $2 \leftrightarrow 7$ ineffective. We may even hold that there are three degrees of inefficiency, the second degree being exemplified by $2 \leftrightarrow 7$, which throws one of the elements out of its standard place, and leaves the other, which was not in its standard place at the outset, still in need of removal.

If the interchanges be all effective, the number, it will be found, is quite definite, and it is this number which is denoted by v. Thus, for the permutation

2 4 1 5 3

we have $v = 4$, 1 being brought into its standard place by the interchange $2 \leftrightarrow 1$, 2 by the interchange $4 \leftrightarrow 2$, 3 by $4 \leftrightarrow 3$, and both 4 and 5 by $4 \leftrightarrow 5$.

Of course we might vary this procedure by putting the elements in a different order into their standard places, but, as will be seen later, the number of necessary interchanges would not be altered. Thus, for the permutation 2 4 1 5 3, if we took the reverse order, 4 interchanges would still be needed, viz., $5 \leftrightarrow 3$, $4 \leftrightarrow 3$, $3 \leftrightarrow 1$, $2 \leftrightarrow 1$.

(33) Where there are n elements in the permutations, the number of needful interchanges cannot be greater than $n - 1$ in any case. For, supposing $n - 2$ effective interchanges have been neces-

sary to put the first $n-2$ elements into their standard places, the remaining two elements must at the worst be in each other's place, and therefore the interchange of them will be doubly effective.

(34) The fundamental proposition in regard to "interchanges" is as follows :—

If λ *and* μ *be any two elements of any permutation of* 1, 2, 3, . . . , n, *and* d *be the number of elements lying between them and intermediate to them in value, the interchange* $\lambda \leftrightarrow \mu$ *will increase or diminish the number of inverted-pairs by* 2d + 1 *according as* λ *is less or greater than* μ.

Let the given permutation be

$$\ldots\ldots\lambda\ldots\ldots\ldots\mu\ldots\ldots\ldots.$$

and consider first the case where $\lambda > \mu$. Then in comparing the numbers of inverted-pairs before and after the interchange, the elements preceding λ and those following μ may clearly be left out of account; that is to say, we have only got to consider the portion

$$\lambda\ldots\ldots\mu.$$

Now, if λ be moved so as to follow immediately after μ, thus giving the permutation

$$\ldots\ldots\mu,\lambda,$$

the number of inverted-pairs is thereby diminished by $d+1$: and if to effect the interchange spoken of in the enunciation μ be next moved to the former place of λ, thus giving the desired permutation

$$\mu\ldots\ldots\lambda,$$

the number of inverted-pairs is further diminished by d. Hence the total diminution of the number is $2d+1$, as was to be proved.

The reasoning for the case where $\lambda < \mu$ is exactly similar.*

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* This proposition is a modification of one of Rothe's. See p. 268 of his Memoir, or Muir's "History of Determinants," p. 56. Rothe's proof is very lengthy.

CIRCULAR SUBSTITUTIONS.

(35) Before proceeding further with the subject of "interchanges," or "transpositions" as Cauchy called them, it will be convenient to take up that of "circular substitutions" and Cauchy's rule of 1812.

On referring back to § 3 it will be seen that the first thing needful in connection with the latter is to write down the substitution which is required for the transformation of the given permutation into the standard permutation, and then decompose or factorise it into substitutions which are circular. The best mode of obtaining the factors will be readily understood by observing the application of it to a particular case, say the case of the permutation 8 7 1 5 4 6 3 2.

The substitution needed to change this into the standard permutation being

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 7 & 1 & 5 & 4 & 6 & 3 & 2 \end{pmatrix}$$

we begin with the first element of the lower line, viz., 8, and learn of course that it has to be changed into 1; proceeding then to 1 in the lower line we find that it has to be changed into 3; similarly that 3 has to be changed into 7, 7 into 2, and 2 into the element with which we started. This process gives us the partial substitution

$$\begin{pmatrix} 1 & 3 & 7 & 2 & 8 \\ 8 & 1 & 3 & 7 & 2 \end{pmatrix},$$

which from its nature is called "circular" or "cyclic." After this there remains the substitution

$$\begin{pmatrix} 4 & 5 & 6 \\ 5 & 4 & 6 \end{pmatrix},$$

which being dealt with in similar fashion is separated into

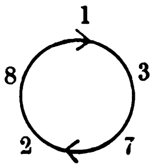
$$\begin{pmatrix} 4 & 5 \\ 5 & 4 \end{pmatrix} \text{ and } \begin{pmatrix} 6 \\ 6 \end{pmatrix},$$

so that we have finally

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 7 & 1 & 5 & 4 & 6 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 7 & 2 & 8 \\ 8 & 1 & 3 & 7 & 2 \end{pmatrix} \begin{pmatrix} 4 & 5 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} 6 \\ 6 \end{pmatrix}.$$

(36) From an examination of this process it will be seen that

(*a*) *A circular substitution is one in which the upper line of elements is got from the lower by removing the first element of the latter to the last place, and that consequently it may be represented by only one line, e.g.,*



(*b*) *There is only one way of decomposing a substitution into circular substitutions*

—a statement which includes the fact that

(*c*) *The number of circular substitutions necessary to transform any permutation into the standard permutation is definite.*

Further, it may be noted that a "two-termed" or "binomial" substitution, like $\begin{pmatrix}4 & 5\\ 5 & 4\end{pmatrix}$, is exactly the same as an "interchange" or "transposition"; that a "one-termed" or "monomial" substitution is, strictly speaking, not a substitution at all: and that a circular substitution may be expressed in the two-line notation in as many ways as there are elements in each line.

(37) We are now prepared for the important propositions which connect the number of "interchanges" necessary to transform a given permutation into the standard permutation with the number of "circular substitutions" required for the same purpose. The first is—

If in the lower line of a circular substitution of m *elements two elements separated by* p *others be interchanged there is produced a substitution which can be resolved into two circular substitutions of* $m-(p+1)$ *and* $p+1$ *elements respectively.*

Let the given circular substitution be

$$\begin{pmatrix}\beta,\gamma,\delta,\epsilon,\,.\,.\,.\,.\,.\,,\,\phi,\chi,\psi,\omega,\alpha\\ \alpha,\beta,\gamma,\delta,\epsilon,\,.\,.\,.\,.\,.\,,\,\phi,\chi,\psi,\omega\end{pmatrix},$$

and γ,χ the pair of elements in the lower line, so that the resulting substitution is

$$\begin{pmatrix} \beta,\gamma,\delta,\epsilon, \ldots\ldots, \phi,\chi,\psi,\omega,\alpha \\ \alpha,\beta,\chi,\delta,\epsilon, \ldots\ldots, \phi,\gamma,\psi,\omega \end{pmatrix}.$$

Then in beginning with α in the latter and telling off in linked fashion the items of the substitution, viz., α-into-β, β-into-γ, ... we necessarily find that, instead of going as before from γ to δ, and so on through the whole of the remaining elements, we go from γ to a more advanced element in the upper line than δ, viz., to ψ, and consequently reach more rapidly the element with which we began, viz., α, thus obtaining the shorter circular substitution

$$\begin{pmatrix} \beta,\gamma,\psi,\omega,\alpha \\ \alpha,\beta,\gamma,\psi,\omega \end{pmatrix},$$

and leaving the substitution

$$\begin{pmatrix} \delta,\epsilon, \ldots\ldots, \phi,\chi \\ \chi,\delta,\epsilon, \ldots\ldots, \phi \end{pmatrix},$$

which also is necessarily circular. Further, as the elements $\delta,\epsilon, \ldots, \phi$ are p in number, the number of elements in the latter circular substitution is $p+1$.

The case where the two interchanged elements are consecutive in the cycle should be noted in passing. One of the two component circular substitutions will then be monomial, the result of the change being to put the second element into its standard place.

(38) From this it follows that—

If in a given permutation of n *elements two elements be interchanged which are in the same cycle of substitution the resulting permutation will have one cycle more than the given permutation.*

Consequently, by continuing to make such interchanges, the resulting cycles may be made all monomial, and therefore be n in number. This means that each element would then be in its standard place, and that the number of such interchanges would be the excess of n over the original number of cycles. We thus learn that—

If a given permutation of n *elements have* κ *cycles of substitution, it can be transformed into the standard permutation by means of* $\mathrm{n}-\kappa$ *interchanges, provided every pair of interchanged elements be, before interchange, in the same cycle.*

The equivalence of Cauchy's rule of "circular substitutions" with the rule of "interchanges" is thus made manifest.

(39) *If in transforming a given permutation into the standard permutation we confine ourselves to effective interchanges only, the number necessary will still be* $\mathrm{n}-\kappa$.

For, an effective interchange, being an interchange which brings at least one of the two elements concerned into its standard place, must be an interchange of two consecutive elements in a cycle, and the only difference will be that a *monomial* cycle will be split off by every operation.

Here, in passing, it may be noted as self-evident that the order in which effective interchanges are made is immaterial.

(40) *The interchange of two elements in different partial circular substitutions destroys the circular character of both substitutions, but makes it possible to form one circular substitution out of the two.*

Let the given circular substitutions be so written that one of the two elements concerned occupies the last place in the substitution to which it belongs, and the other the first place; and let the substitutions so written be

$$\begin{pmatrix} \beta,\gamma,\ldots,\chi,\psi,\omega,\alpha \\ \alpha,\beta,\gamma,\ldots,\chi,\psi,\omega \end{pmatrix} \quad\text{and}\quad \begin{pmatrix} \beta',\gamma',\ldots,\chi',\psi',\omega',\alpha' \\ \alpha',\beta',\gamma',\ldots,\chi',\psi',\omega' \end{pmatrix}.$$

After the interchange these become

$$\begin{pmatrix} \beta,\gamma,\ldots,\chi,\psi,\omega,\alpha \\ \alpha,\beta,\gamma,\ldots,\chi,\psi,\alpha' \end{pmatrix} \quad\text{and}\quad \begin{pmatrix} \beta',\gamma',\ldots,\chi',\psi',\omega',\alpha' \\ \omega,\beta',\gamma',\ldots,\chi',\psi',\omega' \end{pmatrix},$$

which are manifestly non-circular, the break in the chain of the first occurring when we try to leave the item ψ-into-ω. If, however, at this stage we neglect for a little the next item, and move on to the second substitution, we find that ω has to be changed into β', β' into γ', ..., ω' into α'. All that is then wanted to complete the cycle is the omitted item α'-into-α, so that we have only one circular substitution,

$$\begin{pmatrix} \beta,\gamma,\ldots,\chi,\psi,\omega,\beta',\gamma',\ldots,\chi',\psi',\omega',\alpha',\alpha \\ \alpha,\beta,\gamma,\ldots,\chi,\psi,\omega,\beta',\gamma',\ldots,\chi',\psi',\omega',\alpha' \end{pmatrix}.$$

(41) *If in any permutation of* n *elements there be* κ *cycles of substitution,* n $-\kappa$ *is the smallest number of interchanges necessary to transform it into the standard permutation.*

To transform the permutation into the standard permutation implies that the number of cycles is to be increased to n. Now, any two elements of the permutation must be either in the same or different cycles, and we have seen that the interchange of two, which are in different cycles, does not lead to an increase in the number of cycles. To attain our end with the fewest possible number of interchanges, we must, therefore, in every case choose two elements which are in the same cycle. When we do this, however, $n-\kappa$ such interchanges are known to be necessary and sufficient. The lowest possible number is thus $n-\kappa$.

(42) It follows, therefore, from § 39, that *the lowest possible number of interchanges necessary to transform a given permutation into the standard permutation is secured by using only effective interchanges.* This is important from a practical point of view, because effective interchanges are easily recognised. To seek for pairs of elements *which are in the same cycle of substitution* would be much more troublesome. Besides, it implies that the cycles of substitution are known; and, if this be the case, the consideration of interchanges is in practice unnecessary.

As a matter of theory, however, it is curious to note the manner in which, when we follow the more troublesome process and in doing so use an ineffective interchange, we are compensated at a later stage by an additional doubly-effective interchange. Thus, taking the permutation

$$5\ 1\ 7\ 6\ 2\ 8\ 4\ 3$$

with its circular substitutions

$$\begin{pmatrix} 1 & 2 & 5 \\ 5 & 1 & 2 \end{pmatrix}, \begin{pmatrix} 3 & 8 & 6 & 4 & 7 \\ 7 & 3 & 8 & 6 & 4 \end{pmatrix},$$

if we confine ourselves to effective interchanges we shall, in the course of the transformation of the permutation, meet with only two doubly-effective interchanges; but if, in selecting two elements of the second cycle, we make an ineffective interchange—say the interchange 6↔7,—the cycle is split up into

$$\begin{pmatrix} 3 & 8 & 6 \\ 6 & 3 & 8 \end{pmatrix} , \begin{pmatrix} 4 & 7 \\ 7 & 4 \end{pmatrix} ,$$

and then we have three cycles of more than one element each, and therefore must be led later to three doubly-effective interchanges.

(43) We have now dealt with four of the five rules of signs, and have seen how they are related to one another. Summing up, we may say that δ being the number of "inverted-pairs" in a given permutation of n elements, ν the smallest number of "interchanges" necessary to transform the permutation into the standard permutation, κ the number of "circular substitutions" needed for the same purpose, and μ the smallest number of "moves," then

$$\mu = \delta ,$$

and

$$\nu = n - \kappa .$$

Consequently, so far as we know, two of the four numbers are unrelated, namely, δ and κ. Of the former—the number of "inverted-pairs"—a full investigation has been given above. In the case of the latter—the number of "circular substitutions"—such is unnecessary, as may be seen from several papers by Cauchy, the founder of the theory of substitutions.* The theorem of the following § is the only fresh result which has been arrived at in the course of the present investigation.

(44) *If the circular substitutions of any permutation be each reversed in order we obtain those of the conjugate permutation.*

In the substitution necessary to transform the given permutation into the standard permutation, any item, say λ-into-β, must have corresponding to it in the case of the conjugate permutation an item of substitution β-into-λ; for, λ in the former being the element and β its place, there must by definition be in the latter an element β in the λ^{th} place. Consequently if

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* See Muir, "History of Determinants," pages 91 . . . , 234 . . . , 259 . . . , or the papers themselves there referred to.

$$\begin{pmatrix} \beta,\mu,\theta,\ldots,\lambda \\ \lambda,\beta,\mu,\ldots,\xi \end{pmatrix}$$

be a circular substitution for the case of the original permutation, then

$$\begin{pmatrix} \xi,\ldots,\mu,\beta,\lambda \\ \lambda,\ldots,\theta,\mu,\beta \end{pmatrix}$$

must be a circular substitution for the case of the conjugate permutation.

(45) Included in this is the fact that *conjugate permutations have the same number of circular substitutions, and therefore have the same sign.* Also, from § 41 it follows that *conjugate permutations require the same number of interchanges.* Indeed, whichever of the five rules of signs we employ, the number of things to be counted in the case of any permutation is the same as in the case of the conjugate permutation.

(46) Since in a self-conjugate permutation the circular substitution must remain unaltered by reversal, it follows that *the circular substitutions of a self-conjugate permutation must be either monomial or binomial.*

This enables us to determine the total number of self-conjugate permutations of n elements. For, confining ourselves to circular substitutions of these two kinds, we have only to count the number of possible cases with *no* binomial circular substitutions, those with only *one*, those with only *two*, and so on. The result is

$$1 + C_{n,2} + \frac{1}{1\cdot2}C_nC_{n-2,2} + \frac{1}{1\cdot2\cdot3}C_{n,2}C_{n-2,2}C_{n-4,2} + \cdots\cdot$$

or

$$1 + 1\cdot C_{n,2} + 1\cdot3\cdot C_{n,4} + 1\cdot3\cdot5\cdot C_{n,6} + \ldots\ldots$$

as is already known.*

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* See Muir, "On Self-Conjugate Permutations," *Proc. Roy. Soc. Edin.*, xvii. pp. 7–13.

EVEN CIRCULAR SUBSTITUTIONS.

(47) The last of the five rules of signs may be quickly disposed of. Like Cauchy's it makes use of "circular substitutions," but, unlike Cauchy's, is not explicitly dependent upon the number of elements in the permutation. The easiest way, therefore, to connect it with the others is to deduce it directly from Cauchy's.

Denoting then by κ_e the number of "circular substitutions" with an even number of elements in each, and by κ_o the number of "circular substitutions" with an odd number of elements in each, so that

$$\kappa_e + \kappa_o = \kappa ,$$

we see that the total number of elements in the κ_e substitutions is even, and the total number in the κ_o substitutions is even or odd according as κ_o is even or odd. From these it follows by addition that the total number of elements in both kinds of substitutions—that is, n—is even or odd according as κ_o is even or odd, and therefore that

$$n - \kappa_o \text{ is even.}$$

But by Cauchy's rule the sign of the permutation is

$$(-)^{n-(\kappa_e+\kappa_o)}$$

$$i.e., \qquad (-)^{n-\kappa_o}.(-)^{-\kappa_e} ,$$

$$i.e., \qquad (-)^{-\kappa_e} ,$$

which is Jenkins' rule.

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Tidal Currents of the North Sea. By Alexander Buchan, LL.D., F.R.S.

(Read February 20, 1899.)

In the article "Tides" in the *Encyclopædia Britannica*, Professor George Darwin, quoting Sir George B. Airy, remarks that the tides of the North Sea present a very remarkable peculiarity. Along the eastern coast of England as far as the mouth of the Thames, the tide-wave, coming from the Atlantic round the Orkney Islands, flows towards the south. Thus, on a certain day, it is high water in the Moray Firth at 11 A.M., at Berwick at 2 P.M., at Flamborough Head at 5 P.M., and so on to the entrance to the Thames. Thus, on the day supposed, it will be high water off the Thames at 11 P.M., the tide having travelled in twelve hours from the Moray Firth.

It is further stated that the North Sea is considerably deeper on the English side than on the German side; so much so, that the tide-wave coming from the north runs into a deep bay of deep water, bounded on the west side by the Scottish and English coasts as far as Newcastle, and on the east side by the great Dogger Bank. As far as the latitude of Hull, the English side is still the deep one; and though a species of channel through the shoal there allows an opening to the east, yet immediately on the south of it is the Wells Bank, which again contracts the deep channel to the English side.

It is not stated here that the deeper water of the North Sea close to the Scottish and English coasts determines the course of the southward tidal flow to be close to these coasts; but for that course taken, no other cause is suggested.

In the *Annual Report of the Fishery Board for Scotland for* 1896 there appeared a paper by Dr Fulton, scientific superintendent to the Board, on "The Currents of the North Sea and their relation to Fisheries."

The method employed in collecting information regarding the currents of the North Sea was substantially that regularly used by the United States Hydrographic Office, the German Seewarte of Hamburg, the Prince of Monaco, and by various others. Two kinds of floats were used by Dr Fulton—namely, bottles and slips of wood—and very full details are given as to the method employed in conducting the experiments.

The number of floats set adrift on the east coast of Scotland, within 20 miles of the shore south of the Pentland Firth, was 1864; in the neighbourhood of the Pentland Firth, 369; on the route to Christiansand, 630; on the route to Hamburg, 520; and between a point 12 miles off Flamborough Head and the Hook of Holland, 200; and others were put into the sea off the east coast at a distance greater than 20 miles. Of the above, the number found on various coasts up to the end of March 1897 was nearly 700, which were forwarded to the Fishery Board.

The results of discussion of the returns received by the Board are thus summarised by Dr Fulton:—

"The surface Atlantic water passes southwards and eastwards from the Shetlands and Orkneys; it then moves southwards along the east coasts of Scotland and England to the neighbourhood of the Wash, impinging more or less on the coasts that run at an angle to it, such as Banff, Aberdeen, Fife, East Lothian, Berwick, and the East of England as far as Spurn Head. Thence the movement of the surface water is eastwards towards the Continent, the main body impinging on the coast of Denmark north of the Horn. The course is then northward to the Skagerrak and the west coast of Norway, as far at least as the Loffodens."

These are stated to be the regular and predominating courses of the currents of the North Sea as arrived at by this system of observation of the movements of the surface water. It will be noted that for the coasts of Scotland and England, as far south as the Wash, it is identical with the course of the tidal currents of the North Sea as given in the *Encyclopædia Britannica* article on the "Tides."

In August 1865, Dr George Keith undertook, at the request of the Council of the Scottish Meteorological Society, to make observations on the temperature of the sea on a cruise in the yacht "St Ursula" from Gravesend to the Faroe Islands.* His observations showed that the temperature fell from 62°·0 near Yarmouth Roads to 58°·0 near the Wash, and to 56°·5 off Scarborough; but from this point northward to Lerwick, the temperature was virtually a constant, being 55°·5 on entering Bressay Sound. The area overspread by this substantially uniform temperature on the east coast of Great Britain is virtually coincident with the southward flow of the tidal current from the Atlantic as determined by Dr Fulton. Observations of the surface temperature of the sea on these coasts amply confirm those made by Dr George Keith in 1865.

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* See *Journal of the Scottish Meteorological Society*, New Series, vol. i. p. 333.

The wind also has an important influence on the currents of the North Sea. The winds have been observed by the Scottish Meteorological Society's observers from 1856, and the averages during these forty-three years drawn from observations at about sixty stations show that the average number of days the different winds have prevailed during the year are:—

N.	30 Days	S.W.	66 Days
N.E.	28 ,,	W.	69 ,,
E.	39 ,,	N.W.	39 ,,
S.E.	32 ,,	Calm	25 ,,
S.	37 ,,		

Thus the prevailing winds are W.S.W.; and the observations further show that these are by far the strongest of the winds. Now these strong W.S.W. winds, considered as producers of surface currents of the sea flowing from the British coasts towards Denmark, the Skaggerrak, and Norway, are necessarily most effective near the southern limits of the tidal currents from the north or about the Wash, and thence eastward.

This is well shown by Dr Fulton's chart of the observed currents, on which it is seen that the floats crossed the North Sea between latitudes 55° to 53° N.; and thence, curving round, impinged on the Danish coast near Farö and round to Skagen; and thence diverted through the Skaggerrak; and thence northward, keeping close to the Norwegian coast as far as the Loffoden Islands. The prominent characteristic of this current chart is that north of latitude 55° N., few or none of the floats are represented as having crossed the North Sea. North of this parallel the strongest inflowing tidal currents keep near the east coast of Great Britain, and the strongest outflowing tidal currents keep near the coasts of Denmark and Norway. The cause of this striking peculiarity admits of a simple explanation. Since, as the tidal currents from the north flow southwards, they advance over regions where the velocity of the earth's rotation constantly increases, and consequently lagging behind, they press in toward the east coast of Great Britain; and on the other hand, as the outflowing tidal currents proceed northwards, they advance over regions where the velocity of the earth's rotation constantly diminishes, and consequently press close on the coasts of Denmark and Norway.

It is a singular circumstance that the North Sea is the only sea in the Northern Hemisphere quite open to large tidal currents from a contiguous ocean, which flow southwards with flowing tides and northwards with ebbing tides; and it is probably owing to this circumstance that hitherto, so far as I am aware, the rotation of the earth has not been adduced in explanation of the most striking peculiarity of these tidal currents.

On the Development and Morphology of the Marsupial Shoulder-Girdle. By R. Broom, M.D., B.Sc. *Communicated by* Sir WILLIAM TURNER.

(Read January 9, 1899.)

(*Abstract.*)

An examination is made of a series of early stages in the development of the shoulder-girdle in the common Phalanger (*Trichosurus vulpecula*), and of early stages in the development of that of the Ring-tailed Phalanger (*Pseudochirus peregrinus*), and of the Rock-wallaby (*Petrogale penicillata*). Though in their main features there is much similarity between the girdles of the different genera, there are certain peculiar features in each.

In a Trichosurus fœtus of 8·5 mm., though the main part of the scapula is chondrified, together with the glenoid portion of the coracoid and a portion of the acromion, the rest of the arch, the sternum, and the greater part of the clavicle, are still mesenchymatous. The coracoid can be traced downwards as a fan-like expansion, which meets the first rib and the sternum. Between the coracoid and the clavicle a feebly-developed thin sheet apparently represents the precoracoid (epicoracoid). The clavicle is partly ossified at its upper part, and it is very manifest that there is no cartilaginous basis.

In the Trichosurus fœtus, at birth (14 mm.), the scapula is well developed. The acromion is a large process which springs from the anterior border of the blade. The spine is not yet formed, though its basis can be distinctly traced as a membranous structure stretching from the acromion upwards along the outer side of the scapula in its anterior third. There is no cartilaginous basis for this part of the spine. The coracoid is of large size, and its lower part, which is somewhat bulbous, articulates with the sternum and with the first rib.

In the later development it is shown that the girdle becomes detached from the sternum, owing to degeneration of a portion of the bulbous end of the coracoid. By the time the fœtus is 23 mm. in length, the girdle is quite detached.

In the Pseudochirus fœtus of 16 mm., the coracoid, though in contact with the presternum, is not structurally continuous with it.

In the early fœtus of Petrogale (21 mm.) the coracoid is of very large size, and is continued backwards for some distance by the side of the first rib. It is for a short distance structurally continuous with the sternum. In this fœtus no distinct omosternum can be detected.

In considering the morphology of the scapular borders, the following conclusions are arrived at:—

(*a*) That the spine proper, having no cartilaginous basis, is the homologue of the cleithrum or epiclavicle of the lower Theromorphs.

(*b*) That in the Monotremes there is no supra-spinous fossa, owing to the spine or cleithrum being applied to the morphological anterior border; while in the higher mammals a supra-spinous fossa has been formed, owing to the spine becoming applied to the outer surface of the scapula, and away from the anterior border.

With regard to the coracoidal elements, the following are the principal conclusions arrived at:—

(*a*) That the well-developed coracoid in the fœtal marsupials, and, consequently, the coracoid process in the higher mammals generally, is the homologue of the posterior coracoidal element in the Monotremes and Theromorphs, and of the coracoid in Reptiles generally.

(*b*) That the anterior coracoidal element ("epicoracoid") in Monotremes and Theromorphs is the homologue of the precoracoid of the Amphibia.

(*c*) That the only representative of the precoracoid remaining in the higher mammals is the coraco-clavicular ligament.

The cartilage which forms round the ends of the clavicle is regarded as of a secondary nature, and of no special morphological significance.

Equilibrium between Sulphuric Acid and Sulphates in Aqueous Solution. By Sydney A. Kay, B.Sc. *Communicated by* Professor WALKER.

(Read December 19, 1898.)

In the year 1847, as one of the results of an investigation to determine the part played by the mass action of water in chemical reactions, H. Rose (*Pogg. Ann.*, lxxxii. 545) showed that an acid sulphate in aqueous solution is progressively decomposed into free acid and neutral sulphate by increasing quantities of water. His observations were confirmed and extended by the thermo-chemical researches of Thomsen (*Pogg. Ann.* 1869, cxxxviii. 72), and Berthelot (*Ann. Chim. Phys.*, 1873, xxix. 433), who indicated more exactly the course and extent of this decomposition, and from whose work it is known that in the solution of an acid sulphate there exist free sulphuric acid, neutral sulphate and acid sulphate. Finally Ostwald, in his first memoir on chemical affinity (*Jour. prakt. Chem.* 1879, xix. 483), showed how to determine the magnitude of this decomposition, and was able to approximately measure the quantity of free acid in solutions of the acid sulphates at different dilutions. In a later paper (*ibid.*, 1880, xxii. 305), Ostwald investigated the question of the influence of water on the action between sulphuric acid and a neutral sulphate. He measured the changes of volume which occurred when solutions of sulphuric acid and sodium sulphate were mixed in varying proportions, and at different dilutions, and obtained results in agreement with his previous work. He also pointed out, that if the mutual action between the acid and neutral sulphate obeyed the general law of mass action, it should follow that, for example, one molecule sodium sulphate plus three molecules sulphuric acid, give the same quantity of acid sulphate as one molecule sulphuric acid, and three molecules sodium sulphate, the volume of the mixture being the same in each case. This, however, he showed was not true, little agreement being found between the numbers which, according to the law, should be equal. The differences were especially large at the highest dilutions.

The following investigation was undertaken with a view to determine the hitherto unascertained nature of the equilibrium which exists in solutions of the acid sulphates, and in solutions containing sulphuric acid and a neutral sulphate in other than equivalent proportions. It is evident from the results obtained by the above mentioned investigators, that the equilibrium is one of peculiar interest, as being disturbed by mere dilution of the acid salt solution.

The free acid, neutral sulphate and acid sulphate, in the solution of an acid sulphate, are partially dissociated, so that, besides these substances, the ions derived from them are concerned in the final equilibrium. Chief among the equilibria existing between the various substances in solution, is that, however, which obtains between the undissociated portions of the free acid, neutral and acid sulphates, and it was the particular aim of the present investigation to determine the nature of this equilibrium. The problem was treated quite empirically, and from the experimental results obtained, it was sought to ascertain the law of the equilibrium, and to formulate an expression by means of which it would be possible to calculate the percentage of free acid in a solution containing sulphuric acid and a neutral sulphate in any proportions.

THE EQUILIBRIUM.

The chemical equation representing the formation of an acid sulphate as a balanced reaction is written thus:—

$$H_2SO_4 + M_2SO_4 \rightleftarrows 2MHSO_4.$$

According to the law of mass action, the equilibrium between the undissociated portion of these three substances in solution is given by the expression

$$C.H_2SO_4(1-\alpha_1) \times C.M_2SO_4(1-\alpha_2) = K\{C.MHSO_4(1-\alpha_3)\}^2$$

where $C.H_2SO_4$, $C.M_2SO_4$ and $C.MHSO_4$ are the concentrations at equilibrium of the acid, neutral sulphate and acid sulphate respectively, $(1-\alpha_1)$, $(1-\alpha_2)$ and $(1-\alpha_3)$ the undissociated proportions of each of these substances, and K is a constant.

It is known, however, that in the simpler case of equilibrium between the undissociated portions of a strong acid or of a salt and its ions, the law of mass action in its original form is not valid; no dissociation constant is obtained, according to Ostwald's well-known "dilution law," which is simply the law of mass action applied to the equilibrium in question. As the equilibrium, the nature of which it is here sought to determine, is that existing between a strong acid, its neutral and its acid salt, it was therefore improbable that it would follow as a consequence of this law, and a brief inspection of the figures calculated from the experimental results justified this conclusion. In order to represent the equilibrium between the ions and the undissociated portion of a highly dissociated electrolyte, empirical formulae have been proposed, as modifications of Ostwald's expression, by Rudolphi (*Zeit. physik. Chem.*, xvii. 385), van't Hoff (*ibid.*, xviii. 301), and others. In the same manner, the expression for the equilibrium under consideration may be made more general and quite empirical if we write it thus:—

$$\{C.\mathrm{H_2SO_4}(1-\alpha_1)\}^m \times \{C.\mathrm{M_2SO_4}(1-\alpha_2)\}^n = \mathrm{K}.C.\mathrm{MHSO_4}.(1-\alpha_3)$$

where m and n are unknown exponents. The expression given above is a particular case of this more general equation, where m and n are the same and equal to 0·5.

In what follows, an experimental method is described by means of which the concentrations of the acid, neutral sulphate and acid sulphate were determined, while the values of $(1-\alpha_1)$ $(1-\alpha_2)$ and $(1-\alpha_3)$ were calculated from Kohlrauch's measurements of the electrical conductivity of acids and salt solutions. (*Wied. Ann.*, 1885, xxvi. 196.) It was then possible, by comparison of the results obtained for the various solutions, to find the values of the exponents m and n, as well as the mean value of the constant K. The expression was then completely determined. Its accuracy was tested by using it to calculate the percentage of free acid in the various solutions, the values so obtained being then compared with those actually observed.

EXPERIMENTAL METHOD.

The concentrations of the free acid, neutral sulphate and acid sulphate in solutions containing sulphuric acid and a neutral sulphate were determined by means of a reaction velocity method, viz., by the catalysis of ethylic acetate. The velocity with which a solution effects the catalysis gives a measure of the free acid which it contains. As the experimental method employed has already been described by several investigators (Ostwald, *Journ. prakt. Chem.*, xxxv. 112; Arrhenius, *Zeitsch. physik. Chem.*, i. 110), it is unnecessary that I should give it in detail here.

Equal volumes of a known strength of ethylic acetate solution and of a solution containing sulphuric acid and a neutral sulphate in definite proportions, were mixed at the temperature of the thermostat. In order that disturbing influences should be reduced to a minimum, the solutions employed were made as dilute as was consistent with a sufficiently rapid catalysis, which would admit of accurate and convenient measurement at 35°·0. It was not advisable to use a higher temperature on account of the volatility of the ester. The mixed solutions were so prepared that they had the correct concentrations at the temperature of the thermostat. One example will suffice to indicate the procedure.

It is wished to determine the velocity of catalysis with a solution containing 0·2 equivalent normal* sulphuric acid, and 0·1 equivalent normal potassium sulphate. Forty c.c. normal sulphuric acid were mixed with 20 c.c. potassium sulphate solution of the same concentration, and made up to 100 c.c. at 35°. Fifty c.c. of this solution were then mixed with an equal volume of 0·2 normal ethylic acetate, which was previously heated to the temperature of the thermostat, and measured at that temperature. At 35°, and at the beginning of the reaction, the mixture was then 0·2 normal with respect to sulphuric acid, and 0·1 normal with respect to potassium sulphate and to ethylic acetate. It was assumed that the volume of the mixture was equal to 100 c.c. The concentration of the ethylic acetate was 0·1 normal in all the experiments.

At intervals, portions of 5 or 10 c.c. were withdrawn by means of a pipette and titrated as expeditiously as possible. The end point was determined in the same way after a sufficient time had elapsed.

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* All concentrations are given in gram-equivalents per litre.

As it was not found possible to correctly titrate solutions containing sulphuric acid and a neutral sulphate with baryta water, a solution of sodium hydrate was used, to which sufficient baryta solution had been added to precipitate the carbonate, with a slight excess. The concentration of the alkali was 0·1 to 0·05 normal. Phenol-phthaleïn was used as indicator.

THE VELOCITY CONSTANTS.

(*a*) SULPHURIC ACID AND POTASSIUM SULPHATE.

Three series of experiments were made with these solutions.

In the first, the concentration of the sulphuric acid was varied, and that of the potassium sulphate was constant; in the second the concentration of the potassium sulphate was varied, and that of the acid remained constant. The third series was made with solutions containing equivalent quantities of the acid and neutral sulphate at varying dilutions. Each series consists of five different experiments. The constants were calculated according to the expression

$$k = \frac{1}{t} \cdot \log_{10} \frac{E}{E - x},$$

where k is the velocity constant, t the time in minutes, x the quantity of acetic acid produced in time t, and E is the end-point. In the tables below the values of x and $E - x$ are given in terms of the sodium hydrate solution. To avoid unnecessary ciphers the constants have been multiplied by 10^5 in every case.

First Series.

I.

H_2SO_4 0·025 normal.
K_2SO_4 0·1 „

t	x	$E-x$	k
984	272	1608	6·96
2366	596	1284	7·00
2800	679	1201	6·95
3764	856	1024	7·01
5271	1070	810	6·94
∞	1880	—	6·96

II.

H_2SO_4 0·05 normal.
K_2SO_4 0·1 „

t	x	$E-x$	k
177·5	110	1800	14·5
1209	645	1265	14·8
1604	799	1111	14·7
2621	1127	783	14·8
3028	1225	685	14·7
∞	1910	—	14·7

III.

H_2SO_4 0·1 normal.
K_2SO_4 0·1 ,,

t	*x*	E − *x*	*k*
120·5	160	1762	31·4
216	280	1642	31·6
359·5	445	1477	31·8
515	604	1318	31·8
1378	1220	702	31·7
∞	1922	—	31·7

IV.

H_2SO_4 0·2 normal.
K_2SO_4 0·1 ,,

t	*x*	E − *x*	*k*
60	171	1781	66·4
140	378	1574	66·8
281·5	688	1264	67·2
438	960	992	67·1
602	1135	817	67·3
∞	1952	—	67·0

V.

H_2SO_4 0·35 normal.
K_2SO_4 0·1 ,,

t	*x*	E − *x*	*k*
68	166	791	121·6
124	286	671	124·4
180	388	569	125·4
225	459	498	126·1
349·5	605	352	124·3
∞	957	—	124·4

Second Series.

I.

H_2SO_4 0·1 normal.
K_2SO_4 0·4 ,,

t	*x*	E − *x*	*k*
126	100	1766	19·0
317	244	1622	19·1
435	327	1539	19·2
1250	790	1076	19·1
1739	999	867	19·1
∞	1866	—	19·1

II.

H_2SO_4 0·1 normal.
K_2SO_4 0·2 ,,

t	*x*	E − *x*	*k*
214	208	1658	24·0
340	328	1538	24·7
1167	899	967	24·5
1685	1147	719	24·6
2636	1450	416	24·7
∞	1866	—	24·5

III.

H_2SO_4 0·1 normal.
K_2SO_4 0·1 ,,

This experiment is given under III. above; it is the same in all three series.

IV.

H_2SO_4 0·1 normal.
K_2SO_4 0·05 ,,

t	*x*	E − *x*	*k*
118·5	186	1743	37·2
249	373	1556	37·5
397·5	557	1372	37·3
1250	1265	664	37·1
1757	1497	432	37·0
∞	1929	—	37·2

V.

H_2SO_4 0·1 normal.
K_2SO_4 0·025 „

t	x	$E - x$	k
145	243	1679	40·5
228·5	371	1551	40·8
372	566	1356	40·7
478	694	1228	40·7
1370	1381	541	40·2
1856	1574	348	40·0
∞	1922	—	40·5

Third Series.

I.

H_2SO_4 0·2 normal.
K_2SO_4 0·2 „

t	x	$E - x$	k
200	214	754	54·3
240	250	718	54·1
251	259	709	53·9
260	266	702	53·9
271	276	692	53·8
275	280	688	53·9
∞	968	—	54·0

II.

H_2SO_4 0·15 normal.
K_2SO_4 0·15 „

t	x	$E - x$	k
860	560	408	43·6
871	567	401	43·9
890	575	393	44·0
910	582	386	43·9
990	610	358	43·6
1060	634	334	43·6
∞	968	—	43·7

III.

H_2SO_4 0·1 normal.
K_2SO_4 0·1 „

Compare first series.

IV.

H_2SO_4 0·05 normal.
K_2SO_4 0·05 „

t	x	$E - x$	k
1345	406	532	18·3
1372	415	523	18·5
1406	424	514	18·6
1425	427	511	18·5
1475	432	506	18·2
1545	449	489	18·3
∞	938	—	18·4

V.

H_2SO_4 0·025 normal.
K_2SO_4 0·025 „

t	x	$E - x$	k
2760	477	466	11·0
2825	484	459	11·0
2870	490	453	11·1
2920	497	446	11·1
2966	502	441	11·1
2995	505	438	11·1
∞	943	—	11·1

(*b*) SULPHURIC ACID AND SODIUM SULPHATE.

One series only was made with these solutions, viz., a series in which the concentration of the acid varied, and that of the sulphate was constant. The velocity constants, it will be seen, are very similar to those obtained in the first series for solutions containing acid potassium sulphate. They are in every case somewhat higher than the latter.

I.

H_2SO_4 0·025 normal.
Na_2SO_4, 0·1 „

t	x	$E-x$	k
345·5	106	1825	7·09
1354	381	1550	7·05
1824	492	1439	7·00
2849	711	1220	7·00
5790	1167	764	6·96
∞	1931	—	7·01

II.

H_2SO_4 0·05 normal.
Na_2SO_4 0·1 „

t	x	$E-x$	k
177	113	1785	15·1
284	177	1721	15·0
1202	645	1253	15·0
1640	820	1078	15·0
∞	1898	—	15·0

III.

H_2SO_4 0·1 normal.
Na_2SO_4 0·1 „

t	x	$E-x$	k
107·5	149	1779	32·5
192·5	260	1668	32·7
355·5	444	1484	32·0
443·5	544	1384	32·5
535	635	1293	32·4
∞	1928	—	32·4

IV.

H_2SO_4 0·2 normal.
Na_2SO_4 0·1 „

t	x	$E-x$	k
82	231	1688	67·9
140·5	378	1541	67·8
227·5	574	1345	67·9
391	878	1041	67·9
480	1017	902	68·3
∞	1919	—	68·0

V.

H_2SO_4 0·35 normal.
Na_2SO_4 0·1 „

t	x	$E-x$	k
67	170	805	124·2
128·3	300	675	124·5
220	457	518	124·8
364	628	347	123·2
467	719	256	124·4
∞	975	—	124·5

(*c*) SULPHURIC ACID AND LITHIUM SULPHATE.

With these solutions one series of experiments was made, similar to that with solutions containing acid sodium sulphate. The details are as follows:—

I.

H_2SO_4 0·025 normal.
Li_2SO_4 0·1 „

t	*x*	E − *x*	*k*
1659	455	1485	7·00
4272	971	969	7·06
4310	974	966	7·03
4380	988	952	7·07
4423	996	944	7·07
	1940	—	7·05

II.

H_2SO_4 0·05 normal.
Li_2SO_4 0·1 „

t	*x*	E − *x*	*k*
1393	723	1152	15·2
1409	731	1144	15·2
1429	741	1134	15·3
1450	749	1126	15·3
1470	759	1116	15·3
	1875	—	15·3

III.

H_2SO_4 0·1 normal.
Li_2SO_4 0·1 „

t	*x*	E − *x*	*k*
890	489	515	32·5
912	495	509	32·3
945	509	495	32·5
965	518	486	32·6
985	527	477	32·8
1005	533	471	32·7
∞	1004	—	32·6

IV.

H_2SO_4 0·2 normal.
Li_2SO_4 0·1 „

t	*x*	E − *x*	*k*
225·3	295	693	68·4
231	305	683	69·4
235	308	680	69·1
240	315	673	69·5
252	325	663	68·8
265	337	651	68·4
∞	988	—	68·9

V.

H_2SO_4 0·35 normal.
Li_2SO_4 0·1 „

t	*x*	E − *x*	*k*
151	357	648	126·2
160	378	627	128·0
170	393	612	126·7
180	407	598	125·3
190·3	431	574	127·8
200·6	448	557	127·8
∞	1005	—	126·9

As it was more convenient, for the purpose of calculation, to use the velocity constants per equivalent of sulphuric acid, these have been obtained from the observed values by dividing by the concentration of the acid, and are collected in the following table:—

Velocity Constants per equivalent H_2SO_4.

Number of Experiment.	K_2SO_4			Na_2SO_4	Li_2SO_4
	1st Series	2nd Series	3rd Series	1st Series	1st Series
I.	278	191	270	280	282
II.	294	245	291	300	306
III.	317	317	317	324	326
IV.	335	372	368	340	344
V.	355	405	444	356	363

The percentage of free acid in a solution containing sulphuric acid and a neutral sulphate was obtained by comparison of the velocity with which it effected the catalysis of the ester, with the velocity measured in the case of sulphuric acid alone, of the same concentration. I therefore determined the velocity constants with sulphuric acid of varying concentrations. The following are the experiments:—

I.

H_2SO_4 0·01 normal.

t	x	$E-x$	k
1334	314	1542	6·05
2741	589	1267	6·06
4245	829	1027	6·06
5764	1029	727	6·09
7104	1167	689	6·06
∞	1856	—	6·06

II.

H_2SO_4 0·1 normal.

t	x	$E-x$	k
136	246	1646	44·5
286	474	1418	43·8
448	693	1199	44·2
1280	1377	515	44·2
1767	1576	316	44·0
∞	1892	—	44·1

III.

H_2SO_4 0·5 normal.

t	x	$E-x$	k
57	222	732	201·7
170	528	426	205·9
213	598	356	201·0
246	650	304	201·9
371	782	172	200·6
∞	954	—	201·5

From these observed velocity constants, the values per equivalent of the acid were calculated. The velocity constant at any intervening dilution was then obtained from a curve in which the velocity constants per equivalent were plotted against the corresponding concentrations. The following are the results obtained in this way for a series of dilutions. C is the concentration of the acid:—

C	k per equivalent.
0·01	606
0·025	545
0·05	493
0·1	441
0·2	411
0·35	407
0·5	403

CORRECTION OF THE VELOCITY CONSTANTS.

As the neutral salts in the solutions employed in these experiments do not of themselves bring about the catalysis of the ethylic acetate, this being affected only by the hydrogen ions arising from the free acid present at the same time, the velocity constant obtained with any solution gives a measure of the free acid which it contains. The standard of comparison is the velocity constant measured in the case of sulphuric acid alone, of the same concentration. A constant obtained for sulphuric acid alone, on the one hand, and that for a solution containing sulphuric acid and a neutral sulphate on the other, are not, however, immediately comparable. Each must be corrected in a manner which I will now indicate.

It is known that when a neutral salt is added to a solution of the corresponding monobasic acid, the velocity with which the acid effects the catalysis of an ester is increased. As this influence of the neutral salt is considered to be of a physical nature, and due in the first instance to the action of the dissociated part of the added salt, it must be supposed, when the catalysis is performed with a dibasic acid like sulphuric acid, to which a neutral sulphate has been added, that the influence exists here also; and that, although in this latter case the velocity actually measured is *less* than that obtained with sulphuric acid alone, (owing to the formation of the acid sulphate) the neutral salt must exert an accelerative effect on the catalysis, in the same manner and of the same magnitude as in the case of a monobasic acid.

We must therefore correct the velocity constants obtained with solutions containing sulphuric acid and a neutral sulphate, for the accelerating influence of the latter.

This influence has been measured in the case of cane sugar inversion by Sphor (*Zeit. physik. Chem.*, 1888, ii. 194), and by Arrhenius (*loc. cit.*, 1889, iv. 226). The latter has shown that the increase is nearly equal for all salts, or is at least of the same order of magnitude, and amounts to about 6·5 per cent. for an addition of 0·1 equivalent neutral salt per litre.

Trey (*Jour. prakt. Chem.*, 1886, xxxiv. 353), on the other hand, found in the catalysis of methylic acetate, an increase in the velocity of about 3·5 per cent. when the same quantity of neutral salt was added.

As the influence of neutral salts on the velocity of catalysis of ethylic acetate had not been measured, I made special experiments with hydrochloric acid as the catalysing agent, to determine the influence of neutral chlorides in this reaction. These were carried out in the same manner as already described for sulphuric acid and sulphates, the increase in the velocity constant obtained with acid alone being determined for two concentrations of the neutral salt in each case. The details are given below, and are arranged as before.

I.

HCl 0·2 normal.

t	*x*	E − *x*	*k*	*t*	*x*	E − *x*	*k*
235	528	446	144·4	120·5	322	638	147·3
261	568	406	145·7	130	340	620	146·1
270	581	393	146·0	166·3	410	550	145·5
280	596	378	146·8	175	432	528	148·4
290	607	367	146·2	185·3	446	514	146·4
300	621	353	147·0	195	465	495	147·2
315	636	338	146·0	205	480	480	146·9
∞	974	—	146·3	∞	960	—	147·1

A third experiment gave *k* equal to 147·3. The mean value of the velocity constant for 0·2 acid is therefore 146·9.

HCl 0·1 normal.

t	x	$E-x$	k
231	311	661	72·7
243	322	650	72·1
250	327	645	71·4
260·5	339	633	71·7
267	347	625	72·0
278	356	616	71·6
	972	—	71·9

t	x	$E-x$	k
201	556	1366	(73·7)
218	581	1341	71·8
228	603	1319	71·8
338	627	1295	72·0
248	644	1278	71·5
259	667	1255	71·5
∞	1922	—	71·7

II.

HCl 0·2 normal.
NaCl 0·1 ,,

t	x	$E-x$	k
230	526	436	150·3
250	555	407	150·3
260	570	392	150·8
270	586	376	151·9
280	598	364	151·6
290	608	354	150·6
308	629	333	150·5
∞	962	—	150·9

HCl 0·2 normal.
NaCl 0·2 ,,

t	x	$E-x$	k
210	507	454	155·1
220	524	437	155·6
225	533	428	156·1
235	546	415	155·2
240	556	405	156·4
250	569	392	155·8
260	582	379	155·4
∞	961	—	155·7

III.

HCl 0·1
KCl 0·05

t	x	$E-x$	k
210	284	682	72·0
221·5	296	670	71·7
230·5	308	658	72·5
241	318	648	72·0
251	330	636	72·4
262·5	343	623	72·6
267·5	346	620	73.1
∞	966	—	72·3

HCl 0·2 normal.
KCl 0·2 ,,

t	x	$E-x$	k
205	498	456	156·4
210	504	450	155·4
220	522	432	156·4
225	528	426	155·6
235	542	412	155·2
246	558	396	152·2
252	565	389	154·6
∞	954	—	155·6

IV.

HCl 0·2 normal.
LiCl 0·1 ,,

t	x	$E-x$	k
190	466	492	152·3
211	497	461	150·5
221	515	443	151·6
230	526	432	150·4
240	545	413	152·3
252	562	396	152·3
260	574	384	152·7
∞	958	—	151·6

HCl 0·2 normal.
LiCl 0·2 ,,

t	x	$E-x$	k
200	496	414	155·6
222	530	440	154·7
230	541	429	154·1
240	556	414	154·2
250	569	401	153·6
260	585	385	154·4
270	599	371	154·9
∞	970	—	154·5

V.

HCl 0·2 normal. $MgCl_2$ 0·1 „				HCl 0·2 normal. $MgCl_2$ 0·2 „			
t	*x*	E − *x*	*k*	*t*	*x*	E − *x*	*k*
175	444	523	152·5	171	433	523	153·2
180	450	517	151·0	180	452	504	154·5
190	467	500	150·7	190	470	486	154·7
200	481	486	149·4	207	499	457	154·9
226	527	440	151·3	215	518	438	157·7
235	543	424	152·3	227	531	425	155·1
246	558	469	152·0	235	547	409	156·9
∞	967	—	151·3	∞	956	—	155·6

In order to calculate from these experimental results, the real influence of the neutral salts on the velocity of reaction, the following consideration must not be neglected. The velocity with which an acid effects the catalysis is dependent on its degree of dissociation. In general, when a neutral salt is added to an acid, the degree of dissociation of the latter is changed, and therefore before comparing a constant obtained when a neutral salt is present, with that measured in the case of the acid alone, the value of the latter must be made equal to what would be found if the dissociation of the acid were the same as it is after the addition of the salt. An example will show how the correction was applied.

The velocity constant for 0·2 normal hydrochloric acid, as experimentally determined, had the value 146·9. When 0·1 normal sodium chloride was added, the constant obtained is 150·9. But by this addition of the neutral salt, the degree of dissociation of the acid is diminished from 0·896 to 0·882 (Arrhenius, *Zeit. physikal. Chem.*, 1888, ii. 286). The velocity constant with hydrochloric acid, dissociated to the latter extent only, is therefore $146·9 \times \frac{·882}{·896} = 144·6$. The true increase in the velocity of catalysis produced by the addition of 0·1 normal sodium chloride is now $150·9 - 144·6 = 6·3$, or 4·35 per cent. of the calculated value for hydrochloric acid (144·6).

The results obtained for the different neutral chlorides employed and calculated as above are exhibited in the following table:—

	Dissociation of Acid.	k (Observed.)	k(HCl) (Calculated.)	Percentage Increase.	Percentage Increase per 0·1 Salt.
0·2 HCl	·896	146·9	—	—	—
,, ,, +0·1 NaCl	·882	150·9	144·6	4·35	4·35
,, ,, +0·2 NaCl	·872	155·7	143·0	8·88	4·44
0·1 HCl	·924	71·8	—	—	—
,, ,, +0·05 KCl	·910	72·3	70·7	2·26	4·52
0·2 HCl+0·2 KCl	·871	155·6	142·8	8·96	4·48
,, ,, +0·1 LiCl	·882	151·6	144·6	4·84	4·84
,, ,, +0·2 LiCl	·873	154·5	143·1	8·00	4·00
,, ,, +0·1 $MgCl_2$	·884	151·3	144·9	4·42	4·42
,, ,, +0·2 $MgCl_2$	·875	155·6	143·5	8·44	4·22

They are similar in character to those already found for the inversion of cane sugar, and are mainly three, viz.:—

The addition of a neutral chloride to hydrochloric acid increases the velocity with which the latter effects the catalysis of ethylic acetate; the increase produced by any one salt is nearly proportional to the quantity added; and the increase produced by equal quantities of all the salts examined is the same, within the experimental error, its mean value being 4·4 per cent. for the addition of 0·1 normal chloride.

To return again to the constants observed for solutions containing sulphuric acid and a sulphate, we must correct them for the accelerating influence of the latter, and, according to the above results, at the rate of 4·4 per cent. for each 0·1 equivalent present per litre.

Before these corrected constants are compared with those obtained for sulphuric acid alone, the latter must be subjected to a reduction for the same reason and in exactly the same manner as that described above for the hydrochloric acid constants, viz., for the change in dissociation produced by the addition of a neutral sulphate. The following example indicates (1) the correction of a velocity constant obtained with a solution of acid and neutral salt, for the accelerating influence of the latter; (2) the reduction of the corresponding velocity constant for acid alone, to the value it has when its degree of dissociation is equal to that which obtains in (1).

Acid Solution.	Velocity Constant per Equivalent Acid.	Dissociation of Acid.
(1) $0{\cdot}2H_2SO_4+0{\cdot}1K_2SO_4$	335	·524
(2) $0{\cdot}2H_2SO_4$	411	·539

$$\text{Corrected velocity constant for (1)} = \frac{335 \times 100}{104{\cdot}5} = 320$$

$$\text{Reduced} \quad ,, \quad ,, \quad (2) = \frac{411 \times {\cdot}524}{{\cdot}539} = 400$$

The corrected and reduced constants thus obtained are then comparable, and the quotient, multiplied by 100, gives the percentage of free acid in solution (1), viz., 80·0.

The table below gives the percentages of free acid and the data for the calculation of the same, in the three series for sulphuric acid and potassium sulphate. They are arranged according to the following scheme.

K_1 = velocity constant for H_2SO_4 (per equivalent).
k_1 = „ „ „ when K_2SO_4 is present.
a = dissociation degree of acid.
a_1 = „ „ when K_2SO_4 is present.
$K = K_1 \times \frac{a_1}{a}$ = reduced constant for H_2SO_4.
k = corrected constant for $H_2SO_4 + K_2SO_4$.
$\frac{k}{K} \times 100$ = percentage of free acid in the solution of acid and salt.

First Series.

Concentration of $H_2SO_4 = c$.
„ „ $K_2SO_4 = 0·1$.

c	K_1	k_1	a	a_1	K	k	$\frac{k \times 100}{K}$
0·025	545	278	·693	·547	432	267	61·6
0·05	493	294	·630	·543	426	281	65·9
0·1	441	317	·560	·536	422	303	71·8
0·2	411	335	·539	·524	400	320	80·0
0·35	407	355	·522	·514	400	339	84·9

Second Series.

Concentration of $H_2SO_4 = 0{\cdot}1$.
,, ,, $K_2SO_4 = c$.

c	K_1	k_1	a	a_1	K	k	$\frac{k \times 100}{K}$
0·4	441	191	·560	·507	399	156	39·1
0·2	,,	245	,,	·523	411	223	54·3
0·1	,,	317	,,	·536	422	303	71·8
0·05	,,	372	,,	·545	429	363	84·7
0·025	,,	405	,,	·552	436	400	91·9

Third Series.

Concentration of acid sulphate $= c$.

c	K_1	k_1	a	a_1	K	k	$\frac{k \times 100}{K}$
0·2	411	270	·539	·515	393	246	62·6
0·15	420	291	·547	·524	402	271	67·4
0·1	442	317	·560	·536	422	303	71·8
0·05	493	368	·630	·556	436	360	82·4
0·025	545	444	·693	·620	488	439	90·0

THE CONCENTRATIONS OF FREE ACID, NEUTRAL AND ACID SULPHATES, IN THE VARIOUS SOLUTIONS.

Having thus determined what fraction of the total sulphuric acid originally present in any solution exists as free acid when a known quantity of potassium sulphate is added to it, the concentrations of the free acid, and of the neutral and acid sulphates in the mixture are easily calculated. In the first series, for example, we find that 80 per cent. free acid exists in a solution which was made 0·2 normal with respect to sulphuric acid and 0·1 normal with respect to potassium sulphate. We therefore have the following concentrations of acid, acid sulphate and neutral sulphate at equilibrium:—

$\frac{80{\cdot}0 \times 0{\cdot}2}{100} = 0{\cdot}1600$ gram. equivalent H_2SO_4 per litre.
$0{\cdot}2 - 0{\cdot}1600 = 0{\cdot}0400$ „ $KHSO_4$ „
$0{\cdot}1 - 0{\cdot}0400 = 0{\cdot}0600$ „ K_2SO_4 „

This simple calculation has been performed in every case, and in the next three tables the concentrations of the free acid, neutral and acid sulphates in the various solutions are given. The numbers in the first column indicate the concentrations of that substance, either acid, neutral sulphate, or acid sulphate, the amount of which was varied in the different experiments of each series.

First Series.

Concentration of $K_2SO_4 = 0{\cdot}1$.

H_2SO_4	Free Acid.	Neutral Sulphate.	Acid Sulphate.
0·025	0·0154	0·0904	0·0096
0·05	0·03295	0·08295	0·01705
0·1	0·0718	0·0718	0·0282
0·2	0·1600	0·0600	0·0400
0·35	0·2972	0·0472	0·0528

Second Series.

Concentration of $H_2SO_4 = 0{\cdot}1$.

K_2SO_4	Free Acid.	Neutral Sulphate.	Acid Sulphate.
0·4	0·0391	0·3391	0·0609
0·2	0·0543	0·1543	0·0457
0·1	0·0718	0·0718	0·0282
0·05	0·0847	0·0347	0·0153
0·025	0·09186	0·0169	0·00814

Third Series.

$KHSO_4$	Free Acid.	Neutral Sulphate.	Acid Sulphate.
0·2	0·1252	0·1252	0·0748
0·15	0·1011	0·1011	0·0489
0·1	0·0718	0·0718	0·0282
0·05	0·0412	0·0412	0·0088
0·025	0·02250	0·02250	0·00250

These are, however, the total concentrations of the three substances, and as the equilibrium, the nature of which it was sought to determine, is that existing between the undissociated portions of each, we must know the value of their degrees of dissociation. As already stated, these were calculated from the work of Kohlrausch, using his newer values for the conductivity at infinite dilution (*Wied. Ann.*, 1893, l. 408).

The calculation of the degree of dissociation of sulphuric acid, and of a neutral sulphate in a solution containing them in definite proportions was carried out in the following manner.

The dissociations of sulphuric acid and of each neutral sulphate at a number of dilutions were calculated. From these the concentrations of the ions at each dilution was obtained, and a curve was then drawn for each substance, in which ionic-concentration was plotted against dissociation. As a first approximation, the dissociations of the acid and of the salt in any mixture were made equal to the value which they would have, if the concentration of each was equal to the sum of the concentrations of the acid and salt. (Arrhenius, *Zeit. physikal. Chem.*, 1890, v. 1). From the dissociations thus obtained, the concentrations of the ions, due to the acid on the one hand, and to the salt on the other, were calculated. The sum of these gave the total concentration of the ions, which corresponded to a certain dissociation on the ionic-concentration dissociation curves referred to above. This gave a second approximation to the real dissociations of the acid and of the salt (Macgregor, *Phil. Mag.* 1896, xli. 276), which did not, as a rule, differ very much from the first.

If the difference was comparatively great, the above process was repeated, and a third approximation obtained, but this was seldom necessary, and only when the dissociations of the acid and salt were very different. An example is appended.

C = concentration. α = degree of dissociation.

	C	α_1 (when $C = 0·15$).	Concentration of ions.		α_2 (from curves).
H_2SO_4	0·05	0·547	0·0274		0·544
+			+	= 0·0919	
Na_2SO_4	0·1	0·645	0·0645		0·652

A third approximation gives the same result.

The degrees of dissociation of the acid sulphates were assumed to be equal to those of the corresponding acetates. This assumption is justifiable if we consider the acid sulphate to be, in the first instance at any rate, a salt of the weak acid $H^{\cdot}(HSO_4)'$. Any small error which may thus arise is one which affects each dissociation degree equally (Kohlrausch, *Wied. Ann.*, 1893, l. 400), and therefore little uncertainty is encountered when the different experiments are compared with each other, as is done in determining the expression for the equilibrium. In a solution of sulphuric acid and a neutral sulphate, the dissociation of the acid salt was then calculated in the same manner as above indicated. The degrees of dissociation of the free acid, neutral sulphate, and acid sulphate being thus determined, the concentrations of the undissociated portions are given at once by $C(1-\alpha)$, where C is any total concentration, and α the corresponding degree of dissociation.

These concentrations of the free acid, neutral and acid sulphates, viz., $C_1(1-\alpha_1)$, $C_2(1-\alpha_2)$, $C_3(1-\alpha_2)$, as well as the corresponding values of $(1-\alpha_1)$, $(1-\alpha_2)$, and $(1-\alpha_3)$ are given in the following tables :—

First Series.

Concentration of $K_2SO_4 = 0{\cdot}1$.

	Free Acid.	Neutral Sulphate.	Acid Sulphate.			
Concentration of H_2SO_4	$C_1(1-\alpha_1)$	$C_2(1-\alpha_2)$	$C_3(1-\alpha_3)$	$(1-\alpha_1)$	$(1-\alpha_2)$	$(1-\alpha_3)$
0·025	·006976	·02775	·001622	·453	·307	·170
0·05	·01506	·02655	·003069	·457	·320	·180
0·1	·03331	·02434	·005584	·464	·339	·198
0·2	·07616	·02220	·008960	·476	·370	·224
0·35	·1445	·01892	·01320	·486	·401	·250

Second Series.

Concentration of $H_2SO_4 = 0{\cdot}1$.

	Free Acid.	Neutral Sulphate.	Acid Sulphate.			
Concentration of K_2SO_4	$C_1(1-\alpha_1)$	$C_2(1-\alpha_2)$	$C_3(1-\alpha_3)$	$(1-\alpha_1)$	$(1-\alpha_2)$	$(1-\alpha_3)$
0·4	·01926	·1417	·01688	·493	·418	·277
0·2	·02624	·05785	·01071	·477	·375	·234
0·15	·03331	·02434	·005584	·464	·339	·198
0·05	·03851	·01088	·002609	·485	·314	·170
0·025	·04114	·004973	·001245	·448	·295	·153

Third Series.

	Free Acid.	Neutral Sulphate.	Acid Sulphate.			
Concentration of $KHSO_4$	$C_1(1-\alpha_1)$	$C_2(1-\alpha_2)$	$C_3(1-\alpha_3)$	$(1-\alpha_1)$	$(1-\alpha_2)$	$(1-\alpha_3)$
0·2	·06071	·04971	·01885	·485	·397	·252
0·15	·04813	·03771	·01125	·476	·373	·230
0·1	·03331	·02434	·005584	·464	·339	·198
0·05	·01830	·01170	·001301	·444	·284	·148
0·025	·008551	·005332	·0002875	·380	·236	·115

THE EXPRESSION FOR THE EQUILIBRIUM.

It is now possible from these results to calculate the unknown exponents *m* and *n* and the value of the constant K in the general expression for the equilibrium given on p. 486, viz.:—

$$\{C_1(1-a_1)\}^m \times \{C_2(1-a)_2\}^n = K.C_3(1-a_3).$$

If we assume that *m* and *n* are the same and equal to $\frac{1}{x}$, the expression may be simplified, and writing C_1, C_2, C_3 for $C_1(1-a_1)$, etc., we obtain

$$C_1 \times C_2 = K_1.C_3^x.$$

From the first series a fairly good constant is obtained with the expression when $\frac{1}{x} = 0{\cdot}75$.

In the general formula, as a first approximation to its real value, put $n = 0{\cdot}75$. On this assumption one value of *m* can be found from any two experiments. From the first of these we obtain

$$C_1^m = \frac{K_1 C_3}{C_2^{0{\cdot}75}},$$

and from the second

$$C_I^m = \frac{K_1 C_{II}}{C^{0{\cdot}75}}.$$

We therefore have

$$\frac{C_1^m}{C_I^m} = \frac{C_3 \ : C_2^{0{\cdot}75}}{C_{III} : C_{II}^{0{\cdot}75}}$$

and

$$m = \log\left(\frac{C_3 \ : C_2^{0{\cdot}7}}{C_{III} : C_{II}^{0{\cdot}75}}\right) \div \log\left(\frac{C_1}{C}\right).$$

By inspection of the concentrations in the different experiments of the first series, it will be seen that $C_2(1 \quad a_2)$ varies but little, so that an error in the value assumed for the exponent *n*, (0·75) will not affect, to any extent, the magnitude of *m* calculated in this manner.

The mean value of *m* was then obtained from the first three experiments only, for according to the method of calculation employed, the greatest "weight" must be attached to the first experiment in this series, the smallest weight to the last, or in other words, a small experimental error is exaggerated in calculation very much more in the last experiment than in the first. The mean value of m was 0·85. Now, in the second series, the variation in $C_1(1-\alpha_1)$ is small, while $C_2(1-\alpha_2)$ varies considerably from the first to the last experiment. Such conditions are most favourable for the calculation of a new value for the exponent n. When this was done in the same manner as above, m having the value 0·85, the value obtained for n was 0·89. It appears, therefore, then m and n are nearly equal, and if we put $m=n=0{\cdot}87$ the results of the experiments will be expressed with considerable accuracy.

But having decided that $m=n$, their value can be calculated from each series independently according to the simplified expression,

$$C_1 \times C_2 = K_1 C_3^x;$$

for we have

$$C_3^x = \frac{C_1 \times C_2}{K_1}$$

and

$$C_{III}^x = \frac{C_I \times C_{II}}{K_1}$$

and therefore

$$x = \log.\frac{C_1 \times C_2}{C_I \times C_{II}} \div \log.\frac{C_3}{C_{III}}.$$

The mean value of x obtained from the first three experiments of the first series was 1·28; from the second series 1·11; and from the third 1·07. The mean of these is 1·15. We would expect this to be nearly equal to that already found by the first method of calculation, and as it happens, it is exactly the same, for $x=\frac{1}{m}=\frac{1}{0{\cdot}87}=1{\cdot}15$. The expression from the equilibrium may now be written

$$C.H_2SO_4(1-\alpha_1) \times C.K_2SO_4(1-\alpha_2) = K\{C.KHSO_4(1-\alpha_3)\}^{1{\cdot}15}.$$

By introducing into this formula the corresponding concentrations of the undissociated portions of free acid, neutral and acid sulphates given on p. 503, the value of K for each experiment was obtained. In calculating the mean constant in each series, it was necessary, as already stated, to assign to each individual constant its own "weight," which varies very much in the different experiments. It was found in the following manner. An experimental error of 1 per cent. was assumed in the velocity constant, and the corresponding percentage difference produced in K was calculated. The "weight" given to each constant was the reciprocal of this number.

In the following tables is shown the value of K calculated from each experiment by the above formula, the "weight" assigned to each, and the probable value of K for each series.

First Series.

H_2SO_4		K_2SO_4	K	"weight"	K × "weight"
0·025	+	0·1	0·311	0·345	0·1073
0·05		,,	0·310	0·282	0·0874
0·1		,,	0·316	0·197	0·0623
0·2		,,	0·383	0·116	0·0444
0·35		,,	0·396	0·070	0·0277
				1·010	0·3291

$$\text{Mean value of K} = \frac{0·3291}{1·010} = 0·326.$$

Second Series.

H_2SO_4		K_2SO_4	K	"weight"	K × "weight"
0·1	+	0·4	0·298	0·497	0·1481
,,		0·2	0·276	0·345	0·0952
,,		0·1	0·316	0·197	0·0623
,,		0·05	0·392	0·096	0·0376
,,		0·025	0·448	0·045	0·0202
				1·180	0·3634

Mean value of K = 0·308.

Third Series.

H_2SO_4		K_2SO_4	K	"weight"	K × "weight"
0·2	+	0·2	0·290	0·242	0·0701
0·15		0·15	0·316	0·225	0·0711
0·1		0·1	0·316	0·197	0·0623
0·05		0·05	0·446	0·102	0·0454
0·025		0·025	0·539	0·074	0·0398
				0·840	0·2887

Mean value of K = 0·342.

The mean value of K for all three series is 0·325, the greatest difference between this and any of the mean values for each series separately, being rather more than 5 per cent. On the whole, however, the constancy of K is not very satisfactory, the different values obtained exhibiting a marked rise with the increase in the amount of free acid present. This may be seen by comparison with the tables on p. 508. It is evident, therefore, that the formula is not the best which it is possible to obtain from the experimental results, and that by finding an appropriate correcting factor a better expression could be arrived at.

Choosing two of the variable quantities, namely, the free acid and the acid sulphate (the value of the third variable being determined by these), the ratio $\frac{C.\mathrm{H_2SO_4}(1-\alpha_1)}{C.\mathrm{KHSO_4}(1-\alpha_3)}$ was calculated. The change in the value of the constant K corresponding to the change in the above ratio was then found and a new exponent y was thus obtained which is the mean value of the formula

$$\frac{\Delta\ \log.\ \mathrm{K}}{\Delta\ \log.\ \frac{C.\mathrm{H_2SO_4}(1-\alpha_1)}{C.\mathrm{KHSO_4}(1-\alpha_3)}}.$$

The expression for the equilibrium may then be written

$$\frac{C.\mathrm{H_2SO_4}(1-\alpha_1)\times C.\mathrm{K_2SO_4}(1-\alpha_2)}{C\{.\mathrm{KHSO_4}(1-\alpha_3)\}^{1\cdot15}}=\mathrm{K}=\mathrm{A}\left\{\frac{C.\mathrm{H_2SO_4}(1-\alpha_1)}{C.\mathrm{KHSO_4}(1-\alpha_3)}\right\}^{y}$$

where A is a new constant and y a new exponent obtained as above indicated. From each series four values of y can be found, and, assigning to each its own "weight," a mean value of y for each series is obtained, namely, 0·195, 0·128, 0·138. The mean of these is 0·154, and the expression reduces to

$$\frac{\{C.\mathrm{H_2SO_4}(1-\alpha_1)\}^{0\cdot85}\times C.\mathrm{K_2SO_4}(1-\alpha_2)}{C.\mathrm{KHSO_4}(1-\alpha_3)}=\mathrm{A}.$$

The values of A, obtained by introducing into this expression the concentrations of the undissociated portions of the free acid, neutral and acid sulphates are given in the next three tables. They correspond to the tables given on p. 506, the probable mean value of A for each series being found as before by assigning to each individual constant its own "weight."

First Series.

H_2SO_4		K_2SO_4	A
0·025	+	0·1	0·250
0·05		,,	0·244
0·1		,,	0·242
0·2		,,	0·279
0·35		,,	0·277

Mean value of A = 0·252.

Second Series.

H_2SO_4		K_2SO_4	A
0·1	+	0·4	0·292
,,		0·2	0·242
,,		0·1	0·242
,,		0·05	0·262
,,		0·025	0·265

Mean value of A = 0·265.

Third Series.

H_2SO_4		K_2SO_4	A
0·2	+	0·2	0·244
0·15		0·15	0·254
0·1		0·1	0·242
0·05		0·05	0·300
0·025		0·025	0·324

Mean value of A = 0·260.

The steady rise in the value of the constant which was apparent when the calculations were made with the previous expression for the equilibrium, has now to a large extent disappeared, and in each series the constant fluctuates about the mean value. In examining the constancy of A it must be remembered that the percentage of free acid in any solution is the quotient of two velocity constants, and as the experimental error may have the opposite sign in these two constants, the corresponding error in the percentage of free acid may be considerably increased. Further, this error is much exaggerated in the constant A, as for example, in one experiment an error of 1 per cent. in the percentage of free acid produces a change of 22 per cent. in A. This, however, is the worst case, as may be seen by inspection of the "weights" given to the individual constants.

The mean value of A for all three series is 0·259, the greatest difference between this and any of the mean values obtained from each series separately being less than 3 per cent. In view of the fact that the method of calculation emphasises the errors of individual experiments, this will be accounted fairly satisfactory.

The unknown factors in the general expression for the equilibrium are now determined, and for solutions containing sulphuric acid and potassium sulphate it may be written

$$\frac{\{C.\mathrm{H_2SO_4}(1-a_1)\}^{0\cdot85}}{C.\mathrm{KHSO_4}(1-a_3)}=\frac{0\cdot259}{C.\mathrm{K_2SO_4}(1-a_2)}.$$

The accuracy of this formula was then tested by making use of it to calculate the percentage of free acid in the various solutions of sulphuric acid and potassium sulphate, the values so obtained being then compared with those actually observed. In this way we eliminate the exaggeration of experimental error which the formula causes, and can arrive at a better conclusion as to the value of the latter. If the expression is a good one, the differences between the observed and calculated values should not be much more than that due to experimental error.

Let B represent the original concentration of the acid, and C that of the sulphate. At equilibrium let x be the concentration of the acid sulphate. According to the above equation the equilibrium is expressed by

$$\frac{\{(\mathrm{B}-x)(1-a_1)\}^{0\cdot85}}{x(1-a_3)}=\frac{\mathrm{A}}{(\mathrm{C}-x)(1-a_2)}.$$

When B and C are known, the value of x is best calculated by an approximation method. Two values of x were chosen, one of which gave a larger value of A than that observed (0·259), the other a smaller value. The value of x, corresponding to the observed constant was then obtained by interpolation. After a little experience, it was easy to choose one value of x so that the constant corresponding to it differed very little from that observed, under which conditions it was possible to interpolate quite accurately.

The above expression may be written thus:—

$$\frac{(\mathrm{B}-x)^{0\cdot85}.(\mathrm{C}-x)}{x}=\mathrm{A}.\frac{(1-a_3)}{(1-a_1)^{0\cdot85}.(1-a_2)}.$$

As the last factor of the right hand member varies but little with small variations in x, its value was determined for each experiment, and treated as a constant. If we make it equal to a, the expression becomes

$$\frac{(\mathrm{B}-x)^{0\cdot85}(\mathrm{C}-x)}{x\times a}=\mathrm{A}.$$

Having found by means of this equation two pairs of corresponding values of x and A, the required value of x, when A is equal to 0·259, was calculated by the ordinary interpolation formula, viz.:—

$$\frac{x - x_1}{x - x_2} = \frac{A_{\text{obs.}} - A_1}{A_{\text{obs.}} - A_2}$$

where x_1 and A_1, x_2 and A_2, x and $A_{\text{obs.}}$ are pairs of corresponding values, and $A_{\text{obs.}}$ is equal to 0·259. From x the percentage of free acid is easily obtained, being equal to $\frac{(B - x)100}{B}$. In the following tables are exhibited the calculated values of x and of the percentage of free acid in all the solutions, together with the observed values of the same.

First Series.

$H_2SO_4 + K_2SO_4$	x.		Percentage Free Acid.	
	Observed.	Calculated.	Observed.	Calculated.
0·025 + 0·1	·00960	·00938	61·6	62·5
0·05 ,,	·01705	·01646	65·9	67·1
0·1 ,,	·0282	·0271	71·8	72·9
0·2 ,,	·0400	·0415	80·0	79·3
0·35 ,,	·0528	·0543	84·9	84·5

Second Series.

$H_2SO_4 + K_2SO_4$	x.		Percentage Free Acid.	
	Observed.	Calculated.	Observed.	Calculated.
0·1 + 0·4	·0606	·0638	39·1	36·2
,, 0·2	·0457	·0442	54·3	55·8
,, 0·1	·0282	·0271	71·8	72·9
,, 0·05	·0154	·01545	84·6	84·6
,, 0·025	·0081	·00825	91·8	91·8

Third Series.

$KHSO_4$	x.		Percentage Free Acid.	
	Observed.	Calculated.	Observed.	Calculated.
0·2	·0748	·0727	62·6	63·6
0·15	·0489	·0484	67·4	67·7
0·1	·0282	·0271	71·8	72·9
0·05	·0088	·0097	82·4	80·6
0·025	·0025	·0030	90·0	88·0

The general agreement between the two sets of numbers in each series is best seen in the curves of the accompanying diagrams. In these the ordinates are the percentages of free acid, while the abscissae are the concentrations of sulphuric acid, potassium sulphate, and potassium hydrogen sulphate respectively, in the first, second, and third series. The continuous line in each diagram is the curve given by the observed results, and the dotted line is the curve obtained by plotting the results calculated by the mathematical expression.

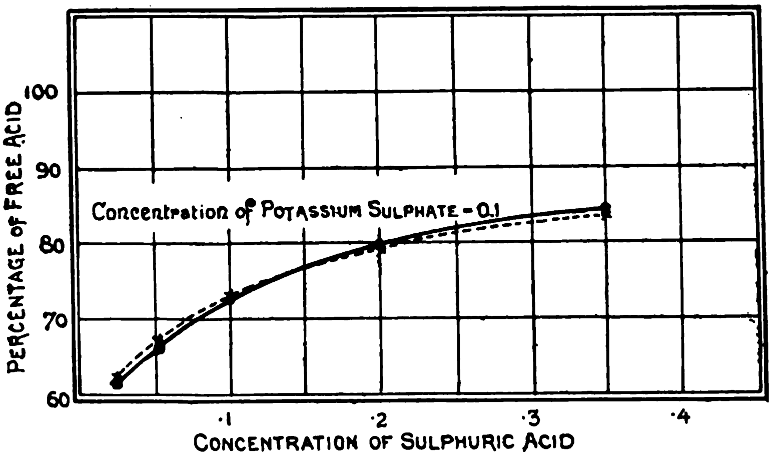

FIG. 1.

It appears that the calculated results are too low when the amount of free acid in the solution is higher than about 80 per cent., and too high when the percentage is less than this. The second series certainly does not exhibit this feature in the same regular way as the other two, the agreement between the two curves being particularly good with the exception of one point

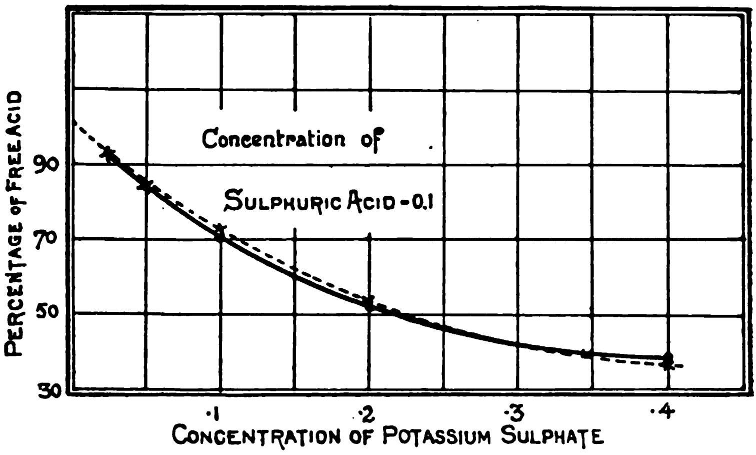

FIG. 2.

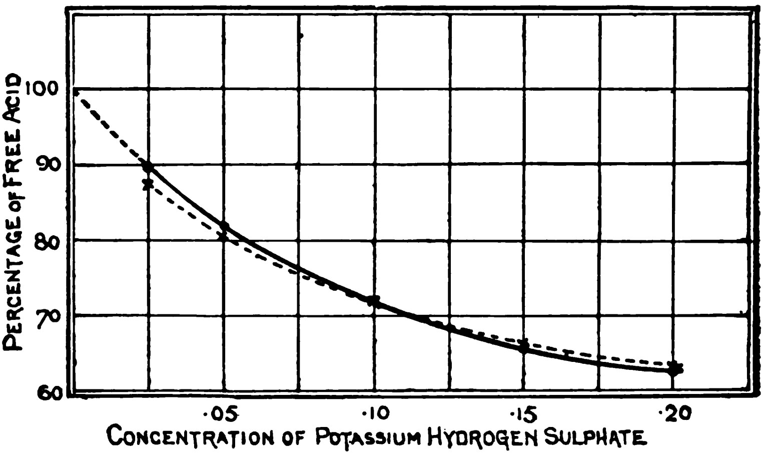

FIG. 3.

where the calculated percentage is below the observed, instead of above it as might be expected.

On the whole, the agreement between the observed and calculated values is fairly satisfactory. The mean real percentage difference between them is about 1·5, and while the experimental error is probably less than this, it may in some cases amount to as much; so that if we take this into consideration, and at the same time the fairly wide range of the experiments, the expression may be said to give at least a good approximation to the equilibrium actually existing in such solutions.

From the percentages of free acid in the various solutions other interesting results may be obtained. The variation of this magnitude with the dilution, and with the ratio of acid to neutral sulphate is seen at once in the curves.

The curve in fig. 1 shows the results of the first series of experiments, in which the concentration of the acid only was varied. If this curve is produced till it meets the origin of co-ordinates, it appears that the percentage of free acid rises at first very rapidly as the concentration of the sulphuric acid increases, and only when the amount of free acid in the solution has reached about 55 per cent. does the rise become somewhat slow. The continued curve also shows, for example, that in a $\frac{N}{100}$ solution of sulphuric acid, at least 50 per cent. of the acid still remains uncombined when as much as ten times the concentration of neutral sulphate is added to it.

If the curve in fig. 2 is continued as shown by the dotted line, it meets the axis of y at the point 100, expressing the fact that when the concentration of the neutral salt is zero, the acid in solution is wholly free.

In fig. 3 the percentages of free acid in solutions of the acid sulphate are plotted against the corresponding concentrations. It is evident that the amount of free acid in the solution increases with increasing dilution of the acid sulphate, a result which is in accordance with that of previous observers. If the curve is produced, it also meets the axis of y at the point 100, which simply means that the acid sulphate at infinite dilution is completely decomposed into free acid and neutral sulphate.

SODIUM AND LITHIUM ACID SULPHATES.

On account of the close resemblance of the alkali metals, and the general analogy which exists between their corresponding salts, it was probable that the equilibria in solutions containing their respective acid sulphates would also be somewhat similar in nature. This conclusion is supported by the fact that the velocity constants obtained for the corresponding solutions of sulphuric acid and the sulphates of potassium, sodium and lithium are very nearly the same, those for the sodium salt being somewhat higher than those for the corresponding potassium salt solutions, and those for the lithium salt rather higher than either. This simply means that the percentage of free acid in corresponding solutions increases slightly in the same order, or that most acid salt is formed when sulphuric acid is added to a solution of potassium sulphate, and least when added to a solution of lithium sulphate. The experimental results were treated exactly as before, and I give therefore only the various tables for sodium and lithium sulphates. These correspond to the tables given for the first series of experiments with potassium sulphate and sulphuric acid, and are arranged in the same way.

SODIUM SULPHATE AND SULPHURIC ACID.

I.

Arranged as on page 499.
Concentration of $H_2SO_4 = c$.
" " $Na_2SO_4 = 0{\cdot}1$.

c	K_1	k_1	a	a_1	K	k	$\frac{k \times 100}{K}$
0·025	545	280	·693	·548	432	268	62·1
0·05	493	300	·630	·544	427	287	67·3
0·1	441	324	·560	·537	423	310	73·2
0·2	411	340	·539	·525	401	324	80·9
0·35	407	356	·522	·515	400	340	84·9

In a paper on the influence of neutral salts on the catalysis of methylic acetate by acids, Trey (*loc. cit.*) gives the results of several series of experiments with solutions of sulphuric acid and neutral sulphates. As it was interesting to know in how far the percentages of free acid calculated from his measurements agreed with mine, I have made use of a few of his numbers for that purpose. Only in the case of sodium sulphate and sulphuric acid has Trey made a long series of experiments, and few of these correspond to the solutions which I employed. I obtained, however, several values of this percentage, for solutions corresponding to mine, by graphic interpolation. These, as well as others calculated directly from Trey's experimental results, are shown in the accompanying table, the figures obtained by interpolation being indicated by brackets. The agreement between the two series of numbers is even better than was to be expected.

			Percentage of Free Acid.	
H_2SO_4		Na_2SO_4	Trey.	Kay.
0·35	+	0·1	(84·5)	84·9
0·2		0·2	63·7	60·6 (for K_2SO_4)
0·2		0·1	80·0	80·9
0·15		0·15	(67·7)	67·4 (for K_2SO_4)
0·1		0·1	(72·7)	73·2
0·08		0·08	74·9	(75·2) (for K_2SO_4)

The concentrations of the free acid, neutral and acid sulphates in the various solutions were calculated as before from $\frac{k \times 100}{K}$, the percentage of free acid in the same.

Concentration of $Na_2SO_4 = 0 \cdot 1$.

H_2SO_4	Free acid.	Neutral sulphate.	Acid sulphate.
0·025	0·01552	0·0905	0·00948
0·05	0·03363	0·0836	0·01637
0·1	0·07323	0·0732	0·02677
0·2	0·1618	0·0618	0·0382
0·35	0·2973	0·0473	0·0527

The product of these concentrations and the undissociated proportions gives the concentrations of the undissociated portions of free acid, neutral sulphate and acid sulphate, viz.:—$C_1(1-\alpha_1)$, $C_2(1-\alpha_2)$ and $C_3(1-\alpha_3)$. These are shown in the next table.

Concentration of $Na_2SO_4 = 0 \cdot 1$.

H_2SO_4	$C_1(1-\alpha_1)$	$C_2(1-\alpha_2)$	$C_3(1-\alpha_3)$	$(1-\alpha_1)$	$(1-\alpha_2)$	$(1-\alpha_3)$
0·025	0·007015	0·02996	0·002180	·452	·331	·230
0·05	0·01534	0·02911	0·003745	·456	·348	·241
0·1	0·03391	0·02761	0·007093	·463	·377	·265
0·2	0·07606	0·02583	0·01146	·475	·418	·300
0·35	0·1442	0·02171	0·01776	·485	·459	·337

From these results a first approximation to the expression for the equilibrium was then obtained in the same manner as for solutions containing potassium sulphate and sulphuric acid. The concentrations were first introduced into the expression deduced directly from the experimental results with the latter solutions, and from the constants thus obtained a correcting factor was calculated as before.

The values of K which satisfy the expression

$$C.H_2SO_4(1-\alpha_1)\times C.Na_2SO_4(1-\alpha_2)=K\{C.NaHSO_4(1-\alpha_3)\}^{1\cdot15}$$

are as follows:—

H_2SO_4		Na_2SO_4	K
0·025	+	0·1	0·242
0·05		,,	0·260
0·1		,,	0·277
0·2		,,	0·338
0·35		,,	0·323.

They exhibit the same marked and almost regular increase from the beginning to the end of the series.

The above expression may be written

$$\frac{C.H_2SO_4(1-\alpha_1)\times C.Na_2SO_4(1-\alpha_2)}{C.\{NaHSO_4(1-\alpha_3)\}^{1\cdot15}}=K=A\left\{\frac{C.H_2SO_4(1-\alpha_1)}{C.NaHSO_4(1-\alpha_3)}\right\}^y$$

where y, as before, is the mean value of the quantity

$$\frac{\Delta\log.K}{\Delta\log.\dfrac{C.H_2SO_4(1-\alpha_1)}{C.NaHSO_4(1-\alpha_3)}}$$

and in this case is equal to 0·370.

On simplification the expression reduces to

$$\frac{\{C.H_2SO_4(1-\alpha_1)\}^{0\cdot63}}{\{C.NaHSO_4(1-\alpha_3)\}^{0\cdot78}}=\frac{A}{C.Na_2SO_4(1-\alpha_2)},$$

and in order that it may be symmetrical with that already found for potassium sulphate solutions, the exponent belonging to the sulphuric acid was made equal to 0·85, and the expression then becomes

$$\frac{\{C.H_2SO_4(1-\alpha_1)\}^{0\cdot85}}{C.NaHSO_4(1-\alpha_3)}=\frac{A_1}{\{C.Na_2SO_4(1-\alpha_2)\}^{1\cdot35}}$$

The values of the new constant A_1, obtained by introducing the concentrations into this expression are given in the next table. The increase in A_1, from the first to the last experiment, has not yet altogether disappeared, but it is not nearly so marked nor so regular. Of course the exaggeration of the experimental errors which the expression causes and which has already been pointed out, holds good here also, and a really good constant is hardly to be expected.

H_2SO_4		Na_2SO_4	A_1
0·025	+	0·1	0·0594
0·05		,,	0·0600
0·1		,,	0·0624
0·2		,,	0·0708
0·35		,,	0·0617

The most probable value of A_1, calculated in the same manner as before, by assigning a "weight" to each individual constant, is 0·0618, and the complete expression for the equilibrium between the undissociated portions of the free sulphuric acid, sodium sulphate and sodium acid sulphate which exist in mixed solutions of the acid and neutral salt is accordingly given by the equation

$$\frac{\{C.H_2SO_4(1-a_1)\}^{0{\cdot}85}}{C.NaHSO_4(1-a_3)} = \frac{0{\cdot}0618}{\{C.Na_2SO_4(1-a_2)\}^{1{\cdot}35}}.$$

The real test of the accuracy and value of this expression is the comparison of the results obtained by means of it with those actually observed, for example, by calculating the percentage of free acid which ought to exist in the various solutions and noting the agreement between this series of numbers with those found by experiment. This was done, and the results are shown in the next table. It will be seen from a comparison of the two series of numbers, and also at once in the curves of the accompanying diagram, that the agreement within the limits of the experiments is particularly good. The same general feature which was noticed in the case of potassium sulphate solutions is present here also, namely, that at the lower percentages the calculated values are too high, and vice versa. The mean real percentage difference between them is, however, considerably less than 1 per cent.

H_2SO_4		Na_2SO_4	Percentage Free Acid. (Observed.)	Percentage Free Acid. (Calculated.)
0·025	+	0·1	62·1	62·9
0·05		„	67·3	67·4
0·1		„	73·2	73·1
0·2		„	80·9	79·6
0·35		„	84·9	84·9

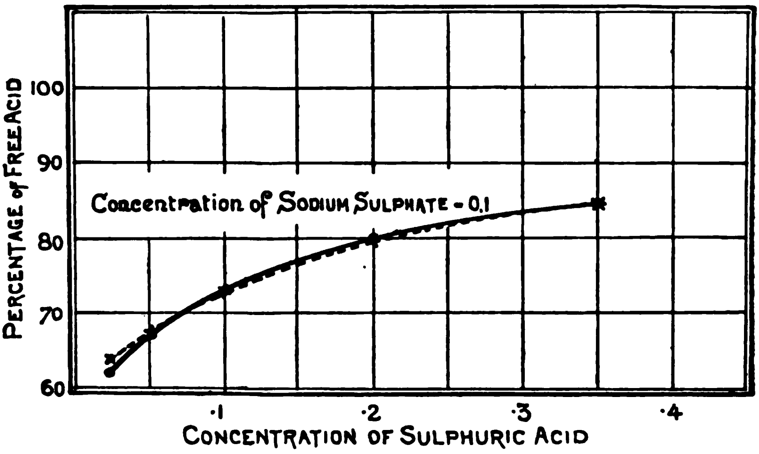

Fig. 4.

LITHIUM SULPHATE.

The experimental results for solutions containing sulphuric acid and lithium sulphate were treated exactly as the preceding. The values of the constant K were first obtained by means of the same expression with the same exponent, and from them a correcting factor determined as before. The value of the exponent y in the latter was 0·365, practically identical with that for sodium sulphate. It appears, therefore, that the equilibrium in this case is represented by an expression of the same form and containing the same exponents as before.

The next tables give all the data for the calculation of the most probable value of the constant A_1, which completely determines the expression for the equilibrium.

Concentration of $H_2SO_4 = c$.
" " $Li_2SO_4 = 0·1$.

C	K_1	k_1	α	α_1	K	k	$\frac{k \times 100}{K}$
0·025	545	282	·693	·550	433	269	62·2
0·05	493	306	·630	·545	427	292	68·4
0·1	441	326	·560	·538	423	311	73·6
0·2	411	344	·539	·526	401	328	81·9
0·35	407	363	·522	·515	401	346	86·3

H_2SO_4	Free Acid.	Neutral Sulphate.	Acid Sulphate.
0·025	0·01555	0·09055	0·00945
0·05	0·03421	0·08421	0·01579
0·1	0·07362	0·07362	0·02638
0·2	0·1638	0·0638	0·0362
0·35	0·3020	0·0520	0·0480

$C_1(1-\alpha_1)$	$C_2(1-\alpha_2)$	$C_3(1-\alpha_3)$	$(1-\alpha_1)$	$(1-\alpha_2)$	$(1-\alpha_3)$
0·006997	0·03350	0·002703	·450	·370	·286
0·01556	0·03267	0·004802	·455	·388	·304
0·03402	0·03064	0·008756	·462	·416	·332
0·07763	0·02947	0·01361	·474	·462	·376
0·01465	0·02621	0·02011	·485	·504	·419

H_2SO_4		Li_2SO_4	A_1	"weight"
0·025	+	0·1	0·0556	·351
0·05		"	0·0597	·262
0·1		"	0·0583	·186
0·2		"	0·0718	·106
0·35		"	0·0712	·065

The values of A_1, given in the above table, were calculated by means of the same expression as that used for sodium sulphate. The probable mean value is 0·0600, which does not differ very much from that found for the sodium salt. According to this result, the equilibrium between the undissociated portions of the sulphuric acid, lithium sulphate, and lithium acid sulphate in the various solutions is given by the expression

$$\frac{\{C.H_2SO_4(1-\alpha_1)\}^{0·85}}{C.LiHSO_4(1-\alpha_3)} = \frac{0·0600}{\{C.Li_2SO_4(1-\alpha_2)\}^{1·35}}.$$

The calculated values of the percentages of free acid given in the next table were obtained by means of this formula. The agreement between them and the observed values is fairly satisfactory, as may be seen in the curves in fig. 5. The general aspect of the two sets of numbers is, on the whole, the same as in the preceding cases, the mean difference between them being somewhat over 1 per cent.

H_2SO_4		Li_2SO_4	Percentage Free Acid. (Observed.)	Percentage Free Acid. (Calculated.)
0·025	+	0·1	62·2	63·9
0·05		,,	68·4	68·5
0·1		,,	73·6	74·0
0·2		,,	81·9	80·2
0·35		,,	86·3	85·3

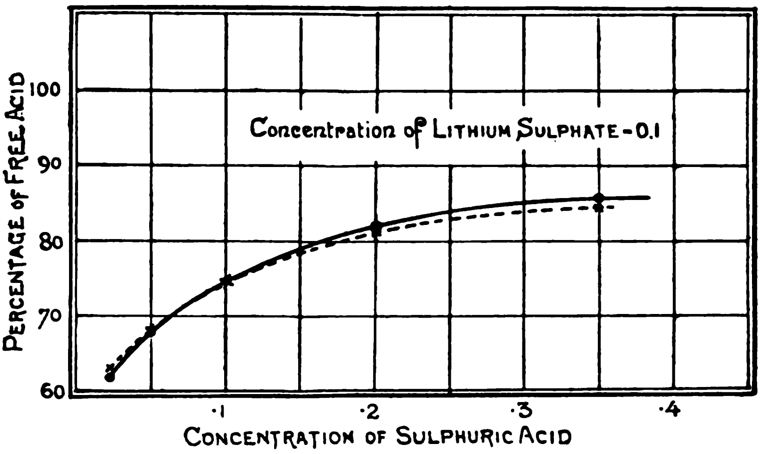

FIG. 5.

EQUILIBRIUM IN SOLUTION BETWEEN SULPHURIC ACID, THE NEUTRAL AND THE ACID SULPHATES OF THE ALKALI METALS.

It may be said, then, as the result of all the experiments and the deductions from them, that the equilibrium between sulphuric acid, the neutral and the acid sulphates of potassium, sodium, and lithium respectively, may be represented in each case by an expression of the form

$$\frac{\{C.H_2SO_4(1-\alpha_1)\}^{0.85}}{C.MHSO_4(1-\alpha_3)} = \frac{A}{\{C.M_2SO_4(1-\alpha_2)\}^x},$$

where x is equal to unity when M is K, and equal to 1·35 when M is Na or Li. The constant A has a characteristic value in each case, and in the expressions involving potassium, sodium, and lithium sulphates it is equal to 0·259, 0·0618, and 0·0600 respectively.

The expression has been deduced quite empirically from the experimental results, and within the limits of the experiments it has been shown to give results which compare favourably with those actually observed.

I hoped to be able to extend this investigation to determine the nature of the equilibrium in solutions of other acid sulphates, but did not succeed in finding a suitable indicator for the titrations, and was therefore unable to measure the velocity constants.

Using litmus as an indicator, I made a few determinations with solutions containing sulphuric acid and ammonium sulphate, but the titrations were unsatisfactory and the results untrustworthy. They showed, however, that sulphuric acid has less action on ammonium sulphate than on the sulphates of the alkali metals.

	Percentage Acid Salt.
$0·35H_2SO_4 + 0·1K_2SO_4$	15·10
„ $+ 0·1Na_2SO_4$	15·06
„ $+ 0·1Li_2SO_4$	13·72
„ $+ 0·1(NH_4)_2SO_4$	12·62?

It appeared from a few measurements, that the action of sulphuric acid is still less in the case of magnesium sulphate.

Ostwald (*Journ. prakt. Chem.*, 1879, xix. 483) determined approximately the amount of free acid in solutions of some acid sulphates by their action on zinc sulphide. Here, of course, the equilibrium is complicated and disturbed by the presence of foreign substances, and although the results differ considerably from those I have obtained, their arrangement in order of magnitude is the same.

Concentration of Acid Sulphate.		Percentage Free Acid.				
		K.	Na.	Li.	NH_4	Mg.
0·125	(Ostwald: Action on zinc sulphide)	79	82	—	91	93
0·1	(Kay: Catalysis of ethylic acetate)	71·8	73·2	73·6	72·6?	79·4
0·2	(Trey: Catalysis of methylic ,,)	63·6	64·0	64·6	—	71·4

I have also added a series calculated from Trey's measurements of the rate of catalysis of methylic acetate. With the exception of one very uncertain value for ammonium acid sulphate, the three horizontal columns show the same arrangement throughout.

This research was carried out during the winter 1897–98 in the physico-chemical laboratory of Stockholms Högskola, Stockholm. I should like to avail myself of the opportunity to express my warmest thanks to Professor Arrhenius for the valuable assistance which he so willingly gave me, and for the interest he took in my work.

Application of Sellmeier's Dynamical Theory to the Dark Lines D_1, D_2 produced by Sodium-Vapour. By Lord Kelvin, *G.C.V.O.*, P.*R.S.E.*

(Read February 6, 1899.)

§ 1. For a perfectly definite mechanical representation of Sellmeier's theory, imagine for each molecule of sodium-vapour a spherical hollow in ether, lined with a thin rigid spherical shell, of mass equal to the mass of homogeneous ether which would fill the hollow. This rigid lining of the hollow we shall call the sheath of the molecule, or briefly the sheath. Within this put two rigid spherical shells, one inside the other, each movable and each repelled from the sheath with forces, or distribution of force, such that the centre of each is attracted towards the centre of the hollow with a force varying directly as the distance. These suppositions merely put two of Sellmeier's single-atom vibrators into one sheath.

§ 2. Imagine now a vast number of these diatomic molecules, equal and similar in every respect, to be distributed homogeneously through all the ether which we have to consider as containing sodium-vapour. In the first place, let the density of the vapour be so small that the distance between nearest centres is great in comparison with the diameter of each molecule. And in the first place also, let us consider light whose wave-length is very large in comparison with the distance from centre to centre of nearest molecules. Subject to these conditions we have (Sellmeier's formula)

$$\left(\frac{v_e}{v_s}\right)^2 = 1 + \frac{m\tau^2}{\tau^2 - \kappa^2} + \frac{m_{,}\tau^2}{\tau^2 - \kappa_{,}^2} \quad . \quad . \quad . \quad . \quad . \quad (1);$$

where m, $m_{,}$ denote the ratios of the sums of the masses of one and the other of the movable shells of the diatomic molecules in any large volume of ether, to the mass of undisturbed ether filling the same volume; κ, $\kappa_{,}$ the periods of vibration of one and the other of the two movable shells of one molecule, on the supposition that the sheath is held fixed; v_e the velocity of light in pure undisturbed ether; v_s the velocity of light of period τ in the sodium-vapour.

§ 3. For sodium-vapour, according to the measurements of Rowland and Bell,[*] published in 1887 and 1888 (probably the most accurate hitherto made), the periods of light corresponding to the exceedingly fine *dark* lines D_1, D_2 of the solar spectrum are ·589618 and ·589022 of a michron.[†] The mean of these is so nearly one thousand times their difference that we may take

$$\kappa = \tfrac{1}{2}(\kappa + \kappa_{,})\left(1 - \frac{1}{2000}\right); \qquad \kappa_{,} = \tfrac{1}{2}(\kappa + \kappa_{,})\left(1 + \frac{1}{2000}\right) . \quad (2).$$

Hence if we put

$$\tau = \tfrac{1}{2}(\kappa + \kappa_{,})\left(1 + \frac{x}{1000}\right) \quad . \quad . \quad . \quad . \quad . \quad (3);$$

and if x be any numeric not exceeding 4 or 5 or 10, we have

$$\left(\frac{\kappa}{\tau}\right) \doteqdot \; - \frac{1}{1000}(2x+1) ; \quad \left(\frac{\kappa_{,}}{\tau}\right)^2 \doteqdot 1 - \frac{1}{1000}(2x-1) . \quad (4);$$

whence

$$\frac{\tau^2}{\tau^2 - \kappa^2} \doteqdot \frac{1000}{2x+1} ; \quad \frac{\tau^2}{\tau^2 - \kappa_{,}^2} \doteqdot \frac{1000}{2x-1} \quad . \quad . \quad . \quad . \quad (5).$$

Using this in (1), and denoting by μ the refractive index from ether to an ideal sodium-vapour with only the two disturbing atoms m, $m_{,}$, we find

$$\left(\frac{v_e}{v_s}\right)^2 = \mu^2 = 1 + \frac{1000m}{2x+1} + \frac{1000m_{,}}{2x-1} \quad . \quad . \quad . \quad . \quad (6).$$

§ 4. When the period, and the corresponding value of x according to (3), is such as to make μ^2 negative, the light cannot enter the sodium-vapour. When the period is such as to make μ^2 positive, the proportion, according to Fresnel and according to the most probable dynamics, of normally incident light which enters the vapour is

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[*] Rowland, *Phil. Mag.*, 1887, first half-year; Bell, *Phil. Mag.*, 1888, first half-year.

[†] "Michron" is the name which I have given to a special unit of time such that the velocity of light is one mikrom of space per michron of time, the mikrom being one millionth of a metre. The best determinations of the velocity of light in undisturbed ether give 300,000 kilometres, or 3×10^{14} mikroms, per second. This makes the michron $\frac{1}{3} \times 10-^{14}$ of a second.

$$1-\left(\frac{\mu-1}{\mu+1}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

if the transition from space, where the propagational velocity is v_e, to medium in which it is v_s, were infinitely sudden.

§ 5. Judging from the approximate equality in intensity of the bright lines D_1, D_2 of incandescent sodium-vapour; and from the approximately equal strengths of the very fine dark lines D_1, D_2 of the solar spectrum; and from the approximately equal strengths, or equal breadths, of the dark lines D_1, D_2 observed in the analysis of the light of an incandescent metal, or of the electric arc, seen through sodium-vapour of insufficient density to give much broadening of either line; we see that m and $m_,$ cannot be very different, and we have as yet no experimental knowledge to show that either is greater than the other. I have therefore assumed them equal in the calculations and numerical illustrations described below.

§ 6. At the beginning of the present year I had the great pleasure to receive from Professor Henri Becquerel, enclosed with a letter of date Dec. 31, 1898, two photographs of anomalous dispersion by prisms of sodium-vapour,* by which I was astonished and delighted to see not merely a beautiful and perfect demonstration of the "anomalous dispersion" towards infinity on each side of the zero of refractivity, but also an illustration of the characteristic nullity of absorption and finite breadth of dark lines, originally shown in Sellmeier's formula † of 1872, and now, after 27 years, first actually seen. Each photograph showed dark spaces on the high sides of the D_1, D_2 lines, very narrow on one of the photographs; on the other much broader, and the one beside the D_2 line decidedly broader than the one beside the D_1 line; just as it should be according to Sellmeier's formula, according to which also the density of the vapour in the prism must have been greater in the latter case than in the former. Guessing from the ratio of the breadths of the dark bands to the space between their D_1, D_2 borders, and from a slightly greater

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* A description of Professor Becquerel's experiments and results will be found in *Comptes Rendus*, Dec. 5, 1898, and Jan. 16, 1899.

† Sellmeier, *Pogg. Ann.*, vol. cxlv. (1872) pp. 399, 520; vol. cxlvii. (1872) pp. 387, 525.

breadth of the one beside D_2, I judged that m must in this case have been not very different from ·0002; and I calculated accord-

FIG. 1. $m = ·0002$.

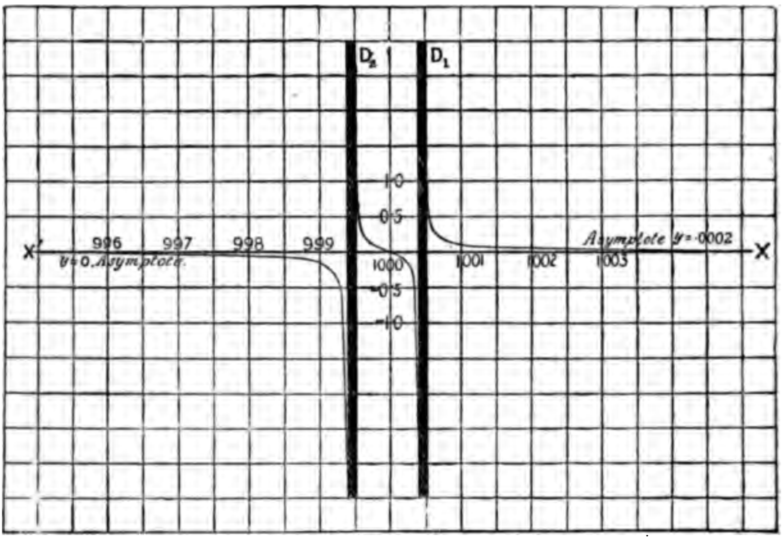
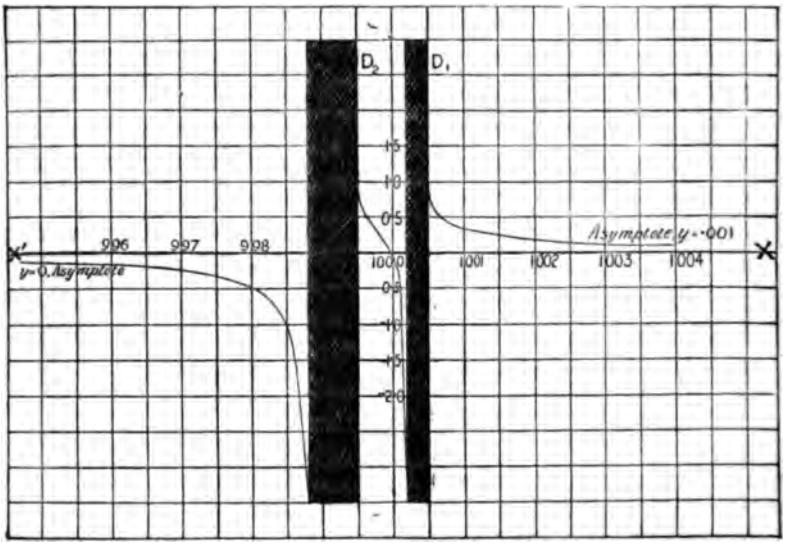


FIG. 2. $m = 001$



ingly from (6) the accompanying graphical representation showing the value of $1-\frac{1}{\mu}$, represented by y in fig. 1. Fig. 2 represents

similarly the value of $1 - \frac{1}{\mu}$ for $m = \cdot 001$, or density of vapour five times that in the case represented by fig. 1. Figs. 3 and 4 represent the ratio of intensities of transmitted to normally incident light for the densities corresponding to figs. 1 and 2; and fig. 5 represents the ratio for the density corresponding to the value $m = \cdot 003$. The following table gives the breadths of the dark bands for densities of vapour corresponding to values of m from ·0002 to fifteen times that value; and fig. 6 represents graphically the breadths of the dark bands and their positions relatively to the bright lines D_1, D_2 for the first five values of m in the table.

FIG. 3. $m = \cdot 0002$.

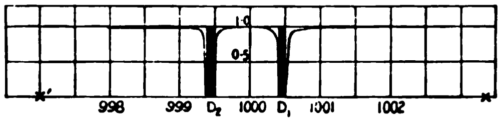


FIG. 4. $m = \cdot 001$.

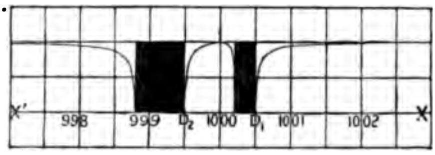
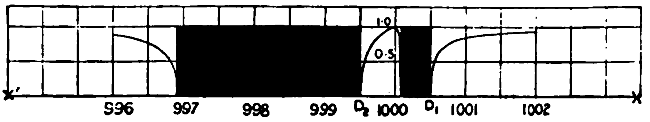


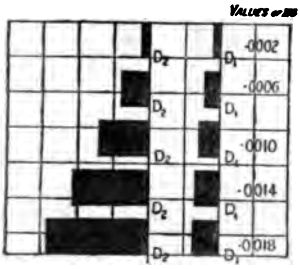
FIG. 5. $m = \cdot 003$.



Values of m.	Breadths of Bands.	
	D_1.	D_2.
·0002	·09	·11
·0006	·217	·383
·0010	·293	·707
·0014	·340	1·060
·0018	·371	1·429
·0022	·392	1·808
·0026	·408	2·192
·0030	·419	2·581

§ 7. According to Sellmeier's formula the light transmitted through a layer of sodium-vapour (or any transparent substance to which the formula is applicable) is the same whatever be the thickness of the layer (provided of course that the thickness is many times the wave-length). Thus the D_1, D_2 lines of the spectrum of solar light, which has travelled from the source through a hundred kilometres of sodium-vapour in the sun's atmosphere, must be identical in breadth and penumbras with those seen in a laboratory experiment in the spectrum of light transmitted through half a centimetre or a few centimetres of sodium-vapour, of the same density as the densest part of the sodium-vapour in the portion of the solar atmosphere traversed by the light analysed in any particular observation. The question of temperature cannot occur except in so far as the density of the vapour, and the clustering in groups of atoms or non-clustering (mist or vapour of sodium), are concerned.

FIG. 6.



§ 8. A grand inference from the experimental foundation of Stokes' and Kirchhoff's original idea is that the periods of molecular vibration are the same to an exceedingly minute degree of accuracy through the great differences of range of vibration presented in the radiant molecules of an electric spark, electric arc, or flame, and in the molecules of a comparatively cool vapour or gas giving dark lines in the spectrum of light transmitted through it.

§ 9. It is much to be desired that laboratory experiments be made, notwithstanding their extreme difficulty, to determine the density and pressure of sodium-vapour through a wide range of temperature, and the relation between density, pressure, and temperature of gaseous sodium.

[Added, April 7, 1899.]

§ 10. Passing from the particular case of sodium, I add an application of Sellmeier's formula, (1) above, to the case of a gas or vapour having in its constitution only a single molecular period κ. Taking $m_{,}=0$ in (1), we see that the square of the refractive index for values of r very large in comparison with κ is $1+m$. And remembering that the dark line or band extends through the range of values for which $(v_e/v_s)^2$ is negative, and that $(v_e/v_s)^2$ is zero at the higher border, we see from (1) that the dark band extends through the range from

$$\tau=\kappa \text{ to } \tau=\frac{\kappa}{\sqrt{1+m}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

§ 11. As an example suitable to illustrate the broadening of the dark line by increased density of the gas, I take $m=a\times 10^{-4}$, and take a some moderate numeric not greater than 10 or 20. This gives for the range of the dark band from

$$\tau=\kappa \text{ to } \tau=\kappa(1-\tfrac{1}{2}a\times 10^{-4}) \quad . \quad . \quad . \quad . \quad . \quad (9);$$

and for large values of τ it makes the refractive index $1+\frac{1}{2}a\times 10^{-4}$, and therefore the refractivity, $\frac{1}{2}a\times 10^{-4}$. If for example we take $a=6$, the refractivity would be ·0003, which is nearly the same as the refractivity of common air at ordinary atmospheric density.

§ 12. Taking $\kappa=1000$, we have, for values of τ not differing from 1000 by more than 10 or 20,

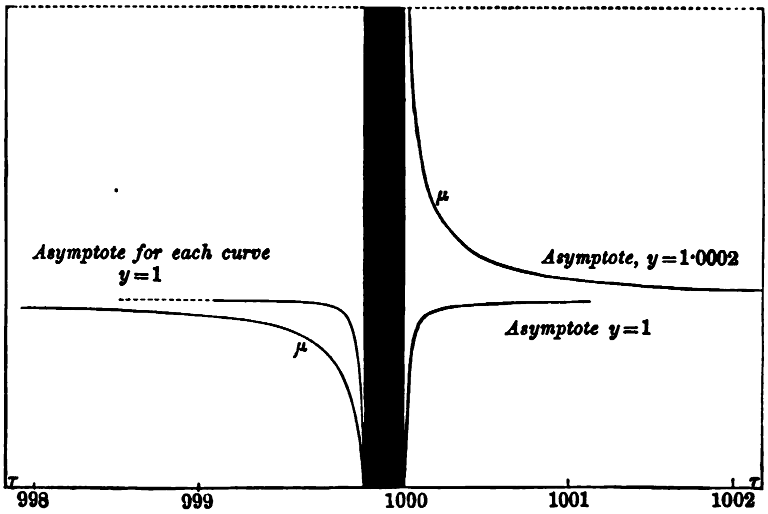
$$\frac{\tau^2}{\tau^2-\kappa^2}\doteq\frac{1000}{2x}, \text{ where } x=\tau-1000 \quad . \quad . \quad . \quad . \quad (10).$$

Thus we have

$$\mu=\sqrt{1+\frac{a}{20x}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

In fig. 7 the curve marked μ represents the values of the refractive index corresponding to values of τ through a small range above and below κ, taking $a=4$. The other curve represents the proportionate intensity of the light entering the vapour, calculated from these values of μ by (7) above.

FIG. 7.



§ 13. The following table shows calculated values for the ordinates of the two curves; also values (essentially negative) for the formula of intensity calculated from the negative values of μ algebraically admissible from (11).

The negative values of μ have no physical interpretation for either curve; but the consideration of the algebraic prolongations of the curves through the zero of ordinates on the left-hand side of the dark band illustrates the character of their contacts. The physically interpreted part of each curve ends abruptly at this zero; which for each curve corresponds to a maximum value of x. The algebraic prolongation of the μ curve on the negative side is equal and similar to the curve shown on the positive side. But the algebraic prolongation of the intensity curve through its zero, as shown in the table, differs enormously from the curve shown on the positive side. To the degree of approximation to which we have gone, the portions of the intensity curve on the left and right hand sides of the dark band are essentially equal and similar. This proves that so far as Sellmeier's theory represents the facts, the penumbras are equal and similar on the two sides of a single dark line of the spectrum uninfluenced

x	μ	$1-\left(\frac{\mu-1}{\mu+1}\right)^2$	
		μ positive.	μ negative.
− 1·0	·894		
− ·9	·882		
− ·8	·866		
− ·7	·845	·993	− 141·9
− ·6	·816	·988	− 82·3
− ·5	·774	·984	− 61·5
− ·4	·707	·971	− 33·5
− ·3	·577	·928	− 12·9
− ·2	0·	0	0
− ·1	imaginary	imaginary	imaginary
− ·0	∞	0	0
+ ·1	1·732	·928	− 12·9
+ ·2	1·414	·971	− 33·5
+ ·3	1·291	·984	− 61·5
+ ·4	1·247	·988	− 82·3
+ ·5	1·183	·993	− 141·9
+ ·6	1·155		
+ ·7	1·134		
+ ·8	1·118		
+ ·9	1·105		
+ 1·0	1·095		

by others. It is also interesting to remark that according to Sellmeier as now interpreted, the broadening of a single undisturbed dark line, produced by increased density of the gas or vapour, is essentially on the high side of the finest dark line shown with the least density, and is in simple proportion to the density of the gas.

Note on Polarisation Phenomena observed in Quantitative Electrolytic Determinations. By Hugh Marshall, D.Sc.

(Read March 6, 1899.)

The phenomena to which it is desired to call attention were first observed in the course of an investigation into the electrolytic methods of determining nickel, carried out in connection with the work of the British Association Committee on the Electrolytic Methods of Quantitative Analysis. The results of the investigation are given in the Report of last year's meeting (Bristol, 1898, p. 300), but the present matter is not referred to there, as it was first noticed only during the preparation of the report.

The apparatus employed for most of the determinations consisted of the usual Classen form of cathode basins, with disc anodes. The current was drawn from storage batteries at 12 volts, variable resistances being used in the circuits. A Davies' ammeter could be introduced easily into any circuit at will, and a Davies' voltmeter was also used to indicate the difference of potential at the electrodes. These measuring instruments were so arranged and connected that the operation which introduced the ammeter into a circuit simultaneously brought the voltmeter into action; hence it happened that voltmeter readings were always taken conjointly with ammeter ones, although attention was paid almost entirely to the effects of varying the current density, and to any changes which might occur during the course of the experiment, with no special consideration of the potential involved.

For the determination of quantities of nickel varying between 0·1 and 0·5 gram, the conditions adopted as most generally suitable when employing the usual method were those given below:—

Volume of solution, about 135 c.c. (giving about 100 sq. cm. of cathode area), containing 5 grams each of ammonium sulphate and of ammonia; current density at cathode (amp. per sq. decim.), 0·6–0·7; ordinary temperature; time necessary, about 4 hours.

As it was found that the method of testing for complete deposition of the metal by withdrawing a portion of the liquid and examining it chemically was inconvenient and unsatisfactory, a considerable number of experiments had been carried out for the purpose of obtaining an idea of the time necessary for complete deposition with varying current strengths and different quantities of metal. As the result of these, the above-mentioned limit of 4 hours was fixed as sufficient, with the other conditions as stated, while not unduly excessive for the smaller quantities of metal; the time necessary for complete deposition is nothing like proportional to the amount of Ni present originally in the solution.

The ammeter readings taken at intervals in the course of an experiment kept very constant. There were slight differences about the beginning, probably due to the change of temperature caused by the current; thereafter the readings remained constant for a considerable time, sometimes until the experiment was stopped, though generally there was a very slight falling off at the end, but the total variation seldom exceeded about 0·03 ampere. On tabulating and comparing a series of experiments in which the action of the current had been continued for varying lengths of time, peculiarities were unexpectedly observed in the voltmeter readings. Here, again, there was always a certain variation at the beginning of an experiment, then a period during which the difference of potential was practically constant, but, in some cases only, there was a considerable rise in the final reading, amounting occasionally to fully 0·5 volt; in other cases there was no evidence of a distinct rise in the voltmeter readings. On further comparing the various experiments it was found that, as a rule, those belonging to the second class were the determinations in which deposition had not been completed, as evidenced by a somewhat low percentage result and the presence of small quantities of nickel in the decanted liquid. (The latter was always tested by means of potassium thiocarbonate.) Those cases which showed a marked rise of potential corresponded to good final results.

These facts seemed to indicate that the removal of the last small quantities of nickel from solution corresponded to a considerable increase in the resistance between the electrodes. The change could not be caused by diminished conductivity of the solution, due to the removal of nickel, because of the large quantity of ammonium sulphate present, and the small quantity of nickel involved. It could only be ascribed to an increase of polarisation, similar to that observed in the electrolysis of mixed salts.

By leaving the ammeter and voltmeter continuously in circuit during several experiments, the progress of the change was studied more fully. It was found to take place in a somewhat peculiar manner. The rise of potential began very gradually, but before long the needle sprang back to its former position; this was followed by another slow, steady rise, again followed by a sudden lapse. These changes continued for some time, the increase becoming more and more marked, and the lapses gradually less and less frequent, and finally the instrument remained steady at about 0·4–0·5 volt above the former level; prolonged electrolysis had very little further effect, if any.

In order to test the value of this effect as an indicator in quantitative determinations, a number of experiments were carried out with varying quantities of nickel, employing the methods already indicated, but stopping the electrolysis as soon as the voltmeter indicated that the potential had risen to the higher level, instead of waiting the standard time of 4 hours, as was done previously. In every case the results were quite satisfactory, and were no lower than those obtained in duplicate determinations in which electrolysis was continued an hour or two longer. A selection of typical results is given below.

	Weight of Salt.	Weight of Deposit.	Percentage.	Time.	Volt.	Amp.	Temp.
				h.m.			°C.
I.	0·9075	0·1342	14·78	0.0	3·15	0·62	14
				1.0	3·2	·63	22
				1.45	3·2	·64	24
				2.0	3·2	·63	25
				2.15	3·35	·63	
				2.30	3·35	·63	
				2.40	3·35	·63	
				2.50	3·5	·62	
				3.0	3·6	·61	25
II.	0·4378	0·0646	14·76	0.0	3·3	0·62	12
				1.45	3·35	·62	22
				2.0	3·4	·61	
				2.15	3·45	·60	
				2.30	3·5	·60	
				2.45	3·5	·60	
				3.0	3·7	·59	22

III.	2·0977	0·3096	14·76	0.0	3·05	0·61	12
				2.30	3·1	·58	23
				2.45	3·2	·58	24
				3.0	3·3	·56	
				3.15	3·3	·56	
				3.30	3·5	·55	
				3.45	3·5	·55	24
IV.	3·0809	0·4593	14·73	0.0	2·9	0·60	19
				1.55	3·0	·58	24
				2.25	3·1	·58	
				2.45	3·2	·58	
				3.10	3·3	·58	
				3.30	3·4	·57	24

It was next sought to get a more accurate idea of the actual quantity of nickel involved in causing the difference of potential; this was effected by adding a nickel solution of known strength to a nickel-free solution undergoing electrolysis in a basin coated with nickel. In some experiments the deposit had been previously washed and dried in connection with a quantitative experiment. In these cases a solution of 5 grams each of ammonium sulphate and ammonia was poured into the basin so as not quite to reach the edge of the deposit, and this was electrolysed for half-an-hour before beginning the experiment proper. In other cases nickel was completely deposited from a solution in the ordinary way, and then the nickel solution of known strength added without any further treatment. The results obtained were exactly alike in the two cases. The nickel solution employed contained 11 m.g. of Ni. per c.c., and one drop of it corresponded approximately to 0·45 m.g. It was added drop by drop to the electrolytic solution, thoroughly mixed with the latter by blowing gently through the liquid, and the effect noted. The results obtained are illustrated below.

	V.		VI.	
	Volume of Solution, 135 c.c.		Volume of Solution, 130 c.c.	
Nickel added.	Volt.	Amp.	Volt.	Amp.
0·	4·25	0·60	3·75	0·57
0·45 m.g.	4·1	·63	3·65	·58
0·9	4·0	·64	3·5	·59
1·35	3·9	·65	3·4	·60
1·8	3·8	·65	3·35	·60
2·25	3·7	·65	3·3	·60
2·7	3·65	·66	3·3	·60
4	3·65	·66	3·3	·60
Next morning	4·1	·64	3·8	·53

In each of the above experiments, electrolysis was continued all night, so that it may be assumed that the nickel had been completely removed again when the readings were taken next morning. In other experiments, greater quantities of nickel, up to 20–30 m.g., were added, without producing any further appreciable change.

These results seem to indicate that when the potential begins to rise in an ordinary determination, only two or three milligrams of nickel remain to be deposited. The time necessary to effect this is out of all proportion, as may be seen by referring to experiments I.–IV. The current employed to eliminate these last traces is very greatly in excess of that theoretically necessary, and it is therefore very questionable whether any advantage is secured by increasing the current towards the end of an electrolysis, as is sometimes recommended; the increased escape of gas would certainly have the effect of stirring up the liquid more, but probably a much better result would be obtained by diminishing the current and agitating the liquid.

When small quantities of nickel have to be determined, the proportion of current wasted becomes greater, and there is therefore greater risk of the deposit being affected by secondary actions; and it is precisely in such cases where a slight error, due to incomplete deposition on the one hand, or to impurity of deposit on the other, has to be specially guarded against. In the Report already referred to, it was therefore recommended to employ small vessels and small volumes of solution—other quantities being in proportion—when determining small quantities of nickel, as better results are then obtained, though the benefit is not nearly so marked in the case of nickel as in that of cobalt. The standard current density then means a much diminished current strength, but, owing to the much smaller volume of liquid from which the nickel has to be deposited, deposition was shown to be completed at least as soon as in duplicate determinations on the larger scale. By making use of the voltmeter readings as indicator, it has now been shown by a number of experiments that the advantage is distinctly in favour of the smaller vessel. For these experiments wide and somewhat shallow platinum crucibles were employed, with an anode of coiled platinum wire. The volume of solution was about 18 c.c., giving about 25 sq. cm. of cathode surface, which was one-fourth of that with the large basins, although the volume was only about one-seventh. Three-quarters of a gram each of ammonium sulphate and ammonia were added, and a current of 0·16 to 0·18 ampere employed.

Duplicate determinations carried out in the two ways, with the same quantity of nickel, showed that deposition was always completed sooner in the smaller vessel, despite the feebler current, while the completion was indicated more clearly on account of the greater rapidity with which the rise of potential took place. The following results were obtained with about 0·17 gm. of nickel ammonium sulphate in each case, corresponding to about 25 mg. of nickel.

Small vessel, 18 c.c.

	Time h.m.	Volt.	Amp.
VII.	0.0	2·85	0·15
	0.30	2·95	·15
	0.45	3·0	·15
	1.0	3·1	·15
	1.15	3·3	·14
	1.30	3·4	·14
VIII.	0.0	3·0	0·17
	0.20	3·0	·17
	0.50	3·1	·17
	1.10	3·3	·17
	1.25	3·4	·17
	1.30	3·5	·16

Large basin, 135 c.c.

	Time. h.m.	Volt.	Amp.
IX.	0.0	3·1	0·60
	0.30	3·15	·58
	0.45	3·15	·58
	1.0	3·15	·58
	1.30	3·2	·58
	1.45	3·25	·58
	2.0	3·3	·57
	2.15	3·4	·57
X.	0.0	3·05	0·65
	1.0	3·15	·68
	2.10	3·35	·68
	2.30	3·5	·66
	2.40	3·6	·65

Experiments similar to V. and VI., but with the smaller vessels, further showed that the actual quantity of nickel involved in causing the change is less than with the larger volume of solution; as was to be expected, it is the strength of solution which determines the effect, so that when the rise of potential sets in with an experiment on the smaller scale the quantity of nickel still to be deposited is only a fraction of what it is in the large apparatus. This is shown by the following results :—

XI.

Volume of Solution, 18 c.c.

Nickel added.	Volt.	Amp.
0·	3·55	0·16
0·45 m.g.	3·10	·18
0·9	3·05	·18
1·35	3·0	·18
1·8	3·0	·18
2·7	3·0	·18

It would appear, from the experiments above described, that when the amount of nickel in solution is reduced to a very small amount—say 1 part in about 50,000—a film of some other substance is deposited, giving a higher polarisation effect. At first this is replaced more or less by nickel, and occasionally breaks down, as shown by the lapses to the original potential, possibly on account of richer solution being brought against it by convection currents, but it ultimately becomes permanent when the nickel deposition is completed. As the deposition of nickel from ammoniacal solutions must be a secondary action (the nickel forming part of a complex ion), it would probably be more correct to assume the formation of a film, other than nickel, from the very commencement; owing to the nickel solution acting as depolariser, however, the polarisation effect due to this primary deposit is only observable when the layer of solution next the cathode is free from nickel. If this is the case, it is probable that with a much feebler current density, the proportion of nickel remaining in the bulk of the solution would fall still further before the rise in polarisation set in distinctly.

The phenomenon, when observed, appears to be an infallible indication of complete deposition, but whether or not it may be trusted to appear in all cases remains to be seen; probably the presence of certain substances may interfere and prevent it.

These experiments have been carried out simply from the point of view of the practical application to quantitative work, and it is proposed to extend them to the other kinds of solution sometimes employed, and to other metals.

The Multiplication of an Alternant by a Symmetric Function of the Variables. By Thomas Muir, LL.D.

(Read February 6, 1899.)

(1) As is well known, the simplest form of alternant is

$$| a^0b^1c^2d^3 \ldots . |$$

and the problem of multiplying it by any symmetric function of $a, b, c, d, \ldots$ has been in a manner fully solved.

(2) When the symmetric function is linear in each of the variables—that is to say, when it takes any of the forms Σa, Σab, $\Sigma abc, \ldots .$—the result is an alternant got from the multiplicand by increasing the last index, the last two indices, the last three indices, ... respectively by 1. Thus, writing for shortness' sake five variables only, we have

$$\begin{aligned}
| a^0b^1c^2d^3e^4 | \,.\, \Sigma a &= | a^0b^1c^2d^3e^5 |, \\
| a^0b^1c^2d^3e^4 | \,.\, \Sigma ab &= | a^0b^1c^2d^4e^5 |, \\
| a^0b^1c^2d^3e^4 | \,.\, \Sigma abc &= | a^0b^1c^3d^4e^5 |, \\
| a^0b^1c^2d^3e^4 | \,.\, \Sigma abcd &= | a^0b^2c^3d^4e^5 |, \\
| a^0b^1c^2d^3e^4 | \,.\, \Sigma abcde &= | a^1b^2c^3d^4e^5 |.
\end{aligned}$$

This was first established in 1825 by Schweins in his *Theorie der Differenzen und Differentiale*, p. 378; but it is also barely possible that it was known to Prony in 1795 (see *Journ. de l'Ec. Polyt.*, i. pp. 264, 265), and Cauchy in 1812 (see *Journ. de l'Ec. Polyt.*, x. pp. 49, 50).

(3) When the symmetric function is non-linear, the result takes the form not of one alternant, but of an aggregate of alternants. These cannot be so readily specified, but the mode of obtaining them can be made clear without any difficulty. Let us take the case of the function Σa^3b, the multiplicand being $| a^0b^1c^2d^3 |$. Since the term $a^3b = a^3b^1c^0d^0$ we may specify it by the four indices alone, viz., 3 1 0 0, and in this way shall have

$$\begin{aligned}\Sigma a^3b &= 3100\\ &\quad+3010\\ &\quad+3001\\ &\quad+\ldots\end{aligned}$$

Now the required multiplication is performed by adding each of these twelve sets of four indices to the indices 0 1 2 3 of the multiplicand, the first index to the first, the second to the second, and so on, the product being the sum of twelve alternants whose indices are the indices resulting from the twelve additions. It will be found that a considerable number of the twelve alternants vanish by reason of the equality of two of the four indices, and that, in fact, we have finally

$$\begin{aligned}|a^0b^1c^2d^3|.\Sigma a^3b &= |a^3b^1c^2d^4| + |a^1b^4c^2d^3| + |a^0b^2c^5d^3|\\ &\qquad + |a^0b^1c^5d^4| + |a^0b^1c^3d^6|,\\ &= 2|a^1b^2c^3d^4| - |a^0b^2c^3d^5| - |a^0b^1c^4d^5| + |a^0b^1c^3d^6|.\end{aligned}$$

(4) The nature of this process is seen to be such that if we were told beforehand that $|a^0b^2c^3d^5|$ was one of the alternants appearing in the expression of the product, and were asked to find its coefficient, we could do so only by finding all the other alternants as well. With a view to ascertaining whether there is no exception to this, let us not use the multiplier as it stands, but take instead the expression for it in terms of Σa, Σab, Σabc, ... , viz.,

$$(\Sigma a)^2\Sigma ab - 2(\Sigma ab)^2 - \Sigma a\Sigma abc + 4\Sigma abcd.^*$$

We then find

$$\begin{aligned}|a^0b^1c^2d^3|.(\Sigma a)^2\Sigma ab &= |a^0b^1c^2d^4|.\Sigma a.\Sigma ab = \{|a^0b^1c^3d^4| + |a^0b^1c^2d^5|\}.\Sigma ab\\ &= |a^1b^2c^3d^4| + 2|a^0b^2c^3d^5| + |a^0b^1c^4d^5| + |a^0b^1c^3d^6|,\\ -|a^0b^1c^2d^3|.2(\Sigma ab)^2 &= -2|a^1b^2c^3d^4| - 2|a^0b^2c^3d^5| - 2|a^0b^1c^4d^5|,\\ -|a^0b^1c^2d^3|.\Sigma a\Sigma abc &= -|a^1b^2c^3d^4| - |a^0b^2c^3d^5|,\\ +|a^0b^1c^2d^3|.4\Sigma abcd &= 4|a^1b^2c^3d^4|,\end{aligned}$$

and thus by addition obtain

$$|a^0b^1c^2d^3|.\Sigma a^3b = 2|a^1b^2c^3d^4| - |a^0b^2c^3d^5| - |a^0b^1c^4d^5| + |a^0b^1c^3d^6|$$

as before.

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* See Hirsch's Tables, given in Salmon's *Modern Higher Algebra*, 4th ed., pp. 350–356.

Now, if we look carefully at the four partial products obtained during this process, we see that one alternant occurs in every one of them, viz., the alternant $|a^1b^2c^3d^4|$, and further, that its coefficient in each is simply the coefficient of the corresponding partial multiplier; so that its coefficient in the complete product is the aggregate of the coefficients in the expression for Σa^3b in terms of Σa, Σab, Σabc, . . . viz., $1 - 2 - 1 + 4$.

(5) The general theorem, of which the foregoing is a particular case, may be formulated thus:—

If the alternant $|a^0b^1c^2 \ . \ . \ . \ k^{n-2}l^{n-1}|$ *be multiplied by any symmetric function of* a, b, c, d, . . . *of the* t*th* *degree,* t *being not greater than* n, *one term of the product is got from the multiplicand by increasing each of its last* t *indices by* 1, *and the coefficient of this term is the same symmetric function of the roots of the equation*

$$x^n - x^{n-1} + x^{n-2} - \ . \ . \ . \ + (^-1)^n 1 = 0.$$

By way of proof we may reason as follows:—

The symmetric function may be expressed in terms of Σa, Σab, Σabc, ... , the expression being of the form

$$C_1(\Sigma a)^{\alpha_1}(\Sigma ab)^{\beta_1}(\Sigma abc)^{\gamma_1} \dots + \ C_2(\Sigma a)^{\alpha_2}(\Sigma ab)^{\beta_2}(\Sigma abc)^{\gamma_2} \dots + \dots$$

where $\alpha_1 + 2\beta_1 + 3\gamma_1 + \ . \ . \ . \ = \alpha_2 + 2\beta_2 + 3\gamma_2 + \ . \ . \ . \ = \ . \ . \ . \ . \ = t$. Now the multiplication of $|a^0b^1c^2 \ . \ . \ . \ k^{n-2}l^{n-1}|$ by Σa raises the index of l by 1, and in the multiplication of this result by Σa one term of the product will be got by raising the index of k by 1, and so on: consequently, the multiplication by $(\Sigma a)^{\alpha_1}$ will give rise to one term having each of its last α_1 indices increased by 1. Similarly in the multiplication by Σab, which then follows, there must arise a term got from the multiplicand by increasing the $(\alpha_1 + 1)^{th}$ and $(\alpha_1 + 2)^{th}$ indices from the end by 1 each, and in multiplying this result by Σab a term must arise which is got from the multiplicand by increasing the $(\alpha_1 + 3)^{th}$ and $(\alpha_1 + 4)^{th}$ indices from the end by 1 each, and so forth through the remaining multiplications, the final result necessarily containing a term having the last $\alpha_1 + 2\beta_1 + 3\gamma_1 + \dots$ (that is, t) of its indices increased by 1, and having further C_1 for its coefficient. For the same reason a like term must occur with the coefficient C_2, and so on: so that the aggregate of its coefficients will be $C_1+C_2+\ldots\ldots$ Now this is exactly what the expression

$$C_1(\Sigma a)^{\alpha_1}(\Sigma ab)^{\beta_1}(\Sigma abc)^{\gamma_1}\ldots + C_2(\Sigma a)^{\alpha_2}(\Sigma ab)^{\beta_2}(\Sigma abc)^{\gamma_2}\ldots+\ldots$$

when we put

$$\Sigma a = \Sigma ab = \Sigma abc = \ldots = 1\,;$$

and this value each of these functions would have if instead of a, b, c, . . . the roots of the equation

$$x^n - x^{n-1} + x^{n-2} - \;\ldots\; + (-)^n 1 = 0$$

were taken.

This is the theorem of alternants which lies at the bottom of a curious theorem of Sylvester's regarding "zeta-ic" multiplication.*

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* Sylvester, "On Derivation of Coexistence," *Phil. Mag.*, xvi. pp. 37–43 (1839).

Note on a Persymmetric Eliminant.

By Thomas Muir, LL.D.

(Read June 5, 1899.)

1. The fact that the eliminant of the set of five equations

$$\begin{aligned} s_0 &= p_1 + p_2, \\ s_1 &= p_1\lambda_1 + p_2\lambda_2, \\ s_2 &= p_1\lambda_1^2 + p_2\lambda_2^2, \\ s_3 &= p_1\lambda_1^3 + p_2\lambda_2^3, \\ s_4 &= p_1\lambda_1^4 + p_2\lambda_2^4, \end{aligned}$$

is, as shown by Professor Schoute,* the persymmetric determinant

$$\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix}$$

implies a linear relation connecting every three consecutive s's. As a matter of fact it is not difficult to see that we have

$$\left.\begin{aligned} s_0\cdot\lambda_1\lambda_2 - s_1(\lambda_1+\lambda_2) + s_2 &= 0 \\ s_1\cdot\lambda_1\lambda_2 - s_2(\lambda_1+\lambda_2) + s_3 &= 0 \\ s_2\,.\,\lambda_1\lambda_2 - s_3(\lambda_1+\lambda_2) + s_4 &= 0 \end{aligned}\right\}$$

and from these, by elimination of $\lambda_1\lambda_2$, $\lambda_1+\lambda_2$, we obtain the result just mentioned.

2. Similarly, when each s is the sum of three terms, viz.,

$$\begin{aligned} s_0 &= p_1 + p_2 + p_3, \\ s_1 &= p_1\lambda_1 + p_2\lambda_2 + p_3\lambda_3, \\ &\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ s_6 &= p_1\lambda_1^6 + p_2\lambda_2^6 + p_3\lambda_3^6, \end{aligned}$$

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* Schoute, P. H., "A Geometrical Interpretation of the Invariant $\underset{n+1}{\Pi}(ab)^2$ of a binary form a^{2n} of even Degree." *Proceedings Roy. Acad. Amsterdam*, I. pp. 318–321.

we find

$$s_0 \cdot \lambda_1\lambda_2\lambda_3 - s_1(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) + s_2(\lambda_1 + \lambda_2 + \lambda_3) - s_3 = 0,$$

and three other equations like it, and so, by elimination of $\lambda_1\lambda_2\lambda_3$, $\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1$, $\lambda_1 + \lambda_2 + \lambda_3$, obtain

$$\begin{vmatrix} s_0 & s_1 & s_2 & s_3 \\ s_1 & s_2 & s_3 & s_4 \\ s_2 & s_3 & s_4 & s_5 \\ s_3 & s_4 & s_5 & s_6 \end{vmatrix} = 0.$$

The general identity which is at the bottom of the whole matter is thus seen to be

$$s_a - s_{a-1}\Sigma\lambda_1 + s_{a-2}\Sigma\lambda_1\lambda_2 - \dots + (-)^a s_{a-n}\lambda_1\lambda_2\dots\lambda_n = 0,$$

where

$$s_a = p_1\lambda_1^a + p_2\lambda_2^a + \dots + p_n\lambda_n^a.$$

3. On looking into this, however, it will be found not only that the expression on the left vanishes, but that it is the sum of n expressions each of which vanishes. These latter are all of one type, and the proposition to which we are thus led by a further step backwards is—

If the simple symmetric functions of n *elements* $\lambda_1, \lambda_2, \dots, \lambda_n$,—*that is to say, the functions*

$$1,\ \Sigma\lambda_1,\ \Sigma\lambda_1\lambda_2,\ \Sigma\lambda_1\lambda_2\lambda_3, \dots\dots$$

—be multiplied by consecutive descending powers of any one of the elements, and the products be taken alternately + and −, the aggregate vanishes.

For, making the chosen element the last, and denoting the symmetric function of the r^{th} degree by $C_{n,r}$, we have

$$C_{n,r} = \lambda_n C_{n-1,r-1} + C_{n-1,r},$$

and consequently the aggregate in question

$$\lambda_n^a C_{n,0} - \lambda_n^{a-1}C_{n,1} + \lambda_n^{a-2}C_{n,2} - \dots\dots$$

becomes

$$\begin{array}{llll} & \lambda_n^a & & \\ - & \lambda_n^{a-1}(\lambda_n C_{n-1.0} + C_{n-1.1}) & & \\ + & \lambda_n^{a-2}(\qquad\qquad \lambda_n C_{n-1.1} + C_{n-1.2}) & & \\ - & \lambda_n^{a-3}(\qquad\qquad\qquad\qquad\qquad \lambda_n C_{n-1.2} + C_{n-1.3}) & & \\ + & \ldots\ldots\ldots\ldots\ldots\ldots & & \end{array}$$

which vanishes, because the two coefficients of every power of λ_n cancel each other.

4. Distinguishing the chosen element by placing a dot over it, we may write this identity shortly in the form

$$\phi(\lambda_1, \lambda_2, \ldots, \dot{\lambda}_n) = 0 .$$

Now, it is easily seen that

$$p_1\phi(\dot{\lambda}_1, \lambda_2, \ldots, \lambda_n) + p_2\phi(\lambda_1, \dot{\lambda}_2, \ldots, \lambda_n) + \ldots\ldots + p_n\phi(\lambda_1, \lambda_2, \ldots, \dot{\lambda}_n)$$

is the same as the general expression

$$s_a - s_{a-1}\Sigma\lambda_1 + s_{a-2}\Sigma\lambda_1\lambda_2 - \ldots .$$

of § 2; and this means that the latter can, by a mere rearrangement of its terms be expressed as a sum of n different multiples of zero.

5. The same superfluous provision for evanescence appears in another mode of establishing the identity

$$\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix} = 0 .$$

For, on substituting the equivalents of the s's, we change the determinant into

$$\begin{vmatrix} p_1 \quad + p_2 & p_1\lambda_1 + p_2\lambda_2 & p_1\lambda_1^2 + p_2\lambda_2^2 \\ p_1\lambda_1 + p_2\lambda_2 & p_1\lambda_1^2 + p_2\lambda_2^2 & p_1\lambda_1^3 + p_2\lambda_2^3 \\ p_1\lambda_1^2 + p_2\lambda_2^2 & p_1\lambda_1^3 + p_2\lambda_2^3 & p_1\lambda_1^4 + p_2\lambda_2^4 \end{vmatrix} ,$$

and this is seen to be the product of two determinants which *both* vanish, viz., the determinants

$$\begin{vmatrix} 1 & 1 & 0 \\ \lambda_1 & \lambda_2 & 0 \\ \lambda_1^2 & \lambda_2^2 & 0 \end{vmatrix} \text{ and } \begin{vmatrix} p_1 & p_1\lambda_1 & p_1\lambda_1^2 \\ p_2 & p_2\lambda_2 & p_2\lambda_2^2 \\ 0 & 0 & 0 \end{vmatrix} ;$$

so that to say that the persymmetric determinant of the s's vanishes is the same as to say that

$$p_1p_2 \begin{vmatrix} 1 & 1 & 0 \\ \lambda_1 & \lambda_2 & 0 \\ \lambda_1^2 & \lambda_2^2 & 0 \end{vmatrix}^2 = 0 .$$

On the Linear and Vector Function. By Prof. Tait.

(Read May 1, 1899.)

(*Abstract.*)

Three years ago I called the attention of the Society to the following theorem:—

The resultant of two pure strains is a homogeneous strain which leaves three directions unchanged; and conversely.

[It will be shown below that any strain which has three real roots can also be looked on (in an infinite number of ways) as the resultant of two others which have the same property.]

As I was anxious to introduce this proposition in my advanced class, where I was not justified in employing the extremely simple quaternion proof, I gave a number of different modes of demonstration; of which the most elementary was geometrical, and was based upon the almost obvious fact that

If there be two concentric ellipsoids, determinate in form and position, one of which remains of constant magnitude, while the other may swell or contract without limit; there are three stages at which they touch one another.

[These are, of course, (1) and (2) when one is just wholly inside or just wholly outside the other (that is when their closed curves of intersection shrink into points), and (3) when their curves of intersection intersect one another. The whole matter may obviously be simplified by *first* inflicting a pure strain on the two ellipsoids, such as to make *one* of them into a sphere, *next* considering their conditions of touching, and *finally* inflicting the reciprocal strain.]

But the normal at any point of an ellipsoid is the direction into which the radius-vector of that point is turned by a pure strain; so that *for any two pure strains there are three directions which they alter alike.* (These form, of course, the system of conjugate diameters common to the two ellipsoids.) This is the fundamental proposition of the paper referred to, and the theorem follows from it directly.

In the course of some recent investigations I noticed that if ϕ have real roots, so also has

$$\psi\phi\psi^{-1}$$

whatever real strain ψ may be. This is, of course, obvious, for they are $\psi\alpha$, $\psi\beta$, $\psi\gamma$, if α, β, γ be the roots of ϕ. At first sight this appeared to me to be a generalisation of the theorem above, of a nature inconsistent with some of the steps of the proof. But it is easy to see that it is not so. For all expressions of the form

$$\psi\omega\psi'$$

correspond to pure strains if ω is pure. Hence

$$\psi\phi\psi^{-1} = \psi\omega\varpi\psi^{-1} = \psi\omega\psi' \,.\, \psi'^{-1}\varpi\psi^{-1}$$

and is thus, as required by the theorem, the product of two pure strains.

Of course we might have decomposed it into other pairs of factors, thus

$$\psi\omega\psi^{-1} \,.\, \psi\varpi\psi^{-1}, \qquad \psi\omega\chi^{-1} \,.\, \chi\varpi\psi^{-1}, \text{ etc.}$$

In the former case the factors have each three real roots, in the latter they have not *generally* more than one.

A great number of curious developments at once suggest themselves, of which I mention one or two.

Thus, let there be three successive pure strains (which may obviously represent any strain). We may alter them individually, as below, in an infinite number of ways without altering the whole.

$$\begin{aligned}\omega\omega_1\omega_2 &= \omega_1^{-1} \,.\, \omega_1\omega\omega_1 \,.\, \omega_2 = \omega \,.\, \omega_1\omega_2\omega_1 \,.\, \omega_1^{-1}\\ &= \omega_1^{-1}\omega^{-1}\omega_1^{-1} \,.\, \omega_1\omega\omega_1\omega_1^{-1}\omega_1\omega\omega_1 \,.\, \omega_2\\ &= \omega_1^{-1}\omega^{-1}\omega_1^{-1} \,.\, \omega_1\omega\omega_1\omega\omega_1 \,.\, \omega_2 = \text{etc.}\end{aligned}$$

The expression $\omega\varpi$ itself, when its three roots are given, *i.e.*, α, β, γ with g_1, g_2, g_3, gives ω and ϖ separately, with three independent scalars. For we may take

$$\begin{aligned}\omega\rho &= x_1\alpha S\alpha\rho + x_2\beta S\beta\rho + \dots\\ \varpi\rho &= y_1 V\beta\gamma S\beta\gamma\rho + y_2 V\gamma\alpha S\gamma\alpha\rho + \dots\end{aligned}$$

and then obviously there are three conditions only, viz.

$$\frac{g_1}{x_1y_1} = \frac{g_2}{x_2y_2} = \frac{g_3}{x_3y_3} = S\alpha\beta\gamma\,.$$

Another portion of the paper deals with a sort of converse of the above problem:—The relation between two strains (whether with three real roots or with one) when their successive application gives a pure strain; and various questions of a similar kind.

In these inquiries we constantly meet with a somewhat puzzling form, which repeats itself in a remarkable manner under the usual modes of treatment, viz.:—

$$\omega V\epsilon\rho + V\epsilon\omega\rho.$$

A little consideration, however, shows that it can be put into the form

$$V(m_2\epsilon - \omega\dot{\epsilon})\rho$$

which is thoroughly tractable.

Contributions to the Craniology of the People of the Empire of India. Part I.—The Hill Tribes of the North-East Frontier and the People of Burma. By Professor Sir Wm. Turner, D.C.L., F.R.S.

(Read July 3, 1899.)

(*Abstract.*)

The author contributes the first of a series of memoirs on the craniology of the natives of the countries comprising the Empire of India. The skulls of the hill tribes were from the Lushai-Chin hill tracts, the Nágá Mountains near Manipur, and Nepaul. The author was indebted for the majority of the specimens to former pupils engaged in the public service in India. A short account of the geographical position and of the external characters of the tribes is given, compiled from the writings more especially of Captain Butler, Colonel Lewin, Colonel Woodthorpe, Surgeon-Colonel Reid, General Sir James Johnstone, and from notes furnished to the author by Surgeon-Captain D. Macbeth Moir, Dr C. L. Williams, Surgeon-Major D. H. Graves, Surgeon-Major Bannerman, and Surgeon-Colonel F. W. Wright.

Eleven adult skulls from the Lushai-Chin hill tracts were examined—nine of which were those of men, two of women. Their characters and measurements were described in detail. Four specimens were dolichocephalic, index below 75; five were between 75 and 77·5, and two from the South Lushai hill tracts were above 80; the mean of the series was 76·1. When the two brachycephalic skulls are excluded the mean index was 74·6, so that the people are in the main dolichocephalic. As regards the relation of length to height the mean of the series was 73·8, and as a rule the breadth exceeded the height. Generally speaking the face was orthognathous and chamæprosopic, the nose was mesorhine, the orbit was megaseme, and the palato-alveolar arch was brachyuranic. The mean cubic capacity of the skulls of nine men was 1353 c.cm., the range being from 1270 to 1480.

Eight skulls from the Tonkal-Nágá village of Hwining, about forty miles north-east of Manipur, were presented by Lieut.-Col. F. W. Wright. They had been used by a native as decorations for his house, and each skull was enclosed in an open basket-work frame of split cane. Their characters and measurements were described in detail. The mean length-breadth index of the series was 76·4, and of these five, ranging from 72·5 to 75·3, may be regarded as dolichocephalic, one was 77·1 and two were brachycephalic. In two specimens the height slightly exceeded the breadth. The face was orthognathous and chamæprosopic. The nose was mesorhine, the orbit megaseme, and the palato-alveolar arch brachyuranic. The cranial capacity was unusually high for a savage people, and ranged from 1455 to about 1600 in the men, with a mean of 1501 c.cm.

Travellers in the hill ranges occupied by the Lushais (Kukis) and Nágás have recognised differences in the physical characters of the people. There is, however, a general opinion that their narrow oblique eyes, flat broad faces, high cheek bones, flat noses, brown skin of various shades, straight black hair, scanty beard and moustache are Mongolian characters, though Colonel Lewin states that the type of feature of the Lushais is not Mongolian, but more like that of Portuguese half-castes. If the Mongolian type of feature be, however, that which is most characteristic, as is the general opinion, it is interesting to note that it is associated in these hill-men with a form of skull which is as a rule dolichocephalic or approximating thereto, instead of being brachycephalic, or in the higher terms of the mesaticephalic group.

Only one skull from the valley of Nepaul was examined, belonging to the tribe of Magars or Gurungs, who have pronounced Mongolian features. Its cephalic index was 90·5, strongly hyperbrachycephalic. It was flattened behind, probably from artificial pressure.

The skulls from Burma were forty-four in number, the majority of which, presented by Surgeon-Major George Bell, had died as prisoners in the jail at Insein. The mean index of length and breadth was above 80, so that the general type was brachycephalic. It was exceptional to have a skull the height of which was greater than the breadth. In their facial relations they were as a rule chamæprosopic, and very few had a strongly prognathic jaw. The mean nasal index was mesorhine, the orbital index was mesoseme, and the palato-alveolar index brachyuranic. In their cranial capacity one was only 1160 c.cm., and three were exceptionally high, but as a rule they ranged from 1240 to 1250 c.cm.

Two of the crania, from an old cemetery in Upper Burma, were distinctly dolichocephalic; one marked Karen was brachycephalic, and the height was less that the breadth. Two skulls were said to be those of Shans, one was brachycephalic, cephalic index 80·6; the other was mesaticephalic, cephalic index 78·7. They were neither prognathic nor platyrhine.

The Burmese are probably the descendants of a Himalayo-Tibetan race, which migrated in a south-easterly direction until they reached Burma. They are of moderate stature; the mean height of a number of men measured in the jail at Insein was 5 ft. $2\frac{3}{4}$ in. The face is broad and flattish, the nostrils spread out laterally, the eyes are wide asunder and inclined to be oblique and almond-shaped. The hair is black and straight, abundant on the head, scanty on the face. The skin is a light olive brown in the upper classes, but darker in those who are more exposed to the sun. The features show, therefore, the Mongolian type. The Burmese skulls were compared with Chinese and Siamese crania. The paper concluded with some remarks on the intermixture of races.

Decorated and Sculptured Skulls from New Guinea. By Professor Sir Wm. Turner, D.C.L., F.R.S. (Plates I.-VI.)

(Read July 3, 1899.)

The ten skulls to which I would direct attention this evening were collected in the island of New Guinea. The first to come into my possession was given to me in 1895 by one of my pupils, Mr F. N. Johnston,[*] the nine others have been recently purchased from dealers. I am not able to name the tribe or tribes by whom the skulls had been sculptured, neither can I state the precise locality at which they were obtained; but the dealer from whom I bought eight specimens told me that they came from the Purari River district. This river rises in the range of the Albert Victor Mountains, and after a known course of 130 miles, it discharges its waters by several mouths into the head of the great gulf of Papua.[†] It is said to be the largest river in the British territory, next to the Fly River.[‡]

In a valuable memoir by Messrs Dorsey and Holmes, on a collection of sixteen decorated skulls from New Guinea, published in 1897, the authors state[§] that although they cannot give the locality from which the specimens came, it is probable that they were collected on the northern shore of the Papuan Gulf, in the British Protectorate. Mantegazza and Regalia have figured[¶] a skull from Canoe Island in the Fly River, where the frontal bone was sculptured with four concentric circles. Professor Haddon, in his elaborate memoir[||] on the *Decorative Art of New Guinea*, says that in the museum at Florence are seven skulls, collected by D'Albertis in the Fly River district, which have designs carved on the frontal bone. He also refers to a statement made by the Rev. James Chalmers,* who saw at Maipua, a village west of Bald Head in the Papuan Gulf, human and other bones, which were carved and in many instances painted. The evidence therefore now before us points to the Papuan Gulf as the region in which it is customary to carve the skulls of the dead with various designs. The crania which I have seen from the south-east coast of New Guinea, and the large collection brought by Dr A. B. Meyer from Geelvink Bay in the north-west of the island, did not show any examples of decorative sculpturing.

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[*] I have figured and described this skull in the *Journal of Anatomy and Physiology*, April 1898, vol. xxxii. p. 353.

[†] *British New Guinea*, by Sir Wm. Macgregor, London, 1897.

[‡] The mouths of this river were recognised by the Rev. James Chalmers, *Work and Adventure in New Guinea*, p. 143, *et seq.*, 1885, but he did not ascend the main stream or give it a name.

[§] *Field Columbian Museum*, Anthropological Series, vol. ii. No. 1, Chicago, August 1897.

[¶] *Archivio per l'Antropologia e la Etnologia*, vol. xi. pl. iii., 1881.

[||] *Cunningham Memoirs of the Royal Irish Academy*, Dublin, 1894.

Seven of the skulls were those of men, one was probably a woman, and two were youths, possibly girls, about 15 or 16 years of age. The skulls had obviously been preserved in the huts of the natives, for they were blackened with smoke. In five specimens the facial bones had been smeared with a red pigment. Two crania had bunches of red grass tied around the zygomatic arches. In all the specimens, with one exception, the lower jaw was kept in place by a band, formed sometimes of split cane, at others of twisted vegetable fibre, which had been passed through the nose, immediately above the floor, and carried round the hard palate and symphysis menti, in front of the latter of which it was in most instances secured by a knot. Through each ascending ramus a hole had been bored, and a piece of split cane or a string of twisted vegetable fibre had been passed through it, and secured to the zygomatic arch (fig. 1). As a rule the necks of the teeth in each jaw were enclosed in loops of string, which secured them together, and retained them in their sockets.† In some of the skulls the teeth were in place and stained, but in others they had to some extent dropped out or been removed, and pieces of wood or cane had been substituted for them, and stained black like the teeth.

The most interesting features of these skulls were the decorative patterns which had been engraved on the frontal bone, and which may be arranged conveniently in five groups.

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* *Pioneering in New Guinea*, 1887.

† With two exceptions the figures in illustration are from photographs of the skulls kindly taken for me by Mr W. E. Carnegie Dickson, B.Sc. Figs. 3 and 5 are from pen and ink sketches by Dr David Hepburn, which show more clearly than the photographs the scratched character of the designs.

Group 1 was the most simple, and consisted of straight incised lines directed diagonally, and continued into each other at their ends so as to provide a chevron or herring-bone-like pattern. This mode of ornamentation was present in four crania. In three of these it extended across the frontal bone, from one temporal curved line to that on the opposite side. In one specimen three horizontal parallel lines had been cut from one curved line to the other; the lowest line was 15 mm. below the middle one, and the highest line was 7 mm. above it (fig. 2). Each interval contained several diagonal lines, which passed alternately from left to right and from right to left, and formed a complete chevron pattern. Immediately above each supraorbital ridge an incised line had been made, which accentuated the projection of the ridge, and the adjoining part of the frontal bone had been scraped and polished.

In a second specimen three similar parallel lines, at almost equal distances apart, had been cut between the two temporal curved lines; in the upper interval twenty-nine incised lines were directed from right to left, in the lower interval seventeen lines passed from left to right, and to produce the chevron pattern both sets of lines were required. A chevron figure, which consisted of two pairs of diagonals, had also been scratched on the upper part of the frontal bone (fig. 3).

In a third specimen two parallel lines had been cut across the frontal bone, and in the interval a complete chevron pattern, consisting of fourteen diagonal lines, had been incised (fig. 4). Above the upper parallel line the arc of a circle had been cut, and its concavity contained an incised figure, which consisted of two triangular limbs, diverging upwards and outwards from a centre in the middle of the frontal bone. Each limb showed two pairs of incised lines, which met at their upper ends and enclosed an unpaired central line. The two limbs converged below, and by their union formed an enlarged chevron pattern. In the interval between the two limbs two circles, one within the other, had been cut. This figure was transitional to the radiated designs in group 2. In the fourth specimen no parallel lines were cut across the frontal, but about midway between the two temporal ridges a chevron pattern, formed of three pairs of diagonal lines, had been incised (fig. 5). Mr J. Y. Buchanan has told me that the chevron zigzag is known to the natives of New Guinea as the snake pattern, obviously from its undulations.

Diagonal lines, running in opposite directions and meeting at their ends so as to form a zigzag arrangement, is evidently a favourite design with the natives of New Guinea. In Messrs Dorsey and Holmes's memoir on sculptured skulls, six examples are reproduced, twice in association with other patterns and four times in its simple form, though in each instance the diagonal lines were included between horizontal lines, as in the majority of my specimens. One can scarcely conceive a more elementary mode of ornamentation or one more likely to mark an early stage in the evolution of decorative art, and it is by no means confined to existing barbarous races. When we examine the unglazed urns not unfrequently found in the short stone cists, which in Scotland are so characteristic a mode of interment of the people of the Bronze Age, we find that the ornamentation is of this character; and in a specimen now before me the exterior of the urn is covered with horizontal and diagonal lines from the lip to the base.

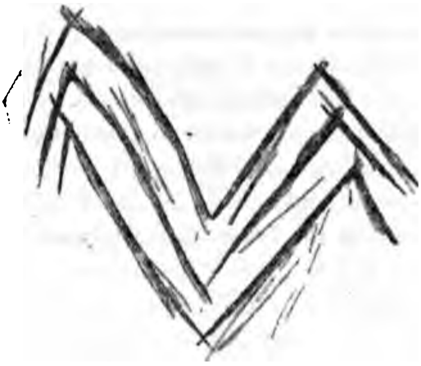

FIG. 5.

The natives of New Guinea by no means limit themselves in the use of the chevron ornament to the decoration of skulls. In Professor Haddon's valuable memoir on the *Decorative Art of British New Guinea*, many examples of the employment of this pattern, under various modifications, are figured as applied to the decoration of combs, pipes, drums, spatulæ, and other articles manufactured by the people in the British part of the island. In the plates which illustrate Professor Mantegazza's memoir* the chevron ornament can be seen to have been used, though sparingly, for the decoration of several of the objects which he has figured.

Group 2 consisted of designs in which limbs radiated from a common centre. The most perfect pattern of this kind was found in one of the young skulls, and consisted of four radiating limbs, each about 30 mm. long (fig. 6). One was directed upwards to about an inch from the bregma, one downwards to about the same distance from the nasion, and the remaining two horizontally outwards,—one in the direction of each temporal curved line. The common centre from which the limbs proceeded was at about the middle of the frontal bone, and was marked by two incised circles, one within the other. Each limb was triangular in shape, and was differentiated from the bone outside it by an incised diagonal line on each side, which met at the apex of the limb, whilst at the base the boundary line of one limb was continuous with that of the adjoining limbs. Within this boundary line two pairs of similar incised lines were present.

In a second specimen the radiated figure had only three triangular limbs, the two longest of which were 22 mm. each. One limb was directed downwards to the front of the glabella, and the others downwards and outwards towards the external orbital processes. The centre of radiation was marked by two incised circles, one within the other. The boundary line of each limb in this specimen enclosed only a single pair of incised lines. Instead of a fourth limb radiating upwards towards the bregma, a pattern, distinct from the radiated figure, had taken its place (fig. 7). It consisted on each side of three incised lines placed one within the other, which, starting close together from the external orbital process, ran upwards and inwards, to become continuous with each other in the middle of the frontal, the outermost line being prolonged upwards for 14 mm. before it joined its fellow in a point. Immediately above the junction three short horizontal and two diagonal lines had been cut in the bone. The radiated designs described in this group, though quite distinctive, are in some respects affiliated to the chevron pattern in Group 1; for, if we trace the boundary line of one limb, it forms a zigzag with that of each of the limbs continuous with its base, and the radiated pattern of the one group may be regarded as an extension and amplification of the chevron design.

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* *Studii Antropologici ed Etnografici sulla Nuova Guinea*, Firenze, 1877.

Mr Holmes, in his joint memoir with Mr Dorsey, figures several radiated designs on the crania in the Field Columbia Museum in Chicago, in some of which there were four limbs, in others three. As a rule the radiations proceeded from a centre marked by one or two incised circles. Whilst the patterns included within the limbs in most instances did not represent any definite natural objects, in a few, especially those reproduced in figs. 7 and 8 of his paper, they were without doubt animal forms conventionally portrayed. Mr Holmes considers that even the simplest designs are significant, "being totems, or having their origin in the crude mythologic conceptions of the people."

It is possible that the designer of these radiating patterns had in his mind to delineate some natural object in a more or less conventional way. Professor Haddon has drawn a number of patterns, carved on wooden belts from the Gulf of Papua, in which figures having two, three, four, or even a greater number of radiations from a common centre have been sculptured. He regards these as degenerate reproductions of the human face, and speaks of the circle carved in the centre of each of the so-called faces in his fig. 27 (p. 115) as an eye inserted by the artist by mistake in the mouth. It is difficult to put this interpretation on the radiated figures on the two skulls described in this group, in both of which circles corresponding to the so-called eye were sculptured in the common centre, which, according to Professor Haddon's view, would have to be regarded as representing the mouth with the contained eye. We can scarcely suppose that in each of these skulls the artist had committed the error of inserting an eye into the middle of the mouth, for the circles which he has interpreted as representing it are obviously a part of the design, and not an accident.* We must look, therefore, for some other interpretation of the radiated figure. If it is to be regarded as a more or less conventional rendering of a natural object, that which it most closely represents is a star-fish. We know from the dredgings of H.M.S. "Challenger" that several species of Asteroidea frequent Torres Strait and the adjoining seas.

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* It should also be kept in mind that the incised circles are not limited to the above skulls, but are found in others where the general design shows a different pattern. Possibly the circles may be intended, as in the hieroglyphical writings of the ancient Egyptians, to represent the sun.

Group 3. Although only one skull belonged to this group, it was in some respects the most interesting of the series, for it showed a decoration which bore a certain resemblance to what is sometimes called the spectacle ornament.* This design has been described as a double disc or circle, connected by lines more or less parallel, and is one of the best known sculptures on crosses and other carved stones in Scotland, which date from the early Christian period of Celtic art.

In the New Guinea skull the ornament was not arranged across the forehead, but in the longitudinal direction of the frontal arc, so that one disc was above the other. The lower circle was placed 33 mm. above the nasion. It was 18 mm. in diameter, and enclosed a smaller circle, 8 mm. in diameter (fig. 8). From the upper part of its circumference two almost parallel lines, 19 mm. long, passed upwards to join the upper circle, which had almost the same diameter as the lower. It presented, however, this peculiarity, that at the upper part the boundary line from the opposite sides of the circle did not become continuous, so that the circle was incomplete above, but it enclosed a smaller circle. The ornament was surrounded by a wavy line, and where it was opposite the parallel lines connecting the two discs, straight incised lines from 4 to 6 mm. long were directed inwards. External to this another wavy line enclosed the whole design. The outermost line had been cut across the glabella and supra-orbital ridges below, whilst above it was involuted, passed through the interval between the incomplete parts of the upper circle, and was continued into the boundary line of the enclosed smaller circle. Within the external boundary line and immediately above the glabella and supra-orbital ridges a wavy incised line followed the outline of the ridges, and five short incised lines were directed upwards from it.

The collection described by Messrs Dorsey and Holmes did not contain a skull with this design. Professor Haddon does not reproduce a similar pattern, but a part of the ornamentation of a wooden belt (fig. 36, p. 121), which contains a pair of concentric circles connected by an intermediate transverse band is the nearest approximation to the design carved on the skull which I have described. Dr Joseph Anderson, in discussing the spectacle ornament sculptured on the Scottish Stones in Ancient Celtic Times, is distinctly of opinion that this symbol was Christian and not pagan. It is therefore very interesting to find that a figure possessing somewhat similar characters had been designed by a pagan artist as far away from Scotland as New Guinea. We do not know the meaning which had been attached to these symbols by those who had engraved them either on the stones or on the skull, but of this we may be sure, that they had not the same significance to the Celtic Christian and the New Guinea savage.

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* Figured in Wilson's *Prehistoric Annals of Scotland*, Stewart's *Sculptured Stones*, and Joseph Anderson's *Scotland in Early Christian Times*, 2nd Series.

Group 4 was represented by only one skull. The design was of large size, and was broadly ovoid in its general form. Its upper limit reached to 13 mm. from the bregma, the lower limit touched the upper part of the glabella, and each lateral boundary was about 12 mm. from the temporal curved line on the frontal bone (fig. 9). The design was fairly symmetrical, and contained five lines concentrically arranged. The lines were not prolonged across the glabella, but the outermost and the third line on each side became continuous below in a point at the inner end of the supra-orbital ridge, and a similar arrangement was present with the fourth and fifth lines. The second concentric line occupied the middle of the interval between the outermost and the third lines. The space enclosed by the fifth line was an elongated ovoid, 50 mm. in longitudinal and 28 mm. in its greatest transverse diameter. A chevron pattern had been cut in the enclosure, about one-third from the upper end was a ring-like figure, 5 to 6 mm. in diameter, and at the lower end opposite points of the fifth line were connected by a shallow zigzag line. The intervals between the first, second, third, and fourth lines were occupied by chevron patterns. One cannot identify this design with any natural object. It may be regarded as a rude geometric pattern, though the zigzags gave it the snake-like undulations referred to by Mr Buchanan, and associate it with the designs in Group 1.

Group 5 possessed two representatives. As the most characteristic was described by me in April 1898,* I need only briefly refer to its design. The area occupied by the sculpture was 71 mm. in transverse and 61 in vertical diameter. It was bounded by an incised line, which passed horizontally across the glabella and supra-orbital ridges, and curved upwards, so that its summit was 35 mm. from the bregma. It enclosed a figure which was a rude representation of a face engraved upside down (fig. 10). Two eyes and a pair of eyebrows were immediately above the glabella; a long straight nose, upper and lower lips, an open mouth, a series of short incised lines to represent teeth, and a tongue were recognisable. On each side of the face an elongated, curved, limb-like object had been sculptured, and from the outside line of enclosure a number of short lines were directed inwards towards the central figure.

In my original description of this specimen I referred to the descriptions and figures in Professor Haddon's treatise on *The Decorative Art of New Guinea*, which showed that the Gulf of Papua was the district in which the human face or designs derived from it seemed to be most frequently employed as an ornament on the articles which they manufactured, and I called attention to a face depicted on a belt in the Berlin Museum (p. 115) as approaching in design to that sculptured on the forehead of this skull.

The second specimen in this group has only recently been acquired. At the first glance it seemed as if it would appropriately fall into the radiated pattern of Group 2, but a more complete examination showed important differences. The design consisted of three limbs, two of which, broadly triangular in shape, were directed horizontally outwards immediately above the glabella, towards the external orbital processes. The third limb, 28 mm. long, and directed upwards, sprang from the upper part of the horizontal limbs at their junction in the middle of the frontal (fig. 11). It terminated 40 mm. from the bregma in a club-like dilatation, in the centre of which a circle from 6 to 7 mm. in diameter had been cut. Between the circle and the outer boundary line an inner faint line, which followed the curve of the latter, was incised. Extending across the front of the bone and in the middle of the horizontal limbs was a series of incised denticulations enclosed within a faintly incised line. This design differed from the radiated pattern in the shape of the three limbs, more especially the ascending, and in the absence of a circle or circles at the centre of radiation. By the exercise of one's imagination one could conceive the club-like limb to be a head, and the circle within it to be a cyclopian eye, whilst the short denticulations might represent the teeth contained within an open mouth.

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* *Journal of Anatomy and Physiology*, vol. xxxii. p. 353.

It is well known that the art of designing is exercised by people who, in many other respects, are primitive in their habits and mode of thought. The possession of this faculty is found in the Bushmen and the aboriginal Australians, as well as in the races of Polynesia. If we go back to prehistoric times in Europe we find that palæolithic man portrayed on his implements and weapons faithful representations of the animals that were cotemporaneous with him, so that the artistic faculty appeared early in the evolution of the human intelligence. One is struck, however, in the study of the decorative art of New Guinea, with the conventional character of the patterns and with the variety of design displayed by the native artists. In the skulls now before me, as well as in those described by Messrs Dorsey and Holmes, no two patterns are exactly alike, and the artist in each case had not copied either his previous productions, or those of his fellows, but had followed the bent of his imagination.

From the appearance presented by the designs, as is well shown in figs. 3 and 5, it is evident that they had not been cut in the bone by a sharp instrument, such as would be used by a European engraver, but had been scratched or scraped by a more primitive tool, made probably from a piece of shell.

The question may now be considered whether the decorated crania were those of relatives or of enemies. With one exception, no skulls were fractured, or otherwise injured, as if from blows received during life. Ample evidence is given by travellers that the people of New Guinea attach much importance to the preservation of human crania in and adjacent to their houses, although the practice of sculpturing designs on the frontal bone seems to be confined to a comparatively small area. The famous traveller and missionary, the Rev. James Chalmers,* stated that at Teste Island, off the south-east point of New Guinea, the chiefs had skulls on the posts of their houses which they said belonged to enemies they had killed and eaten. In another village, skulls said to be those of eaten enemies were hung about the house of the chief man, and the war canoe was decorated with painted skulls. One of the guides to the village wore as an armlet the jawbone of a man he had killed and eaten, whilst others had human bones attached to the hair and neck. The people bury their dead relatives, and place houses over the graves. Mr Chalmers says that the inhabitants of the inland villages in the Aroma district cook the heads of the slain enemies to secure clean skulls to put on sacred places. At Maiva, in the Gulf of Papua, numerous human skulls were suspended from a pole close to a temple or dubu. The Rev. W. Wyatt Gill, in the same volume, wrote that he saw at Hula, in Hood Bay, a widow carrying about in a basket the skull of her deceased husband. He stated that at Suau, near the South Cape, the dead are buried in a shallow grave in a sitting posture, with an earthen pot covering the head. After a time the pot is removed and the skull cleansed, to be eventually hung up in the house in a basket or net over the fire, so as to become blackened with smoke. The Suau people also eat their enemies and preserve the skulls, and trade amongst each other with them. Nothing is said by these authors of designs being sculptured on any of these crania. Sir Wm. Macgregor figures† a native house on Dobu, one of the D'Entrecasteaux group of islands, in front of which was a ledge covered with human skulls. It is obvious that the practice of preserving human skulls in and about the native houses applies both to those of relatives and of enemies slain in battle.

In addition to the study of the decorative features of these crania, I have examined them in order to determine their race characters, and I append in Table I. the measurements which I have made. The general form of the skulls, as seen in the *norma verticalis*, was that of an elongated ovoid. With two exceptions the crania were

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* *Work and Adventure in New Guinea*, 1877 to 1885, by James Chalmers and W. Wyatt Gill, 1885.

† *British New Guinea*, London, 1897.

TABLE I.—*New Guinea—Sculptured and Decorated Skulls.*

Collection,	No. 1.	No. 2.	No. 3.	No. 4.	No. 5.	No. 6.	No. 7.	No. 8.	No. 9.	No. 10.
Age,	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ab. 16.	Ab. 15.
Sex,	M.	M.	M.	M.	M.	M.	F.	M.	F.	F.
Cubic capacity,	1190	1210	1300	1310	1160	1330	1150	1215	1210	1140
Glabello-occipital length,	178	166	177	179	171	182	173	176	174	170
Basi-bregmatic height,	128	133	133	140	128	132	121	129	122	122
Vertical Index,	*71·9*	*80·1*	*75·1*	*78·2*	*74·9*	*72·5*	*69·9*	*73·3*	*70·1*	*71·8*
Minimum frontal diameter,	94	89	95	96	87	94	89	90	85	83
Stephanic diameter,	101	98	111	102	100	104	98	108	96	96
Asterionic diameter,	106	108	103	102	98	105	99	99	98	97
Greatest parieto-squamous breadth,	121	128	134 p.	128 s.	127 p.	133 p.	123 s.	132 p.	122 p.	119 p.
Cephalic Index,	*68·0*	*77·1*	*75·7*	*71·5*	*74·3*	*73·1*	*71·1*	*75·0*	*70·1*	*70·0*
Horizontal circumference,	491	475	505	500	479	508	483	494	474	471
Frontal longitudinal arc,	122	115	124	118	116	122	115	133	116	115
Parietal ,, ,,	135	125	128	135	134	131	128	123	131	126
Occipital ,, ,,	104	103	116	115	106	122	105	107	103	110
Total ,, ,,	361	343	368	368	356	375	348	363	350	351
Vertical transverse arc,	277	285	295	295	285	282	274	288	270	274
Length of foramen magnum,	34	36	37	32	30	34	33	29	36	32
Basi-nasal length,	97	100	96	103	97	99	92	101	91	89
Basi-alveolar length,	100	95	102	103	103	103	101	107	92	95
Gnathic Index,	*103·1*	*95·0*	*106·*	*100·*	*106·2*	*104·*	*109·8*	*105·9*	*101·1*	*106·7*
Interzygomatic breadth,	118	126	122	126	120	128	117	123	108	98
Intermalar ,,	100	115	116	115	111	112	107	110	96	91
Nasio-mental length,	101	103	98	101	94	103	106	109	92	89
Nasio-mental complete facial Index,	*85·6*	*81·7*	*80·3*	*80·1*	*78·3*	*80·4*	*90·6*	*88·6*	*85·1*	*90·8*
Nasio-alveolar length.	61	67	65	61	60	62	64	70	60	55
Maxillary upper facial Index,	*51·7*	*53·1*	*53·2*	*48·4*	*50·*	*48·4*	*54·3*	*56·9*	*55·5*	*56·1*
Nasal height,	46	50	51	50	46	54	40	53	44	43
Nasal width,	24	23	26	22	25	27	22	28	19	22
Nasal Index,	*52·2*	*46·0*	*51·*	*44·*	*54·3*	*50·0*	*55·*	*52·8*	*43·2*	*51·2*
Orbital width,	40	39	39	40	37	37	36	38	35	32
Orbital height,	30	36	33	32	30	32	31	34	32	32
Orbital Index,	*75*	*92·3*	*84·6*	*80·*	*81·1*	*86·5*	*86·1*	*89·5*	*91·4*	*100*
Palato-maxillary length,	57	..	..	60	55	58	57	60	50	50
Palato-maxillary breadth,	60	..	..	60	60	67	66	64	59	58
Palato-maxillary Index,	*105·2*	..	..	*100·*	*109·*	*115·5*	*115·7*	*106·6*	*118·*	*116·*
Lower jaw. Symphysial height,	23	26	20	25	23	28	26	28	23	24
Lower jaw. Coronoid ,,	50	50	53	71	56	55	50	60	49	50
Lower jaw. Condyloid ,,	54	54	60	65	56	56	53	59	49	47
Lower jaw. Gonio-symphysial length,	85	77	84	85	84	93	86	91	83	71
Lower jaw. Inter-gonial width,	90	83	89	103	91	99	94	89	81	72
Lower jaw. Breadth of ascending ramus,	31	27	30	34	31	34	35	35	31	30

not ridged in the sagittal line, and the slope outwards to the parietal eminences was not steep. In most of the crania the greatest width was in the parietal region, near the eminences, and the sides of the crania were almost vertical below these projections. The skull was not flattened in the parieto-occipital region, except in one specimen, in which the length-breadth index was 77·1. In the nine other crania the index ranged from 68 to 75·7, so that the crania were of the dolichocephalic character, and the mean of the entire series was 72·6. In five crania the height exceeded the breadth, in one these diameters were equal, in four the breadth somewhat exceeded the height; the mean vertical index was 73·8. In all the crania, with one exception, the parietal longitudinal arc was longer than either the frontal or occipital. In the dolichocephalic form and proportions, in the height being frequently greater than the breadth, and in the dominancy of the parietal longitudinal arc, the skulls possessed Melanesian characters.

The *norma facialis* showed in the male skulls a moderately projecting glabella and supra-orbital ridges, a forehead not specially retreating, no great depression at the nasion, nasal bones moderately projecting with the osseous bridge concave, anterior nares moderately wide and with a mean nasal index 50. The nasal process of the superior maxillæ was moderate; the floor of the nose was continued into the incisive region of the upper jaw without the intervention of a sharp ridge, except in the skull with the higher cephalic index (No. 2 in Table I.), in which also the jaw was orthognathic; in almost all the other crania the upper jaw was distinctly prognathic, and the mean gnathic index was 104·7. The width of the orbit did not, as a rule, greatly preponderate over the height, and the mean index 86 was mesoseme. In several skulls the palato-alveolar arch was proportionately long and the mean index was 110·6, just above the dolichuranic group. As regards the proportion of the face generally, obtained by taking the proportion of the interzygomatic breadth to the nasio-mental length, the mean complete facial index was 84·1, which places them in the chamæprosopic or low-faced group of Kollmann.

The cubic capacities were measured by my assistant, Mr James Simpson, in accordance with the method which I recommended in my Report on the Human Crania (*Challenger Reports*, Part XXIX., 1884). The mean of the seven adult male skulls was 1245 c.cm., whilst the single skull of an adult female was 1150 c.cm.

The relatively small cranial capacity, the prognathic upper jaw, the mesoseme orbit, the mesorhine nasal region, associated, it may have been, with the prominent nose so frequently described in the natives of the south coast of New Guinea, are all conformable with Melanesian characters, and conjoined with those already referred to as existing in the cranium proper, justify one in saying that these people belonged to the Melanesian race. It seems, however, not improbable that skull No. 2 in Table I. may have been due to an intermixture of another race, for it was definitely mesaticephalic, the jaw was orthognathous, the nose was leptorhine, and the orbit was megaseme.

The series of sculptured skulls described by Mr Dorsey were also dolichocephalic in their length-breadth proportions. They also belonged to the Melanesian race, and it is not unlikely that they may have been collected in or near the district watered by the Purari river.

The Anatomical Museum of the University contains, in addition to the skulls above described, several specimens, none of which had designs engraved on them, from the south coast or the east point of New Guinea, some of which I have described elsewhere. In my *Challenger Report*, Part XXIX., 1884, I gave an account of a brachycephalic skull from D'Entrecasteaux Island, and a hyper-dolichocephalic skull from Possession Bay, presented by Dr Comrie; also a dolichocephalic and a brachycephalic skull from Tomara (Domara ?), Cloudy Bay, presented by Mr A. F. Davenport; also a brachycephalic skull from Warrior Island,* presented by Dr Cox; and a dolichocephalic skull from Jarvis Island, presented by the Rev. Dr Macfarlane. In my *Challenger Report*, Part XLVII. p. 126, 1886, I gave a short description of a hyper-dolichocephalic skull which had been collected by Captain Strachan, and I have subsequently ascertained that it was got at Turituri, a village situated near the mouth of the Katow River, to the west of the Fly River. In April 1898 I described† a dolichocephalic skull from Port Moresby, which was presented by Dr Lamrock.

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* I figured this skull in the *Journal of Anatomy and Physiology*, vol. xiv. p. 479, 1880.

† *Journal of Anatomy and Physiology*, vol. xxxii. p. 359.

Subsequently to the last date, I am indebted to Mr W. E. M'Farlane for four skulls from Kwato,* near Dinner Island, a little to the east of the South Cape; and to Mr D. C. L. Fitzwilliams for a skull from Geelvink Bay, in the Dutch settlement in the north-west of the island, said to have belonged to the Wandessie tribe.

The skulls from Kwato were from the same locality. They are all adult,—two males, and two apparently females. The base around the foramen magnum had been broken, probably to assist in the removal of the brain, or perhaps for room to receive a pole. In three crania so much had been destroyed that the internal capacity, basi-bregmatic height, and several other measurements could not be taken. The lower jaw was absent in each specimen.

One male and one female skull, when seen from the *norma verticalis*, were ovoid in shape, and had dolichocephalic proportions, the mean length-breadth index of the two being 72·6. The height could be measured only in the male, and it exceeded the breadth by 10 mm. In the male the parietal longitudinal arc exceeded the frontal, but the opposite condition was present in the woman's skull. In the man the upper jaw was prognathic, index 105·2; the nasal index was mesorhine, the orbital index was almost in the mesoseme group, and the palato-maxillary index was in the lower term of the mesuranic group. The characters of this skull were definitely Melanesian, and those of the woman were in the main in the same category.

In the other two crania the breadth was proportionately greater, so that the man's skull had a cephalic index 77·2, and the woman's 81·2. In the man the parietal longitudinal arc exceeded the frontal, in the woman the opposite condition was seen. Neither showed any indication of parieto-occipital flattening. In both the nasal bones were depressed at the nasion; they had a concave bridge, their projection forwards was feeble, and the nasal index was leptorhine. The incisive region of the upper jaw inclined very slightly forwards, and the orbital index was microseme in one, mesoseme in the other. The presence of a brachycephalic skull in the Kwato group gives another example to those previously

* These skulls were sent to Mr M'Farlane by the Rev. C. W. Abel.

TABLE II.—*New Guinea—Non-Sculptured Skulls.*

	Kwato, South Cape.				Wandessi, Geelvink Bay.
Collection,	A.	B.	C.	D.	..
Age,	Ad.	Ad.	Ad.	Ad.	Ad.
Sex,	M.	M.	F.	F.	M.
Cubic capacity,	1220	..	..	..	1460
Glabello-occipital length,	171	171	176	165	188
Basi-bregmatic height,	133	..	..	..	134
Vertical Index,	*77·8*	..	..	..	*71·3*
Minimum frontal diameter,	93	92	87	86	101
Stephanic diameter,	110	102	100	104	106
Asterionic diameter,	93	104	104	98	105
Greatest parieto-squamous breadth,	123 p.	132 s.	129	134 p.	132 s.
Cephalic Index,	*71·9*	*77·2*	*73·3*	*81·2*	*70·2*
Horizontal circumference,	485	488	487	476	516
Frontal longitudinal arc,	125	122	130	127	128
Parietal „ „	136	130	124	117	140
Occipital „ „	..	..	..	..	111
Total „ „	..	..	..	..	379
Vertical transverse arc,	287	286	285	288	305
Length of foramen magnum,	..	..	..	..	37
Basi-nasal length,	96	..	..	..	104
Basi-alveolar length,	101	..	..	..	106
Gnathic Index,	*105·2*	..	..	..	*101·9*
Interzygomatic breadth,	127	126	120	122	136
Intermalar „	116	113	108	108	123
Nasio-mental length,	..	..	..	..	128
Nasio-mental complete facial Index,	..	..	..	..	*96·3*
Nasio-alveolar length,	63	59	65	61	74
Maxillary upper facial Index,	*49·6*	*46·8*	*54·1*	*50·*	*54·4*
Nasal height,	48	48	49	46	54
Nasal width,	24	23	26	22	23
Nasal Index,	*50·*	*47·9*	*53·1*	*47·8*	*42·6*
Orbital width,	36	36	35	36	42
Orbital height,	30	29	30	31	34
Orbital Index,	*83·3*	*80·6*	*85·7*	*86·1*	*81·0*
Palato-maxillary length,	54	48	51	51	61
Palato-maxillary breadth,	60	58	62	60	65
Palato-maxillary Index,	*111·1*	*120·8*	*121·5*	*117·6*	*106·5*
Lower jaw. Symphysial height,	..	..	..	..	33
Coronoid „	..	..	..	..	70
Condyloid „	..	..	..	..	71
Gonio-symphysial length,	..	..	..	..	89
Inter-gonial width,	..	..	..	..	100
Breadth of ascending ramus,	..	..	..	..	37

recorded of an intermixture of races in the eastern part of the south coast of New Guinea.

The skull of the native of the Wandessa tribe from Geelvink Bay was that of an adult male. In its appearance and general dimensions it was on a larger scale than the skulls from the south coast of the island, and had belonged apparently to a well-built man. In the *norma verticalis* it was an elongated ovoid with steep side walls. The length-breadth index was dolichocephalic, 70·2, the basi-bregmatic height exceeded the breadth, the parietal longitudinal arc was much longer than the frontal and the occipital, and in all these characters it possessed Melanesian features. The glabella and supra-orbital ridges were distinct, the nasal bones were short, narrow, only slightly projecting, the nasal index was leptorhine, the floor of the nose passed smoothly into the incisive region of the upper jaw, which was mesognathous. The face was high in relation to the interzygomatic breadth, and came into the leptoprosopic group of Kollmann. The palato-alveolar arch was elongated and of dolichuranic proportions. The internal capacity of the cranium was considerable, so that the skull was mesocephalic.

MM. de Quatrefages and Hamy have described in their great work * four skulls of natives of the Wandessa tribe, to which this skull was also said to belong. Three males gave a mean length-breadth index 69·9, whilst the length-height index was 73·5; the height, therefore, exceeded the breadth. In a female skull, however, the length-breadth index was 77·7. Notwithstanding this exception, the cephalic index in the Wandessa tribe would seem generally to be dolichocephalic. In the magnificent series of 135 crania collected by Dr A. B. Meyer,† 23 from Rubi on the mainland at the head of Geelvink Bay, and 112 from Kordo, in the island of Mysore, at the mouth of the bay, the majority were dolichocephalic, several were mesaticephalic, and only a small proportion were brachycephalic. In no specimen of this large series did there appear to be any decorative sculpturing on the skull, and as the crania from the south-east part of the island were also free from carving, it seems as if the practice were in a great measure, if not altogether, restricted to the tribes living on the Gulf of Papua.

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* *Crania Ethnica*, p. 25.

† *Mittheilungen aus dem K. Zool. Mus. zu Dresden*, 1875, 1876, 1878.

No skull described in this communication was metopic. In two skulls a broad tongue-like process of the squamous temporal reached the frontal, and cut off the ali-sphenoid from the parietal. In three specimens there were epipteric bones. Four skulls had small Wormian bones in the lambdoidal suture, in a fifth specimen a large triquetral bone was situated in the right half of the suture. One skull had a third condyle. In several skulls the inner wall of the orbit was broken, but in the uninjured specimens the small size of the os planum of the ethmoid was noted, and in one of these I observed this plate to be triangular in form, and that a tongue-shaped process of the orbital plate of the superior maxilla intervened between the os planum and lachrymal and articulated with the frontal. Some years ago I figured this variety in a Bushman's skull (pl. 1, fig. 4, *Challenger Reports*, 1884), and I have since seen it in the skull of a man from the Lushai Hill tracts to the north of Burma. It is obviously a rare variety even in the crania of savage races.

Through the courtesy of Dr A. B. Meyer of Dresden, I received on July 19th a valuable and interesting memoir, "On the distribution of the Negritos in the Philippine Islands and elsewhere,"* which he has recently published. In the course of this Memoir he discusses the question of the characters of the people of New Guinea; is this island, he says, inhabited by a uniform race, the Papuan, or is the Papuan a mixed race? Especially, do Negritos exist in New Guinea by the side of, or amongst the Papuan population, and can Negritos be racially distinguished from Papuans? In reply he regards the Negritos and Papuans as one race, which exhibits considerable variability in its physical characters. The differences in the form of the skull and the stature do not weigh, he says, against the uniformity in so many other respects, and it is not necessary to look upon brachycephaly and dolichocephaly as constant factors in the determination of racial features. He assumes that certain races vary more in this character than in others. In the course of his argument he refers to my description of the occurrence of brachycephalic skulls in New Guinea,* and cites me as supporting the opinion of MM. de Quatrefages and Hamy that a Negritic race exists in New Guinea side by side with the Papuan race. He attaches a significance to my remarks greater than I intended to convey, for though I referred to the opinion of de Quatrefages and Hamy, I did not commit myself to the view that brachycephalic crania collected in New Guinea were necessarily or exclusively Negritos; nor did I cite any of the brachycephalic crania which I had personally examined as those of Negritos. I did, indeed, say that the island was on the west brought into relation with brachycephalic, short-statured Negritos, and also with brachycephalic Malays; whilst on the east, colonies of brachycephalic Polynesians had reached it from the Louisiade Archipelago; so that it was not unlikely that an intermixture of foreign and native elements had occurred. I gave Dr Meyer as an authority for the intermixture of Malays and Papuans on Waigiou and the smaller Papuan islands to the west of New Guinea.

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* Stengel & Co., Dresden, 1899.

The observations of Mr Dorsey on the sculptured skulls in the Museum at Chicago, and those sculptured crania which I have described in this article, do not support the presence of a brachycephalic people indigenous to the districts where these skulls were collected. Eight adult male skulls in the Dorsey collection had a mean length-breadth index 71, and the index ranged from 65 to 74; seven adult female skulls had a mean index 73, and ranged from 65 to 77, whilst in the skull of a child the index was 78. In my series of ten skulls, as has been stated on p. 565, the mean length-breadth index was 72·6 and the range was from 68 to 77·1. No skull was brachycephalic, or in the upper terms of the mesaticephalic group. If the relatively higher index had been the only feature of the mesaticephalic cranium, I should not have regarded it as in itself expressing a racial difference, but it varied from the other adults in the proportions of the nose and orbits, and the upper jaw was orthognathic (see p. 566). In several important respects, therefore, it differed from the Papuo-Melanesian type. Not unlikely its characters may be owing to a crossing of a Papuan with an individual of another race, who had been captured in war, or had migrated to the Papuan gulf. If the decorated and sculptured crania are to be regarded, in many instances at least, as the skulls of enemies, its original possessor may have come from some place on the south-eastern seaboard, where there is evidence of the mingling of a brachycephalic Polynesian people with the dolichocephalic Papuo-Melanesian race.

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* *Challenger Reports*, 1884.

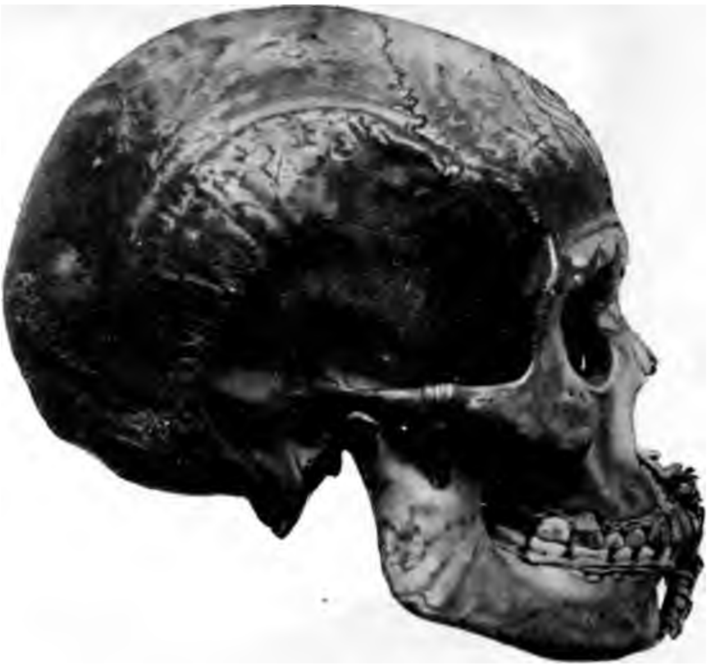

Fig. 1.

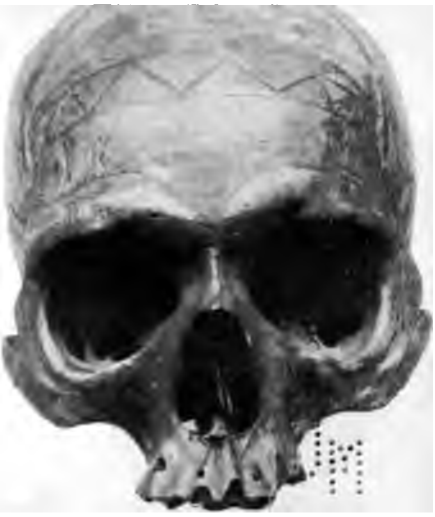

Fig. 2.

Prof. Sir W. Turner on Decorated and Sculptured Skulls from New *Guinea*.—*Plate I.*

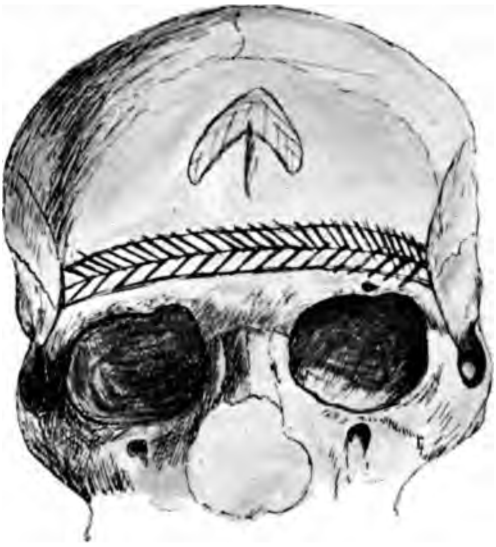

Fig. 3.

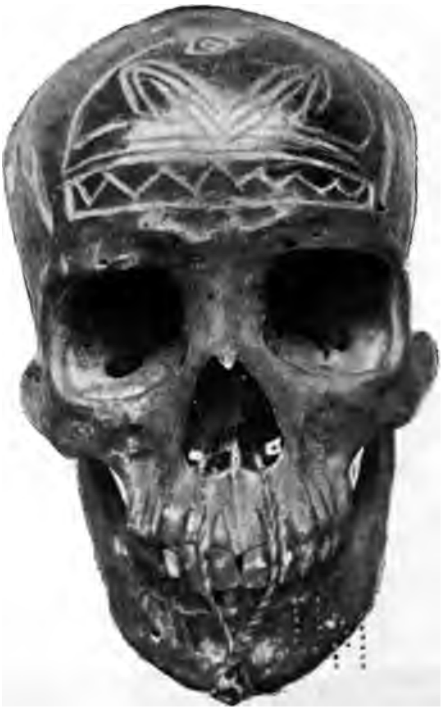

Fig. 4.

Sir W. TURNER.—Plate II.

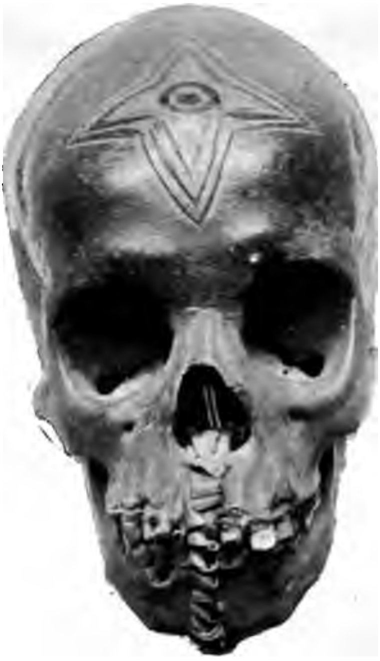

Fig. 6.

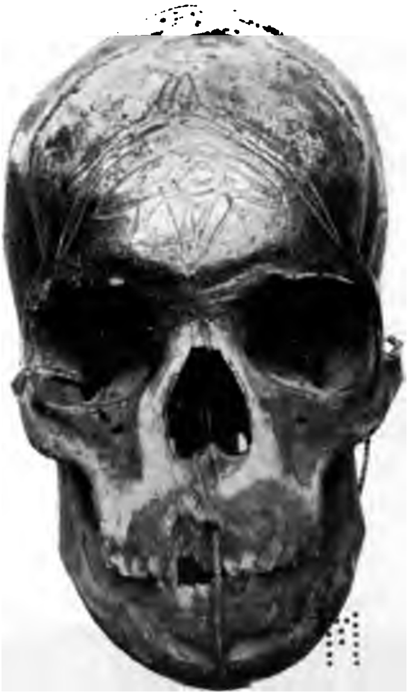

Fig. 7.

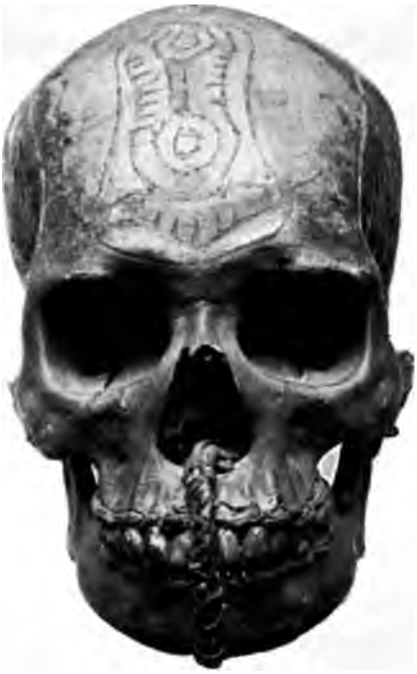

Fig. 8.

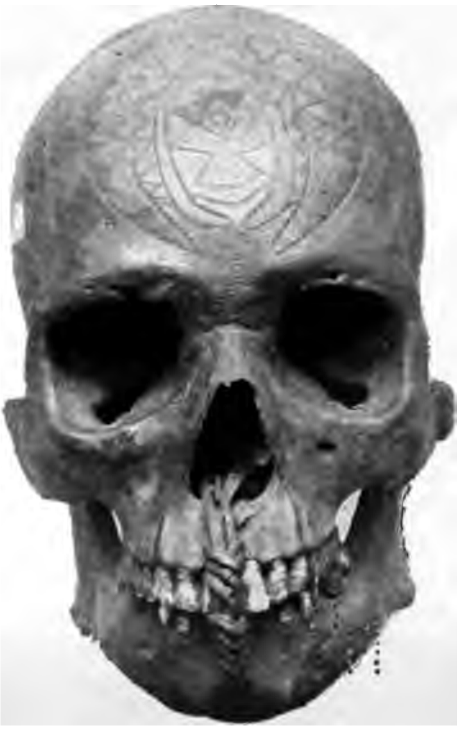

Fig. 9.

Sir W. TURNER.—Plate IV.

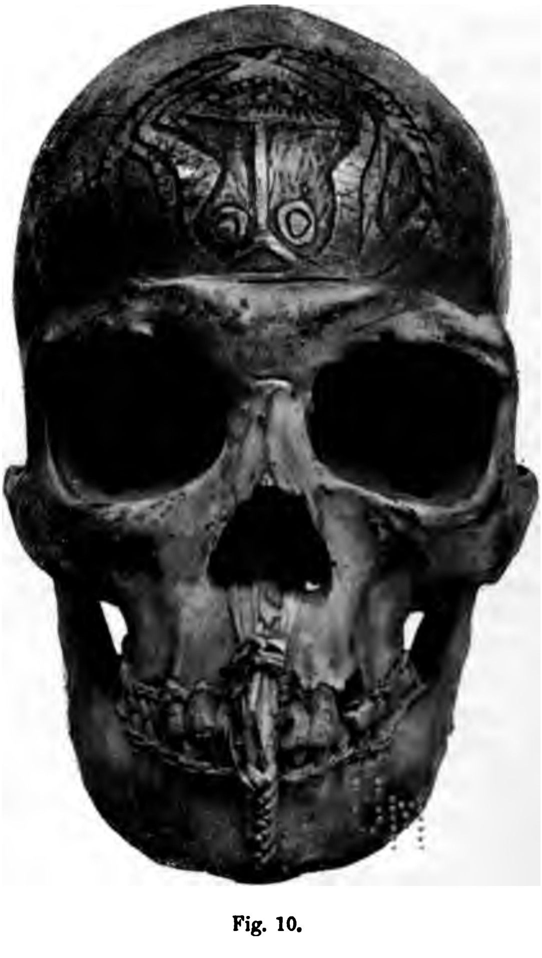

Fig. 10.

Sir W. Turner.—Plate V.

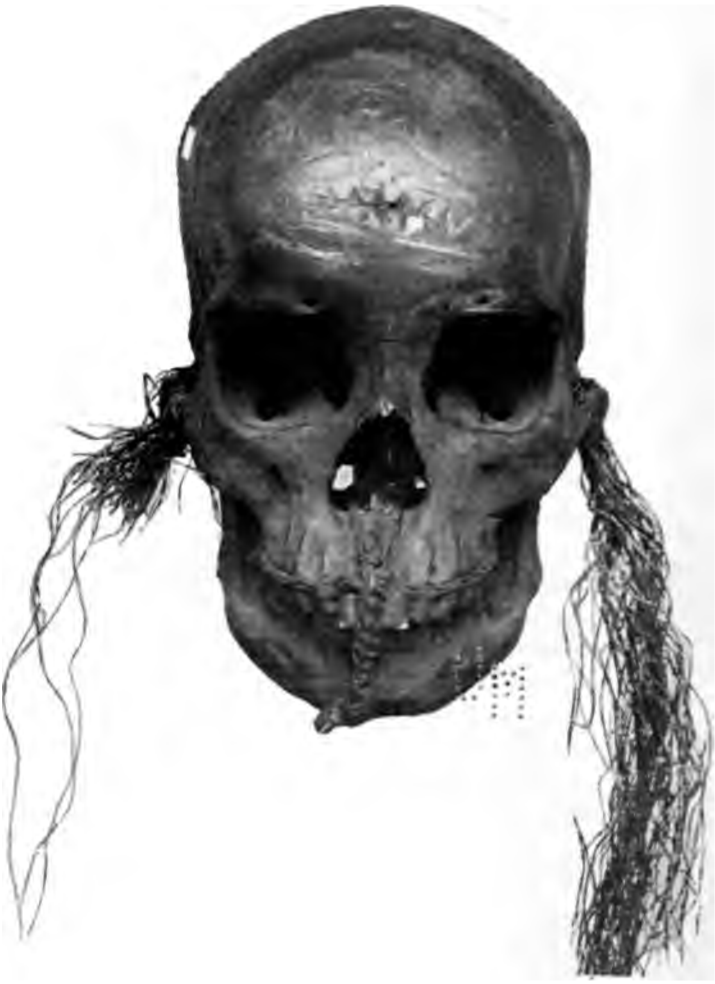

Fig. 11.

Sir W. TURNER.—Plate VI.

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The Propagation of Earthquake Vibrations through the Earth. By Professor C. G. Knott, D.Sc.

Address delivered at the request of the Council, July 10, 1899.

(*Abstract.*)

The history of seismological research and discovery may be conveniently divided into three great epochs. 1. We have the recording of earthquakes in the popular significance of the term, with an enquiry into their character, based almost entirely upon the (usually) destructive results of their visitation. 2. We find investigators beginning to appeal to experiment to elucidate some of the effects noticed, with a growing appreciation of the necessity of recording all palpable earthquakes, whether destructive or not. One of the most honoured names in this connection is that of Mallet, whose two volumes on "The Great Neapolitan Earthquake" form a classic in the literature of the subject. Most of the developments of recent times will be found in embryo in the pages of this monumental work. 3. The introduction of instruments for recording earthquakes, and, as a natural consequence, the recognition of pulsations and tremors and the various kinds of earthquake too feeble to be detected by our senses.

At every stage in this history, geological and physical problems of intrinsic difficulty have been encountered; and it is to the discussion of some of the most recent of these that this address is devoted.

From the days of Mallet and Hopkins, numerous reports on earthquakes and seismological phenomena have been prepared and published by the British Association; and the last of these, from the industrious pen of J. Milne, F.R.S., formerly Professor of Mining in the Imperial University of Japan, has a surpassing wealth of detailed facts and of suggested theories. And yet, before this report was issued in printed form, the careful scrutiny of tremor records had led Professor Milne to the evolution of an altogether novel idea, which may possibly throw light on that much-debated problem—the internal constitution of the earth.

Whatever may be the specific cause of any particular earthquake, there must be at its source a disturbance or abrupt change in the configuration of the material of the earth's crust. A rupture or dislocation occurs, and outwards, in all directions, a disturbance is propagated, accompanied by rupture or dislocation in nearer regions, by vibrations, elastic and quasi-elastic, in these and in more distant regions. For the study of the non-destructive vibratory accompaniments of earthquakes, seismologists have invented various forms of seismometer or seismoscope.

The essential feature of all seismometers is the 'steady point.' A mass is adjusted, by means of various devices, so as to remain steady although its supports move with the ground. A style or pen attached to this steady body is arranged so as to make a tracing on a surface fixed to the moving ground.

To register small earthquakes, it is advisable to have a multiplication of the relative motion of earth and steady body, so that the records may be studied to advantage. In Vicentini's form of seismograph, the motion is magnified 100-fold by means of two delicately-poised levers. The record is made by a light glass fibre resting against a smoked surface. The same method of recording is adopted in other delicate Italian instruments, such as Cancani's horizontal pendulums at Rocca di Papa (*Rep. B.A.*, 1898, p. 266).

In his own form of horizontal pendulum, Milne uses a photographic method of recording the motion of the end of the horizontal boom which bears the approximately steady body, and which is made to have a very long period by being attached by a tie to a point nearly vertically above the point on which the boom pivots. Darwin's delicate bi-filar suspension and other forms of 'tromometer' and horizontal pendulum, though intended primarily for other purposes, may also be classified as seismometers.

The delicacy of a seismometer varies with the function it is intended to perform. The instrument intended to detect and record minute movements which do not affect our senses must be much more delicate than the instrument whose function it is to record veritable earthquakes in the popular significance of the term. The greater the distance from the source at which we hope to detect seismic movements, the more delicate the instrument must be; and, as pointed out by Milne in 1883 (see *Earthquakes*, p. 226), the detection of vibrations which have come from an earthquake occurring at the other side of the earth is a mere question of having sufficiently delicate instruments. It was not, however, till 1889 that a record taken in Europe was identified with an earthquake occurring in Japan. At the present time Milne obtains at his seismological observatory in the Isle of Wight some 70 records per year of vibrations which have travelled over or through the earth from true earthquake origins.

These records of vibrations having a true earthquake origin have certain characteristics which at once distinguish them from other types of tremors and oscillations recorded from time to time. It is generally possible to distinguish in each seismogram two and perhaps three types of vibration. There is first of all a series of small motions or tremors; and after these have lasted for it may be 20 or 30 minutes, much larger motions assert themselves, and these gradually die away into a series of small tremors again. It is usual to call the first set of small motions the Preliminary Tremors, and the subsequent larger motions the Large Waves. It must be remembered, however, that the so-called Large Waves have no effect on our senses, and are large only when compared with the preliminary tremors; also that the term 'preliminary' has reference simply to the fact that the tremors come first to the front. Exactly similar tremors may continue throughout the whole disturbance. (See *B.A. Report*, 1898, pp. 208–218.)

On the assumption that this type of motion has come from an earthquake origin, the conclusion is inevitable that the Preliminary Tremors have outraced the Large Waves. As a general rule, the interval of time between the first appearances of the Tremors and Large Waves increases with the distance of the earthquake origin from the place where the record is taken. In the *B.A. Report* for 1898 (pp. 222–3) Milne has given interesting details in this connection. In considering these, we must bear in mind that all the records were not made with the same type of instrument, and that consequently the comparison of intervals can be only approximate. Still, there is not more discrepancy among the results as a whole than among the results obtained at any one place, and presumably with the same instrument. There is a clear indication that the preliminary tremors outrace the large waves by intervals of time which are proportional to the arcual distances between the place where the record is taken and the place where the earthquake shock is most violent—or, what amounts to very nearly the same thing, proportional to the average depth of the chords connecting these places.

Again, from a knowledge of the instant at which the earthquake really occurred, the approximate times of propagation of the preliminary tremors and of the large waves can be calculated.

From the data given by Milne I have deduced a simple formula * for the speed of propagation of the preliminary tremors in terms of the average depth of chord, on the assumption that the line of propagation is that of the chord. It is

$$v^2 = 2{\cdot}9 + {\cdot}026d,$$

where v is the speed in miles per second, and d the average depth of chord in miles. Expressed in kilometres, the formula becomes

$$v^2 = 7{\cdot}5 + {\cdot}042d.$$

For large depths of chord, this formula approximates to Milne's own statement that the speed varies as the square root of the depth of chord.

When a like calculation is made for the larger waves, the speed is found to be practically constant for arcual distances greater than 60°, its value being 1·7 miles per second.

The formula just given may be used to obtain an approximation to the form of the wave-front of simultaneous disturbance as it passes through the earth. The problem, mathematically stated, is similar in essence to finding the wave-front in a crystal, the difference being that in the optical problem the speed of propagation has a value depending on certain directions in the crystal and is otherwise the same at every point, whereas in the present problem the speed depends on the distance from the earth's centre.

In *Nature* Professor Milne reproduced a rough sketch in which I gave the forms of successive wave-fronts drawn by aid of the formula given above.* It is evident that the paths traversed by any vibration are at right angles to the wave-fronts and can be straight lines only when these are spherical, that is, when the speed is constant at all depths. If the speed increases with nearness to the centre, the paths will be convex towards the centre. In this case the true average speed will be somewhat greater than the value obtained by dividing length of chord by the time taken. Until many more observations have been accumulated, it would probably be a waste of labour to attempt any better approximation to the law of propagation of seismic disturbance through the earth. The mathematical difficulties are considerable, and we can hardly hope for other than approximate solution of the problem.

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* See *Scottish Geographical Magazine*, January 1899, p. 9.

The speed of propagation of an elastic wave depends on a particular coefficient of elasticity and on the density of the material. Assuming—and this seems the most plausible assumption—that both types of waves travel by brachistochronic paths through the earth, we conclude that, since the density is the same in both cases, the coefficients of elasticity must be influenced by the depth in quite different ways. The density is known to increase with the depth; and various formulæ have been given by different investigators. For ordinary purposes, where no depths greater than 2000 miles are considered, we may use the formula,

$$\text{density} = 2{\cdot}75 + {\cdot}0028 \times \text{depth in miles},$$

which agrees very closely with Laplace's historic formula.

In other words, the density may be assumed to increase by $\frac{1}{360}$ per mile descent, or ·28 per cent. The coefficient of elasticity which determines the propagation of the larger waves will therefore increase at nearly the same rate, whereas the coefficient of elasticity which determines the propagation of the preliminary tremors will increase at the rate of nearly 1·2 per cent. per mile descent.

These results seem to have a distinct bearing upon the question of the internal condition of the earth. They indicate that the earth throughout the greater part of its mass is capable of transmitting two types of elastic waves, and is therefore an elastic solid. The only way to escape from this conclusion is to argue that the larger waves are propagated round the crust, and not through the interior of the earth; but this supposition seems to lead to the somewhat incredible conclusion that the speed of these waves increases with the (arcual) distance traversed. (See *B.A. Report*, 1898, p. 220.) This question of the nature of the large waves, however, cannot be regarded as finally settled. What is needed is a more complete discussion of many typical records.

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* I have since found that the problem has been mathematically worked out in a form convenient for application by M. P. Rudzki of Krakau in Gerland's *Beiträge zur Geophysik.*, Bd. iii.

In an isotropic solid there are two types of waves—the condensational-rarefactional and the purely distortional. In the latter, the speed of propagation is determined by the *rigidity* or resistance to change of form; in the former, the *compressibility* comes into play also as a determining factor. The condensational wave travels more quickly than the distortional. Hence it is natural to regard the preliminary tremors as corresponding to the condensational type of wave. Now, it is quite conceivable that an increase of pressure may influence the resistance to compression to a marked degree, and yet have a comparatively slight effect on the resistance to distortion. That is to say, the speed of propagation of the condensational type of wave may be, through its determining elastic constant, affected by change of pressure to a distinctly more appreciable extent than either the density or the rigidity.

If n is the rigidity and k the resistance to compression (the reciprocal of the compressibility), the squares of the speeds of the two types of wave are respectively

$$\frac{k+\frac{4}{3}n}{\rho} \text{ and } \frac{n}{\rho}$$

when ρ is the density. Now, if we suppose k to have at the surface the value $\frac{5}{3}n$, and if we assume n to increase with depth at the same rate as ρ (namely, ·28 per cent. per mile descent) and $k+\frac{4}{3}n$ to increase at 1·2 per cent., we find that k must increase by about 2·2 per cent. per mile descent.

I am not aware of any experimental attempts to measure compressibility of solids at high pressure and temperatures such as exist in the interior of the earth. All we can say is that the relation indicated above, namely, decrease of compressibility with increase of pressure and density, is quite consistent with modern views of the constitution of matter.

From a study of the seismograms obtained in his Isle of Wight observatory, Milne has been led to a very curious result, which seems to point to a *reflection* of trains of waves at some well-marked boundary or barrier. Certain seismograms show a 'repeating' character—the group of 'large waves' being followed at definite intervals by one or more similar groups of much smaller wavelets. The manner in which the 'grouping' is reproduced is, to say the least, very striking. A particularly instructive case is the Shide seismogram of June 29, 1898, here reproduced, with Professor Milne's kind permission.

The first figure shows, on reduced scale, the complete seismogram. The motion recorded lasted for 3 hours. First we have the Preliminary Tremors, increasing in intensity until after a lapse of 20 minutes the Large Waves enter abruptly upon the scene. These then die away in gradually diminishing tremors. Now a careful inspection will show that the following tremors may be divided broadly into two groups, and that each group is not unlike the group of serrations that constitute the large waves. The suggestion is that they are either direct consequences of these large waves, or at any rate referable in their origin to the same original disturbance. To facilitate comparison the corresponding parts of each group are numbered alike—1, 1′, 1″; 2, 2′, 2″; etc.

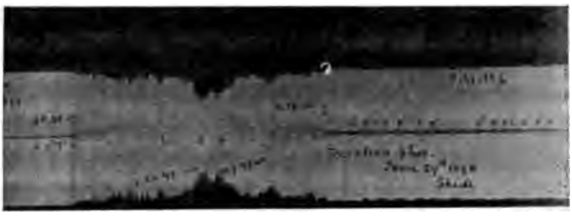

Fig. 1.

It is, however, in the two groups succeeding the Large Waves that the most striking correspondence is observed. This is clearly brought out in fig. 2, which contains enlarged representations of the tail groups of the complete seismogram. Here also, to facilitate comparison, the corresponding parts are numbered similarly, but, as will be readily seen, the numbering in these does not quite correspond with the numbering in the complete seismogram. In considering the possible significance of this apparent correspondence, we must remember that the succession of crests and troughs in the seismogram does not represent the vibration itself, but is due to a fluctuation in the intensity of the vibrations. For example, the well-marked double crest 6–7 is no doubt composed of a great many individual vibrations. This fluctuating intensity has long been known to be characteristic of earthquake motions, and has its origin in the complexity of the original disturbance. We see, then, that a definite fluctuating character belonging to one part of a drawn-out disturbance is almost exactly reproduced in another part. Two explanations of this may be given. We may either suppose these similar groups of disturbances to be brought into existence simultaneously at the earthquake origin but to travel at different speeds or by different paths through the earth; or we may suppose the later disturbance to be a reflection of the earlier at some distant barrier.

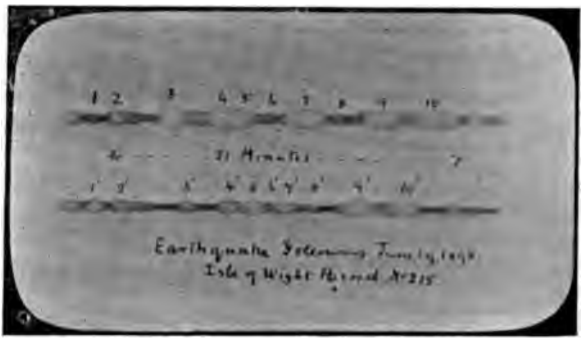

FIG. 2.

If this 'repeating' character of certain seismograms indicate a reverberation within the earth, then we must postulate a comparative uniformity of structure throughout the greater part of our globe, and a fairly abrupt boundary or transition surface. A train of waves of either type impinging on such a surface will, in general, give rise to trains of reflected waves of *both* types, and trains of refracted waves also of both types. The greater the differences in the elastic constants and densities of the two media separated by the boundary, the smaller the amount of energy which passes through into the second medium, and the greater the amount of energy thrown back into the first.

In a seismological paper published * some ten years ago, I discussed at length the reflection and refraction of waves at a surface of rock and water. Some of the main results arrived at in regard to the reflection and refraction of elastic waves at the boundary of rock and water are indicated in the following diagrams (fig. 3). Each figure shows approximately the manner in which the energy of a particular type of wave at a particular angle of incidence is distributed among its derivatives of both types. The incident ray is represented by the broadest line passing downwards from left to right. The upper medium is rock, and the lower water. Since condensational waves

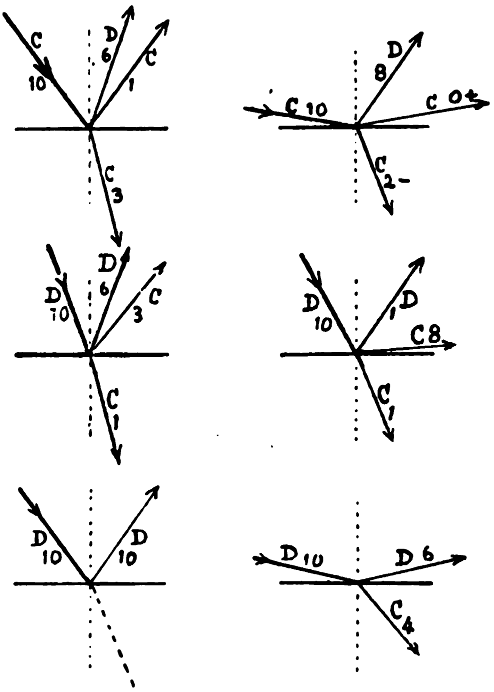

FIG. 3.

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* Republished, with extensions and additions, in the *Philosophical Magazine* for July 1899.

only can exist in a fluid, there is never more than one ray in the lower medium. In all the figures, C represents a condensational-rarefactional wave, and D a distortional. The first two figures show how an incident condensational wave breaks up into two reflected waves and one refracted wave. The angles of incidence are 36° and 80° respectively. In both cases the greater part of the energy is reflected in the distortional form, and in the second case the reflected condensational wave is practically non-existent. The numbers attached to the different rays indicate roughly the amounts of energy associated with them. The accurate values will be found in the article already referred to.

In the four remaining figures the behaviour of an incident distortional wave at various incidences is shown. The condensational wave travels faster than the distortional wave, and is therefore in all cases reflected at a greater angle. When the angle of incidence approaches 35° (see fourth diagram), the reflected condensational wave is sent off at a very high angle, and carries away with it most of the energy, about 80 per cent., while the amount of energy associated with the reflected distortional wave is excessively small. At a slightly greater angle of incidence, namely, 35° 16′, the reflected condensational wave passes off parallel to the surface with zero energy, and at higher incidences has no existence. With the vanishing of the reflected condensational wave at this critical angle, the refracted condensational wave also vanishes—a very remarkable result. Consequently at this angle all the energy is reflected back into the rock in the distortional form. See the fifth diagram, in which the *direction* the refracted ray would have if it existed is indicated by a dotted line. For higher incidences the refracted ray comes strongly into evidence, accounting for approximately half the energy, but becoming less important at very high incidences, until at grazing incidence nothing is left but the reflected distortional wave.

Bearing in mind these broad facts regarding the behaviour of waves at surfaces separating two elastic media, and especially the fact that, in general, each type of wave arriving at such a surface brings into existence the other type as well, we have little difficulty in understanding how an earthquake disturbance may be drawn out in time as the various vibrations, started directly by it or indirectly by its secondary effects in neighbouring regions, travel through the earth towards distant regions. No doubt also the quasi-elastic character of the larger vibrations referred to in my paper of 1888 will make the speed of propagation depend on the frequency. These considerations seem to me sufficient to account for the continuousness and the extension in time of the records at these distant localities.

My original object in discussing the reflection and refraction of waves at the boundary of rock and water was to show that earthquake vibrations, as then understood, could hardly be expected to retain their original characteristics after reflection and refraction at several boundaries. The excessively complex character of the motion of a particle on the earth's surface, when a seismic disturbance is passing, is demonstrated by all good seismograms, but is most completely demonstrated by Sekiya's model, which was built up point by point by a laborious synthetic process from a seismogram giving the vertical and two horizontal components of motion (see *Journal of the College of Science, Imperial University of Japan*, vol. i. p. 361, 1881; also *Nature*, vol. 37, p. 297). In this particular instance, the complexity is largely conditioned by the character of the origin; but even if we assume a comparatively simple original form, reflections and refractions in the heterogeneous crust of the earth must of necessity add complexity.

But in Milne's earthquake 'followers' we have the other aspect presented; and, on the plausible assumption that we are dealing with reverberations or echoes, as they might be called, we are constrained to postulate a comparative simplicity of structure throughout the greater part of the course of the waves. Where the solid nucleus of the earth passes into the somewhat plastic, and, as some believe, fluid magma on which the firm crust rests, the conditions may be favourable for reflection of a large part of the energy of the incident wave. It is probable that for some particular combinations of origin and observing locality the conditions for reflection and subsequent concentration of reflected rays are more favourable than for others, so that the repeating character of certain seismograms may be more marked than that of others.

The elaborate and somewhat fascinating speculation of Ritter, that the earth has a gaseous nucleus highly heated and highly condensed, does not seem to suit these recent seismological discoveries so well as the theory of the solid nucleus. The very high temperatures required by Ritter's theory would necessitate a gradient of temperature in the outer crust which is difficult to bring into accord with Clarence King's recent calculations; and there are other physical difficulties besides. But, from the seismological side, there is the difficulty of accounting for the two or more types of vibrations—the preliminary tremors, and the big waves. On the other hand, the assumption of a solid nucleus necessitates the existence of two such types at least.

It is probable that at a certain depth below the earth's surface materials are in a state bordering on fusion. Lower down the increasing pressure raises the melting point of the material to a higher value than the temperature existing there; nearer the surface, the temperature falls off so rapidly as to be, at any place, well below the melting point of the material there under its particular pressure. We assume, of course, in accordance with Barus's experiments on diabase, that the melting point of most earth-forming materials is raised by increase of pressure.

It is possible, however, that in former ages this critical shell of material just bordering on fusion was really in a state of fusion. The settling down process by which at a particular stage in its history the surface of the earth became cool enough to be habitable is well described by Kelvin in his last article on "The Age of the Earth as an Abode fitted for Life" (*Phil. Mag.*, January 1899). Twenty-four million years ago, according to Kelvin's and Clarence King's calculations, there was no solid crust; but when the solid crust began to form by a kind of crystallisation, it would rapidly cool over its surface. There would soon be an approximation to present conditions, but the crust would be subject to greater and more frequent ruptures and readjustments. Changes of level of the solid superstructure would occur more rapidly than at present, and underground displacements and explosions would occur more frequently and with greater violence. As the earth continued to cool to its present state, the conditions for the production of earthquakes would become less favourable. The causes that operated five million years ago were the same as those that operate now; but they would operate more rapidly and with greater vehemence then than now. We have recognised within the last decade that the earth's crust is everywhere in a constant state of tremor, slight, no doubt, but measurable. In former ages these tremors existed also, and probably in a much more pronounced form. The geologist can to some extent build up the volcanic conditions of the past; but the earthquake seems to leave behind it in the geological record no effect that can be chronologically interpreted. Faults, thrust planes, synclines and anticlines, all tell a story of seismic action; but with what rapidity these changes and distortions were effected we know not. Geologists tell us that there is no evidence of increased volcanic activity in the past; but that does not settle the question of seismic activity. Distribution of volcanic rocks, and faults and contortions of strata *near the surface*, throw practically no light on the seismic phenomena of to-day, nor can we expect them to throw much more on the seismic activity of the past. It is at all events not improbable that in former times, and perhaps not so very long ago, the continual shakings, tremblings, and tremors had an average intensity and a distribution far exceeding what they have now. Under the influence of these, materials would yield more easily and more quickly to the stresses acting on them, just as a steel rod becomes fatigued and deteriorates under intermittent loadings. Is it not conceivable that under such conditions contiguous strata might slip the one over the other without any *apparent* unconformability? And is it not likewise conceivable that strata may have suffered extension, or the reverse, while they were being bent? In the production of these strains or distortions under the influence of appropriate stresses, the mechanical vibration that accompanies seismic phenomena must certainly have played an important rôle.

Magnetic Twist in Nickel Tubes. By Professor C. G. Knott, D.Sc.

(Read February 20, 1899.)

(1.) *Introduction.*—In previous papers* I have had occasion to discuss at some length the relations between Joule's discovery of the elongations accompanying magnetization in iron and Wiedemann's discovery of the twist produced in an iron wire under the combined influence of longitudinal and circular magnetizing forces. Following out a suggestion of Maxwell's† as to the intimate connection between these two phenomena, I turned my attention to nickel, in the hope of finding the Wiedemann effect in it opposite to what it is in iron in low magnetic fields. The fact that, as Barrett‡ had shown, nickel contracted in length when longitudinally magnetized, whereas, in low fields, iron elongated was the ground for this hope, which experiment fully justified.

In a recent paper on the strains produced in iron, steel, nickel, and cobalt tubes in the magnetic field (*Trans. Roy. Soc. Edin.*, vol. xxxix., 1898), I have obtained, on sufficiently reasonable assumptions, values for the strain coefficients at the inner and outer walls of these tubes. Hitherto, work by other experimenters on similar lines had been on ellipsoids, rods, or wires, solid throughout. The theoretical discussion of the Wiedemann Effect (as I have called it) was rendered the more difficult on this account, and also because the current producing the circular magnetization flowed through the magnetized material. In my paper of 1888 referred to above, I make an attempt to get a formula by means of which the Wiedemann effect may be calculated from the Joule effect. The formula is applicable, however, only to a thin-walled tube; and merely a rough first approximation to concordance could be looked for when the formula was applied to a solid wire. It was a natural extension of my recent work on tubes to measure if possible the Wiedemann effect in them also. The comparatively large radius of even the narrowest tubes which had been found suitable for the other work precluded the hope of getting powerful circular magnetizations, so that, according to the view advocated, the Wiedemann effect would be very small. As will be seen below, it was, in fact, little more than just measurable in a very thin-walled nickel tube.

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* *Trans. Roy. Soc. Edin.*, vol. xxix. (1883) pp. 193–203; vol. xxxv. (1888) pp. 377–390; vol. xxxvi. (1891) pp. 485–535; *Phil. Mag.*, January 1894.

† See *Electricity and Magnetism* (2nd edition, vol. ii. p. 448); see also Chrystal's article "Magnetism" in the *Encyclopædia Britannica* (vol. xv. pp. 269–271).

‡ See *Nature*, vol. xxvi., 1882.

The nickel tube used in this investigation was that known as CI in the paper just referred to. It was formed by coiling a piece of sheet nickel ·027 cm. in thickness. The tube was 25 cm. long and 2·5 cm. external diameter. The elongations at the outer surface, when the tube is longitudinally magnetized, are given in Table V., p. 485, of my last paper cited above. For subsequent reference I reproduce certain of the results here. The quantities in the columns headed λ, μ, ν are the elongations multiplied by 10^6 of a surface element; λ representing the elongation parallel to the axis of the tube, μ the "tangential" elongation, and ν the radial elongation.

TABLE I.—*Elongations in Various Magnetizing Fields of Nickel Tube CI.*

Field.	λ	μ	ν	Magnetization.
25	− 4	+ 1·8	+ 2·2	202
50	− 9·5	+ 5·0	+ 4·5	315
100	− 18·2	+ 10·9	+ 7·4	392
150	− 22·5	+ 13·8	+ 8·7	430
200	− 25·4	+ 15·6	+ 9·8	442
300	− 28·2	+ 17·1	+ 11·1	450

(2.) *Measurement of the Wiedemann Effect in the Tube.*—The tube was set vertical in the heart of the magnetizing coil used in the former experiments in magnetic strains, the upper end being fixed, and the rest of the tube being left as free as possible. Near the lower end a mirror was attached, and in this mirror the reflected image of a millimetre scale was viewed through a telescope, the scale and telescope being both at a distance of 256 cm. from the mirror. The circular magnetization was produced by an axial current passed along a rubber-covered wire which threaded the tube 20 times. This sheaf of wires really passed through a glass tube placed loosely inside the nickel tube, so that there was no fear of the wires coming in contact with the nickel.

TABLE II.—*Twists in Nickel Tube under the influence of circular (H′) and longitudinal (H) magnetizing forces.*

H′.	H.	Twist.
1·23	132·5	·4+10^{-6}
	80·1	·8 ,,
	64·4	2·4 ,,
	26	1·4 ,,
2·4	159	1·6+10^{-6}
	132	1·4 ,,
	106	1·6 ,,
	80	2·2 ,,
	53	3-0 ,,
	27	2·0 ,,
4·33	160	2·4+10^{-6}
	133	3·2 ,,
	106	3·8 ,,
	80	4·0 ,,
	53	4·4 ,,
	37	4·7 ,,
	27	3·9 ,,
	16	2·8 ,,
10·9	65	8·2+10^{-6}
	44	8·8 ,,
	22	6·4 ,,
5·74	131	3·2+10^{-6}
	84	4·4 ,,
	61	6 ,,
	36	4·8 ,,
	24	3·2 ,,
6·3	157	3·4+10^{-6}
	131	3·8 ,,
	105	4·6 ,,
	79	5·0 ,,
	53	6·3 ,,
	37	6·2 ,,
	27	5·8 ,,
	16	2·8 ,,
15·8	159	6·8+10^{-6}
	132	7·8 ,,
	106	8·8 ,,
	80	11·0 ,,
	53	11·7 ,,
	32	9·4 ,,
	21	6·0 ,,
	10	2·8 ,,

In the preceding Table of results H is the longitudinally magnetizing field produced by the current in the vertical magnetizing coil, H′ is the circularly magnetizing field acting on the thin wall of the tube, and due to the axial current. The "twist" is the angular displacement of the mirror divided by the length of the tube. The observed deflections were very small, never exceeding 2 of the millimetre divisions of the scale, so that the angular displacement of the mirror was never greater than $1\frac{1}{2}$ minutes of arc. It was impossible to read certainly to less than tenths of a division. Consequently each number, although the average of eight readings, may be out by 2 or 3 in the second significant figure.

These results are shown graphically in the full-line curves of fig. 1, p. 592. In these cases the axial current is kept constant, and the helical current is varied. When the helical current is kept constant, and the axial current varied, the twist is much smaller, being indeed barely measurable. Thus with a longitudinal field of 61, the establishing of a circular field of 5·74 gave a twist of only 1×10^{-6}. This agrees with my former results with wires.

(3.) *Circular and Longitudinal Magnetizations in the Nickel Tube.*—In order to have as complete a knowledge as possible of the various magnetic properties of the tube, the magnetic inductions were carefully determined by the ballistic method, and the magnetizations deduced in the usual way. To the measurements made with the ballistic galvanometer corrections had to be applied for the lines of induction in the air spaces not occupied by the metal. In the case of the circular magnetization this correction was comparatively small; but in the case of the longitudinal magnetization it amounted to a considerable fraction of the whole measured induction—to fully one-half in field 246. To make sure that the ends of the magnetized nickel tube had an inappreciable effect on the field inside the tube, this field was measured directly by means of a secondary coil inserted within the tube. It was from these measured inductions in part of the air space within the tube that the correction for the whole air space was calculated; but it was found that this correction, based on direct measurement, agreed to form significant figures with the correction calculated on the assumption that the thin-walled nickel tube did not appreciably alter the field in the included air space. The magnetizations I and I′ corresponding to the longitudinal field H and the circular field H′ are given in Table III. R and R′ are the corresponding residual magnetizations, and the columns headed R/I, R′/I′ give the ratios of the residual to the total magnetization.

TABLE III.—*Longitudinal and Circular Magnetizations in the Nickel Tube in various fields.*

Longitudinal.			Circular.		
H.	I.	R/I.	H′.	I′.	R′/I′.
246	448	·421	14·5	172	·712
196	441	·412	12·5	148	·710
146	429	·437	10·1	110	·677
99	389	·474	8·44	76	·615
69	352	·495	6·78	36·3	·444
49	309	·528	5·13	15·7	·151
29·5	235	·591	4·33	11·9	·09
19·5	168	·601	3·44	9·2	·044
9·9	50	·419	2·64	6·6	·021
			1·75	4·2	·039

As we should expect, the circular magnetization in a given field is greater than the longitudinal magnetization in the same field, simply because the demagnetizing factor is greater in the latter case.

(4.) *Comparison of the Joule and Wiedemann Effects.*

Let us consider the elongations λ, μ, ν of Table I. as elongations in a strained elastic solid; and let P, Q, R be the principal stresses corresponding to λ, μ, ν. Then P is of the form $M\delta + N\lambda$, where δ $(=\lambda+\mu+\nu)$ is the cubical dilatation; and the corresponding expressions for Q and R are obtained by cyclical permutation of λ, μ, ν. Expressed in terms of Young's Modulus E and the rigidity n, the stresses become

$$P = n\left(\frac{E-2n}{3n-E}\,\delta + 2\lambda\right),\ Q = \text{etc.},\ R = \text{etc.}$$

Now assuming that the direction of the principal elongation λ in the wall of the nickel tube is determined by the direction of the resultant magnetizing force, we see at once that a twist must accompany the combined action of longitudinal and circular magnetic fields. Let the corresponding principal stress P make angle θ with the axis of the tube; then Q is perpendicular to P in the tangent plane to the tube, and R is perpendicular to both, that is, radial.

The tangential stress on the surface at right angles to the axis of the tube is

$$\tfrac{1}{2}(\mathrm{P}-\mathrm{Q})\sin 2\theta = n(\lambda-\mu)\sin 2\theta.$$

But if τ is the twist and r the radius of the tube, this tangential stress is also equal to $n\,\tau\,r$. Hence

$$\tau\,r = (\lambda-\mu)\sin 2\theta.$$

Now introduce the assumption that the direction of P is the direction of the resultant magnetizing force. Then

$$\tan\theta = \frac{\mathrm{H}'}{\mathrm{H}}$$

and

$$\sin 2\theta = \frac{2\mathrm{HH}'}{\mathrm{H}^2+\mathrm{H}'^2}.$$

Thus finally the twist is

$$\tau = \frac{2(\lambda-\mu)}{r}\cdot\frac{\mathrm{HH}'}{\mathrm{H}^2+\mathrm{H}'^2}$$

where λ and μ are the elongations in field $\sqrt{\mathrm{H}^2+\mathrm{H}'^2}$.

This is exactly the same formula I obtained formerly as the result of a much more complicated piece of analysis (see *Trans. Roy. Soc. Edin.*, vol. xxxii. p. 388).

Since H′ is generally much smaller than H, the last factor may, without serious error in most cases, be written H′/H; and in this form we see at once why, with $(\lambda-\mu)$ for nickel increasing in numerical value rapidly at first and then very slowly in higher fields, the value of τ should pass through a maximum in moderate fields.

In the case of iron, λ has itself a maximum value; but the form of the expression above shows that the maximum twist will occur in lower fields than the maximum elongation.

The maximum twist was observed by me both in iron and nickel, and its existence in the latter case in which there is no maximum elongation is explained by the formula just given.

Again, in nickel λ is negative in all fields, whereas in iron it is positive in low and moderate fields. Consequently in iron, when the circular magnetization is right-handed with reference to the longitudinal magnetization, the twist is right-handed also in low and moderate fields and left-handed in high fields. In the case of nickel the twist is left-handed throughout.

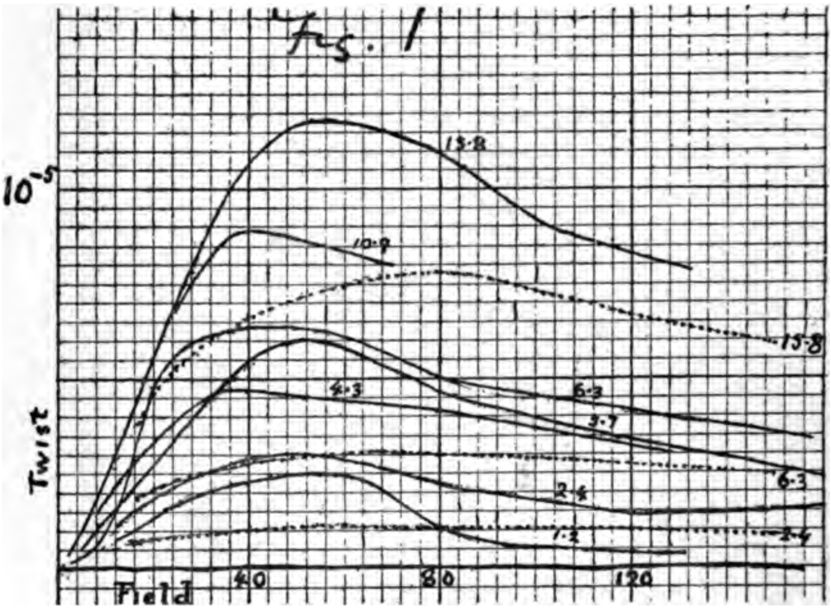


FIG. 1.

Let us now make a quantitative comparison between τ and the observed twist. For this purpose it will suffice to take the most complete series from Table II., namely, the second, third, sixth, and seventh, and tabulate the calculated and observed twists alongside of one another. The columns headed τ contain the calculated twists, and the calculations are made for longitudinal fields 20, 40, 60, 80, 100, and 150. The corresponding values of $-(\lambda-\mu)$, taken by interpolation from Table I., are 4, 11, 17·8, 24, 29, 36·3, multiplied by 10^{-6}. The twists in Table IV. are expressed in millionths of a radian (10^{-6}).

TABLE IV.—*Comparison of Observed and Calculated Twists.*

Longitudinal Field.	H′=2·4.		H′=4·33.		H′=6·3.		H′=15·8.	
	τ	Twist.	τ	Twist.	τ	Twist.	τ	Twist.
20	·78	1·5	1·41	3·4	2·08	4·5	4·6	6·0
40	1·07	2·8	1·86	4·7	2·79	6·3	6·7	10·7
60	1·15	3·0	2·07	4·2	3·00	6·2	7·48	11·7
80	1·17	2·2	2·11	4·0	3·03	5·0	7·8	11·0
100	1·08	2·1	2·01	3·8	2·96	4·6	7·38	8·1
150	·94	1·5	1·69	3·2	2·46	3·5	5·87	7·1

The curves showing the march of τ with magnetic field for H′ equal to 2·4, 6·3, and 15·8 are shown in dotted outline in fig. 1, and may be compared at a glance with the corresponding full-line curves of observed twist.

There is an evident broad similarity. The two outstanding differences are:—

(1.) The values of the observed twist for a given combination of fields are always greater than the values of the calculated twist, the difference being greatest for the lowest circularly magnetizing force and least for the greatest.

(2.) The observed maximum twists occur in lower fields than those indicated in the calculated values, and are distinctly more pronounced.

And to these may be added—

(3.) The twist for a given combination of circular and longitudinal magnetizations is much less when the circularly magnetizing force is superposed on the longitudinally magnetizing force than when the order is reversed.

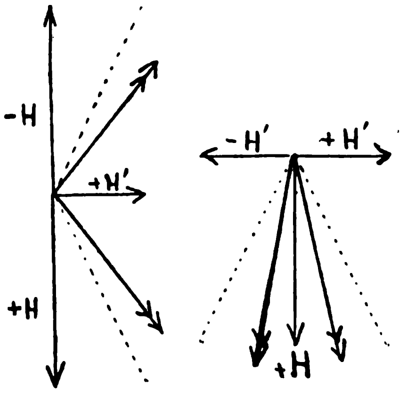
In seeking for an explanation of these quantitative discrepancies we must bear in mind two facts of the first importance.

I. Because of æolotropy pre-existent or produced by the magnetizing forces, the principal direction of strain does not probably coincide with the direction of the magnetizing force.

II. The effects of hysteresis will be to produce a magnetic æolotropy, the main character of which is not difficult to imagine or describe.

Thus let H′ be the circularly magnetizing force acting on a molecule of the tube, and let the longitudinally magnetizing force H be applied first in one direction, then in the opposite direction (+H, −H). Were there no hysteresis, the resultant twist would be as if the principal elongations were first in the direction of one of the dotted diagonals, and then in the direction of the other. But because of the persistence of H′, conjoined with the effects of hysteresis, the direction of principal elongation as the longitudinal

FIG. 2.



(*a*) (*b*)

field changes cyclically from +H to −H and back again to +H will oscillate between the double arrow-headed lines and will be less inclined to H′ than the dotted diagonals (see fig. 2, *a*). In other words, the result is as if a more powerful H′ acted without hysteresis; but this means in general a greater twist, since H > H′ usually, a result in full accord with (1).

On the other hand, if we take the case in which the longitudinal field (H) is constant, while the circular field (H′) is varied cyclically from +H′ to −H′ and back again to +H′, we see at once that the direction of principal elongation will oscillate between the double arrow-headed lines less inclined to H than the dotted diagonals. When H is greater than H′, this gives a smaller twist—in other words, the result is as if a less powerful H acted without hysteresis.

Thus we see the reason why, when H is greater than H′ (as is generally the case), the twist for steady H′ and cyclically-varying ± H is greater than for steady H and cyclically-varying ± H′. On the other hand, for small values of H the contrary should hold. This result could not be established in the present experiment with the nickel tube, because of the minuteness of the effect in small longitudinal fields; but it was obtained eight years ago in my experiments with wires (see *Trans. Roy. Soc. Edin.*, vol. xxxvi., 1891). The curves published with that paper show the result very beautifully; for in the lower fields the curve giving the twists in ascending longitudinal fields, when the circular field is varied cyclically, lies above the curve giving the twists in the same combinations of field when the longitudinal field is varied cyclically. At a certain field the curves cross, and the first-named curve continues ever after to be the lower. The critical field in which the twist is the same, however the combination of fields may be applied, is greater as the circular magnetizing field is increased—a conclusion quite in accord with the views now stated.

The fact that the value of the twist, calculated by means of the formula given above, is intermediate to the two observed values obtained experimentally (1) with cyclical variation of the longitudinal field, (2) with cyclical variation of the circular field, is a result of some importance. It demonstrates the sufficiency of the explanation given by Maxwell that the Wiedemann effect is essentially the Joule effect. In other words, in striving to get an insight into the molecular changes that accompany magnetization, we are not warranted in considering the twist effect observed by Wiedemann as involving any factor that is not prominent in the production of the elongation effect observed by Joule.

The Hydrolysis of Thallic Sulphate.

By Hugh Marshall, D.Sc.

(Read June 5, 1899.)

The alteration of the amount of hydrolysis by change of temperature has been directly observed in connection with various salt solutions. In the case of ferric salts, quantitative determinations have been carried out by Wiedemann, employing magnetic methods. He showed that, in the case of the sulphate, the amount of hydrolysis is considerably increased by rise of temperature. A similar effect is observed in the case of gallic sulphate, and here the change is directly evident; a solution of the salt when heated deposits white basic salt, which re-dissolves on cooling. A somewhat analogous case which has recently come under my notice, and which, so far as I am aware, has not been noted previously, is provided by thallic sulphate solution. The thallic salts generally are decomposed more or less easily by water, with formation of a brown precipitate, but dissolve in dilute acids, giving colourless solutions (Crooke, Lamy, Willm, Strecker). An acid solution of thallic sulphate, provided it does not contain too great an excess of sulphuric acid, may be used to illustrate several points connected with the hydrolysis of salts. Dilution with water produces a brown precipitate. A similar precipitate is formed in considerable quantity when the solution is warmed; when the mixture cools, the precipitate slowly re-dissolves, and a clear colourless solution is again obtained. It is therefore evident that the formation of the precipitate is not due simply to a colloidal substance being rendered insoluble, but to a greatly increased hydrolytic action. A further interesting point observed with some of solutions examined, though not with all, was the effect of other sulphates upon the action. Certain solutions which gave deposits on heating, and cleared again on cooling, were found to give no precipitate after a considerable quantity of solid ammonium sulphate had been dissolved in the solution, indicating that at higher temperatures there is less hydrolysis in presence of the other sulphate. Other solutions gave a precipitate even after the addition of ammonium sulphate, though probably in these cases also the amount of hydrolysis would be diminished.

The brown precipitate produced by the action of water on the thallic salts is stated to be thallic hydrate, not a basic salt. In that case the action might be supposed to be that expressed by the equation

$$Tl_2(SO_4)_3 + 6H_2O = 2Tl(OH)_3 + 3H_2SO_4.$$

According to Lamy, however, thallic hydrate is a metahydrate ($TlO.OH$), though Strecker states that he could obtain only the anhydrous oxide. This might be explained by assuming that the orthohydrate [$Tl(OH)_3$] is first produced, but loses water subsequently. There is, however, another difficulty. Willm states that he experienced great difficulty in preparing normal thallic sulphate, but readily obtained a colourless basic salt, $Tl_2O(SO_4)_2 5H_2O$, by crystallisation from strongly acid solutions; a similar but less hydrated salt was even precipitated by the addition of concentrated sulphuric acid to the solution. Strecker prepared and analysed the normal salt, to which he ascribes the formula $Tl_2(SO_4)_3 7H_2O$, but says nothing about a simple basic salt. By addition of solution of potassium hydrogen sulphate to his thallic sulphate solution, on the other hand, he obtained a white crystalline precipitate, having the composition expressed by the formula $Tl_2O(SO_4)_2 2K_2SO_4$.

There is therefore some doubt as to whether normal thallic sulphate exists in the ordinary solution containing dilute acid. As it would be of considerable interest to have the matter cleared up, I have commenced an investigation of the thallic sulphates and their double salts, and I also propose to investigate quantitatively the action of water upon them.

If it is desired simply to show the action of heat on thallic sulphate solution, without requiring specially to prepare the pure salt, this may be effected by warming thallous sulphate and ammonium persulphate with a little water. If the ammonium persulphate is approximately pure, equal weights of the two salts may be taken. A considerable amount of brown precipitate separates; the mixture should then be allowed to cool, and filtered, or sulphuric acid may be added till the precipitate is just dissolved. The clear solution, obtained in either way, gives a precipitate on heating.

Note on Mr Joseph O. Thompson's Results regarding Vibrating Wires. By Dr W. Peddie.

(Read June 19, 1899.)

In 1865 Lord Kelvin published the results of experiments which first made evident "a very remarkable fatigue of elasticity, according to which a wire which has been kept vibrating for several hours or days through a certain range came to rest much quicker when left to itself than when set in vibration after it has been at rest for several days and then immediately left to itself." On the strength of Lord Kelvin's statement this elastic fatigue of metals has been regarded as a definitely ascertained fact. But, quite recently (*Physical Review*, March 1899), Mr Joseph O. Thompson has published a paper "On the period and logarithmic decrement of a continuously vibrating wire," in which he states that it seems probable that "for constant temperature and constant amplitude the logarithmic decrement is constant." This conclusion is based upon his observation that the logarithmic decrement, in the case of a copper wire, when the amplitude of vibration varied from about 185° to 175°, had the same value after it had been continuously vibrated through the average arc of 180° for fifty consecutive hours as it had at the commencement of that period. Observations upon other metals seemed to support the conclusion.

Lord Kelvin also stated that fatigue caused an increase of the period of vibration. Mr Thompson finds that "no matter what metal was used, no matter whether the arc of vibration was as small as 20° or as high as 200°, no matter whether the wire was long or short, thick or thin (provided of course the breaking strength of the wire was at least twice the weight of the disc), the result was uniformly the same, namely, *that when temperature and amplitude of vibration remained constant, the period of vibration was a constant quantity.*"

Mr Thompson, after having reached the above conclusion regarding the logarithmic decrement, proceeds to determine, if possible, how Lord Kelvin could have obtained results so different. He finds that the value of the decrement at a given range depends essentially upon the magnitude of the maximum oscillation previously given to the wire; and he suggests that the increased value of the decrement may have been due to an accidental increase of maximum range.

I scarcely think that the suggested explanation is probable. I think rather that Lord Kelvin's results and conclusions are unassailable. A considerable number of experiments were made by the students working under Lord Kelvin's directions. In order that the explanation should hold, it would be necessary to assume that, in each case, too large an initial oscillation was given to the fatigued wire while the unfatigued wire was correctly oscillated. There does not seem to be any sufficient reason for assuming such a condition persistently.

In the series of experiments which I have made upon the torsional oscillations of an iron wire (*Trans. Roy. Soc. Edin.*, 1896 and 1898), the results show a distinct effect of fatigue upon the rate of decrease of oscillations and entirely corroborate Lord Kelvin's results. No attempt was made to investigate the effect of fatigue upon the period of oscillation.

If y be the range of oscillation, while x is the number of oscillations made since the commencement of the experiment, the equation

$$y^n(x+a)=b,$$

where a, n, and b are constants in any one experiment, holds with great accuracy throughout a large range. This gives as the value of the logarithmic decrement the quantity

$$-\frac{1}{y}\frac{dy}{dx}=\frac{1}{nb}y^n.$$

In the papers referred to, I have shown that the value of b may, by suitable treatment regarding fatigue, be diminished to one-half of its initial value while n is not much different from unity. Thus the logarithmic decrement may be doubled. In more recent experiments, not yet published, I have obtained still greater variations.

I found that it was possible to have the product nb sensibly constant; that, in general, log. nb is practically proportional to n in various suitably conducted series of experiments, the constant of proportionality being different in different series; and that, in each such case, it is possible to choose a y-unit which will make nb practically constant.

No doubt one cannot conclude, from the behaviour of one wire, anything regarding the possible behaviour of another under similar conditions. Yet the presumably identical cause of the decrease of the range of oscillation in each wire renders it probable that the same law applies in all—at least when the decrement is not excessively large.

I cannot therefore account for the results which Mr Thompson has obtained, unless it be that the treatment to which his wires were subjected was such that

$$\log. nb = \log. A + n \log. y_0$$

where y_0 is the angle of oscillation at which readings were taken on the average, and $1/A$ is the logarithmic decrement, so that a linear relation, similar to those above spoken of, held between log. nb and n.

That this relation did not hold at values of y other than y_0 is evident from Mr Thompson's observations. Indeed, Mr Thompson expressly points out that, when the wire was oscillated for some time through a considerably smaller range than y_0, before observations were made on the decrement at y_0, the decrement at y_0 "as was to be expected, was lower than any hitherto found." This statement seems to be an explicit recognition of fatigue. If a decrease in the value of the decrement at y_0 were essentially produced by previously keeping the wire oscillated for some time through a smaller range, much more should there be a decrease if it were previously kept entirely unoscillated.

An Improved Form of Craniometer for the Segmentation of the Transverse, Vertical, and Antero-Posterior Diameters of the Cranium. By David Hepburn, M.D., C.M., *Lecturer on Regional Anatomy in the University of Edinburgh.* (With a Plate.)

(Read July 3, 1899.)

Introduction.—All craniologists are familiar with the form of craniometer or calliper-compasses at present employed for the purpose of ascertaining the various diameters of the cranial box, and they know that these measurements represent certain facts in connection with the breadth, height, and length of the skull as applicable to a number of accepted fixed points upon the surface of the cranium. In effect these diameters are the direct lengths between two points upon an arched surface, *i.e.*, they are the chords of certain arcs, and by the use of the measuring tape we may ascertain the relative proportions between the arc and the chord which subtends it. When several bones contribute to the formation of the arc we may readily determine the relative proportions between each section of the arc (or the chord of each section) and the chord of the entire arc, but it is a much more difficult matter to *segment* the chord of the entire arc in terms of perpendiculars prolonged to it from definite points upon the arc. For example, by callipers we obtain the glabello-occipital length or chord of the great longitudinal arc, and by measuring tape we may ascertain the frontal, parietal, and supra-inial sections of this arc, but we do not thereby segment the chord of the arc by perpendiculars prolonged to it from the bregma and the lambda. Again, such important transverse diameters of the skull as the minimum frontal, stephanic, greatest parieto-squamous, and asterionic, are recorded without reference to their proportions on opposite sides of the mesial plane, while the almost constant visible asymmetry of the skull shows that the mesial plane does not necessarily bisect these diameters. Further, the basi-bregmatic height might be segmented in terms of a point selected upon the lateral aspect of the skull, so as to provide an approximate representation of the proportions of this diameter which pertain respectively to the cerebrum and cerebellum.

Doubtless opinions may differ as to the value of recording figures which express the segmentation of the different diameters of the skull in terms of certain fixed points situated in their arcs, but that they have not already been recorded is in all probability due to the difficulty of obtaining them without having recourse to trigonometric calculations, a method which has not attracted the craniologist. Various efforts have been made to obtain this kind of information. Thus, an instrument was devised for the purpose of measuring radii drawn from an inter-aural axis to different points upon the surface of the skull, but the results have not warranted its extended use. Again, Sir William Turner,* by bisecting the skull and drawing radii from the basion to different points upon the surface of the skull, as well as by raising a perpendicular to the plane of the foramen magnum from the basion to the vertex, obtained much valuable information, but the extension of this method entailed serious damage to each skull which was bisected.

Professor Cleland † has also followed a plan which was associated with bisection of the skull, and had for its object the calculation of the chords and angles connected with each section of the great longitudinal arch, as well as the distance of various points upon the arch from the base of the skull.

As the result of a conversation with Dr Waterston, Demonstrator of Anatomy in the University of Edinburgh, my attention was directed to the possibility of solving the problem without the necessity of elaborate calculations, and I have accordingly devised a modified form of craniometer which overcomes the former difficulties, and by means of which cranial diameters may be segmented by perpendiculars prolonged from fixed points upon the surface of the skull. Moreover, besides being easily performed, this operation does not necessitate any damage of the

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* Turner, *Chall. Rep.*, "Human Crania," Part xxix., 1884.

† Cleland, "An Inquiry into the Variations of the Human Skull, particularly in the Antero-Posterior Direction," *Phil. Trans.*, 1870, vol. clx. pp. 117–175.

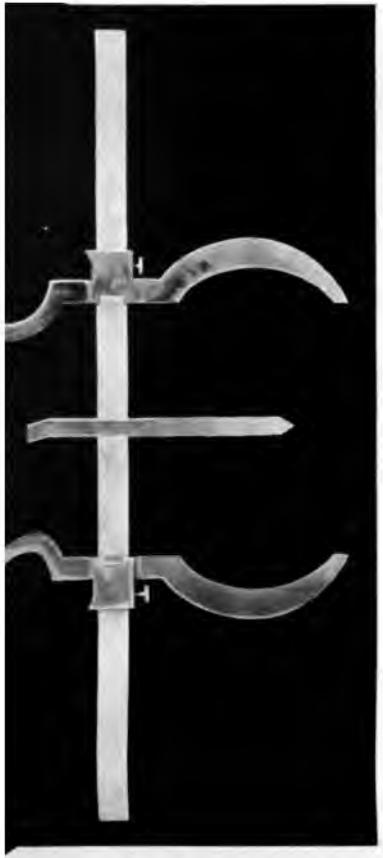

Fig. 1.

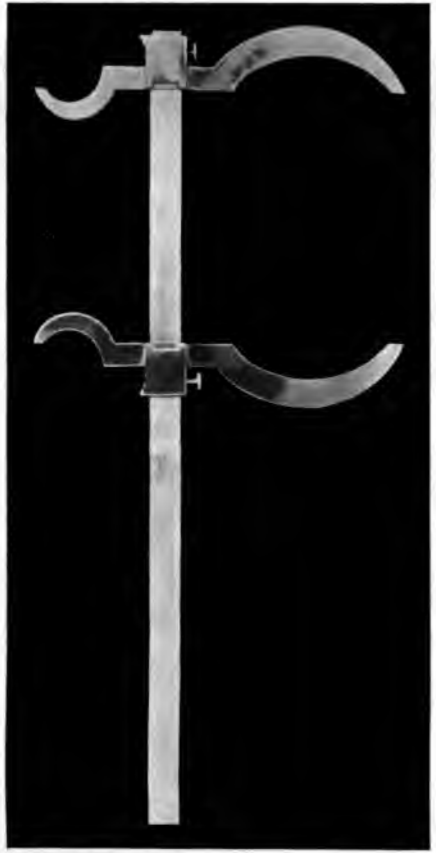

Fig. 2.

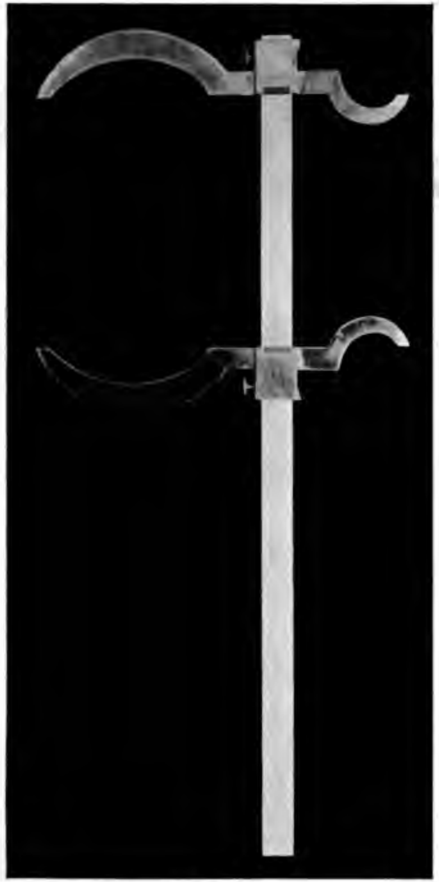

Fig. 3.

skull, and as the results are obtained mechanically, the risk of error is largely obviated.

Description of the Instrument.—The instrument presents various modifications of the craniometer in ordinary use (fig. 1). First, upon one side of the graduated bar, zero is placed at the centre, and the millimetre scale is duplicated from the centre towards opposite ends of the bar; second, opposite zero, a straight calliper leg has been introduced. This leg is placed at right angles to the graduated bar, and being sunk in an undercut flat groove, it may be pushed backwards or forwards so as to alter its length relative to that of the curved legs of the callipers without disturbing its position at right angles to the graduated bar and without deflecting its pointed end from indicating zero on that bar. Further, a small flat spring sunk in the bar at the bottom of the groove assists in retaining this leg in any position to which it has been adjusted. Third, the two curved legs of the callipers are so disposed as to be both capable of gliding upon the graduated bar, and therefore since each of these legs may approach or recede from the central one, the edge of the armature which carries each curved leg indicates, upon the scale, the distance which the point of each curved leg is from the point of the central straight leg, *i.e.*, from zero. Fourth, the central leg may be removed (fig. 2), and the groove closed by a metal slip. Thereafter, since the reverse side of the bar is graduated continuously from one end, by fixing one of the curved legs by means of a screw, the instrument is converted into an ordinary craniometer (fig. 3) available for any of its usual purposes.[1]

It will readily be apparent that this *three-legged callipers* is mathematically correct in its mode of measurement, because since the curved legs of the instrument are of equal length, and as both of them are fixed at right angles to the graduated bar, the chord or diameter which they measure is an imaginary line parallel to that bar; and further, the central straight leg in its imaginary prolongation intersects the chord of the arc at right angles, and therefore segments the chord in the same proportions that it does the graduated bar.

Through the very great kindness of Sir William Turner I have been privileged to test the capabilities of this instrument upon a number of human skulls, all of which had already been described by himself in his "Challenger" Report. These consisted of eleven brachycephalic skulls of Sandwich Islanders, and of twelve dolichocephalic skulls of aboriginal Australians, and were selected because of their extremely typical differences. I have also examined a few skulls of anthropoid apes for purposes of contrast and comparison.

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[1] This instrument is made solely by Mr A. H. Baird, Scientific Instrument Maker, Lothian Street, Edinburgh.

Methods.—In obtaining the principal diameters of the skull, I have been careful to adhere to the methods adopted and approved by Sir William Turner for ascertaining these measurements, and their segmentation has been carried out in terms of well-marked and definite points upon the surface of the cranium.

A. TRANSVERSE DIAMETERS.

1. *Minimum Frontal Diameter.*—The point selected for the segmentation of this chord was the frontal suture in such cases where it was still visible, but when it had entirely disappeared the upper end of the inter-nasal suture was taken instead. In this way we can represent the relative proportions of the frontal region on the right and left sides of the mesial plane, approximately at the level of the roof of the orbits.

2. *Stephanic Diameter.*—The segmentation of this chord may be effected either from the same points as the preceding or from the bregma, which is the more convenient point. It frequently happens, however, that the anterior end of the sagittal suture, *i.e.* the bregma, does not correspond to the highest point in the transverse arch of the vault. In other words, there is such a pronounced *obliquity* in the direction of the sagittal suture, between its posterior and anterior ends, that it becomes difficult to regard it—the sagittal suture—as an accurate expression of the mesial plane, since it may not run in a true antero-posterior direction. At the same time, there is no reason to suppose that the falx cerebri does not correspond with the direction of the sagittal suture, and therefore the position of this suture may fairly be taken as representative of intra-cranial developments.

3. *Maximum Parieto-squamous Breadth.*—This diameter is most readily segmented from the lambda, and it is the almost

constant discrepancy between the result obtained from the bregma and that obtained from the lambda, which emphasises the obliquity of the sagittal suture.

4. *Asterionic Diameter.*—This measurement has reference to the region of the cerebellum, and for the segmentation of the chord a well-marked part of the external occipital crest near to the foramen magnum was selected.

By the segmentation of these four transverse diameters, in terms of different points representing the mesial plane, my object was to indicate, if possible, the amount of divergence from true bilateral symmetry of the skull, and inferentially, of the cerebrum and cerebellum.

B. VERTICAL DIAMETER.

Basi-bregmatic Height.—The examination of a skull in vertical transverse section shows that the upper margin of the petrous bone (to which the tentorium cerebelli is attached), is practically upon the same horizontal plane as the posterior or supra-auditory root of the zygoma. Therefore by selecting the edge of this root immediately superior to the external meatus as a basis for segmenting the basi-bregmatic height, my intention was to express the approximate proportions of this chord which pertain to cerebrum and cerebellum respectively.

C. ANTERO-POSTERIOR DIAMETER.

Glabello-occipital Length.—This diameter, which expresses the length of the cerebrum between its anterior and posterior poles plus the thickness of the skull at each end, may be segmented in reference to several surface points. Thus, by selecting the bregma, we may ascertain the præ-bregmatic and post-bregmatic proportions of the total length. Again, by segmenting the chord in terms of the lambda, we may obtain not only the præ-lambdoidal and post-lambdoidal proportions, but by taking the sum of the præ-bregmatic and post-lambdoidal segments and deducting it from the total length, we may express, in the difference, the sagittal or parietal segment of the total length. In this way we may ascertain approximately how much of the total length is under cover of the frontal, parietal, and occipital bones, and thus provide a series of facts which are complementary to those ascertained by measuring the frontal, parietal, and supra-inial sections of the great longitudinal arch.

At the same time, the lambda is occasionally so close to the occipital end of the glabello-occipital diameter that the post-lambdoidal segment can only be stated approximately.

I have also segmented the glabello-occipital length in terms of the anterior margins of the occipital condyles, and it will be found that the anterior margin of the foramen magnum closely corresponds to a line drawn transversely between the anterior margins of the two condyles. This measurement is of much value in connection with the degree of erect attitude assumed by the various races of men as well as by anthropoid apes and other mammals. These segments are defined by the terms præ-condyloid and post-condyloid.

Lastly, by using the external auditory meatus, I have represented præ-auditory and post-auditory segments for contrast with those derived from the position of the condyles. Approximately, these segments are calculated from the centre of the external meatus by inserting the V-shaped end of the middle limb of the callipers.

The details of the measurements of the various skulls, taken in the manner above indicated, are given in the accompanying Tables.

ANALYSIS OF THE MEASUREMENTS.

Minimum Frontal Diameter.—(*a*) Brachycephalic skulls—Sandwich Islanders.—In three cases the right and left segments were alike; in one case, there was a preponderance of 2 mm. in favour of the left segment; while in seven cases the excess lay to the right side of the mesial plane. This excess varied from 2–5 mm., the average being 3·2 in favour of the right side. (*b*) Dolichocephalic skulls—Aboriginal Australians.—In one case there was equality between the right and left segments. In eight cases the right segment predominated, the variation being from 2–6 mm., with an average of 3·2 mm. In three cases the left segment was in excess, giving an average of 2·3 mm.

(*c*) Anthropoid Apes.—The skulls of the orang and of a young chimpanzee produced equality in the two segments, while in the skull of a young gorilla there was an excess of 3 mm. in favour of the left side, and an excess of 1 mm. in favour of the right side in the skull of a small Gibbon.

Stephanic Diameter.—(*a*) Brachycephali.—In seven cases the right segment was in excess, and in four cases the left segment was the greater. On the right side, the variation lay between 2–9 mm., the average difference being 4·8 mm. On the left side, the figures fluctuated from 1–11 mm., the average being 4·5 mm.

(*b*) Dolichocephali.—In nine cases the right segment was the greater, its fluctuations being from 1–16 mm., and the average excess being 6 mm. In three cases in which the left segment was the greater, the average excess was 4 mm.

(*c*) Anthropoids.—In the four skulls examined, the right held the advantage in two cases, and the left in two cases.

In the frontal region, therefore, viewed as a whole, the evidence seems to show that asymmetry is practically a constant feature; that it does not exclusively select one side in preference to the other; and that its amount tends to a higher average increase in the stephanic region as compared with the minimum frontal area. While, however, the asymmetry is not confined to one side of the skull in particular, yet there is, in both of the regions above referred to, a decided preponderance in favour of the right segment. This is shown by the fact that, of the forty-six measurements of human skulls taken in these regions, no fewer than thirty-one gave the excess to the right side. Of the remaining fifteen measurements, in eleven the excess was on the left side, and only in four cases were the segments equal, and each of these four pertained to the minimum frontal diameter.

In association with this definite asymmetry in the frontal region, it would be interesting to know whether these figures could be made to bear any conclusions with reference to the question of right- and left-handedness.

Maximum Parieto-squamous Breadth.—(*a*) Brachycephali.—In six skulls the excess was on the right side, and varied from 1–8 mm., the average being 4·1 mm. In the other five skulls the excess followed the left segment, and varied from 2–12 mm., the average being 5·2 mm.

(*b*) Dolichocephali.—In one case the segments were equal; in four cases the right segment was in excess, giving an average variation of 2·2 mm.; in seven cases the left segment predominated, with an average excess of 3·5 mm.

(*c*) Anthropoids.—In two cases the right side predominated, and in two cases the left.

In this region of the skull, asymmetry was also a noteworthy feature, and in its proportions it was very much more pronounced among the brachycephalic skulls. It appears to be quite exceptional to have the asymmetry confined to the same side of the skull throughout the series of transverse diameters, although in certain skulls this condition may prevail.

Asterionic Diameter.—(*a*) Brachycephali.—Bearing in mind the variable nature of the points which are the extreme ends of this diameter, it is surprising that four of the skulls indicated equality between the right and left segments. In other four, the excess was in the right segment, with a variation from 1–3 mm., and an average excess of 2 mm. In the remaining three cases, the excess belonged to the left segment, and also varied from 1–3 mm., with an average of 2 mm.

(*b*) Dolichocephali.—In three cases the right and left segments were equal; in five, the right segment held an average excess of 1·6 mm.; and in four, the left held an average excess of 4 mm.

(*c*) Anthropoids.—In all four the excess lay with the right segment.

Of the twenty-three measurements made in this region on the two groups of human skulls, nine gave the excess to the right side, seven to the left side, and seven were neutral. This result, taken in conjunction with the small averages of the variations, and the variable character of the sutures which determine the asterionic point, scarcely warrants any deduction concerning the value of the asymmetry in this region, although, as the asterionic diameter suggests approximately the width of the cerebellum, it is perhaps not to be wondered at, that there should be so much uniformity between the right and left halves of the occipital region of the skull.

Basi-bregmatic Height.—(*a*) Brachycephali.—In this group of skulls the height varied from 132 mm. to 144 mm., giving an average height of 140 mm.

The supra-zygomatic segment varied from 109–120 mm., the average being 113·9 mm.; the infra-zygomatic segment varied from 21–30 mm. Representing the supra-zygomatic segment in the terms of a percentage of the entire height, it varied from 78·5–84 per cent., the average percentage for the group being 81·3 per cent., leaving an average of 18·6 per cent. for the infra-zygomatic segment.

(*b*) Dolichocephali.—In this set the height varied from 128–146 mm., giving an average of 135·5 mm.

The supra-zygomatic segment varied from 98–114 mm., the average being 106·1 mm.; the infra-zygomatic segment varied from 22–35 mm., the average being 29·4 mm. Again, reducing the supra-zygomatic segment to a percentage of the total height, we find that it varied from 74 per cent. to 83·8 per cent., giving an average of 78·1 per cent., and leaving an average of 21·8 per cent. for the infra-zygomatic segment.

(*c*) Anthropoids.—The figures showed a steady reduction of the supra-zygomatic segment, with a corresponding increase in the infra-zygomatic segment, so that upon the figures referred to, these apes formed a sequence from man in the order orang, gorilla, young chimpanzee, adult chimpanzee.

In view of the fact that the segmentation of the basi-bregmatic height into supra- and infra-zygomatic segments suggests approximately the relative proportions between the height of the cerebrum and the height of the cerebellum, the results above referred to are extremely interesting, whether considered as averages or percentages.

In the human skulls whose measurements have been given, the average supra-zygomatic height is 113·9 mm. or 81·3 per cent. of the total height for the brachycephali, and 106·1 mm. or 78·1 per cent. of the total height for the dolichocephali. Approximately, therefore, the cerebrum, with the skull-cap, is 7·8 mm. or 3·2 per cent. higher in the brachycephali as compared with the dolichocephali. On the other hand, the infra-zygomatic segment in brachycephali has an average of 26 mm. or 18·6 per cent. as compared with 29·4 mm. or 21·8 per cent. in dolichocephali, apparently indicating that the depth of cerebellum, with thickness of skull base near basion, is 3·2 per cent. greater in dolichocephali than in brachycephali.

In the anthropoids, the steady diminution of the supra-zygomatic segment, and the correspondingly steady increase in the infra-zygomatic segment, are equally noteworthy.

Glabello-occipital Length.—The results obtained by segmenting this diameter in terms of the bregma and lambda are instructive, although probably less distinctive of the two extreme varieties of skull than might have been expected.

(*a*) Brachycephali.—The glabello-occipital length averaged in the eleven skulls 175·2 mm.

(1) The præ-bregmatic segment varied from 62–88 mm., giving an average of 72 mm., or an average of 40·9 per cent. of the total length.

(2) The parietal segment varied from 90–105 mm., giving an average of 97·9 mm., or an average of 55·3 per cent. of the total length.

(3) The post-lambdoidal segment varied from 3–9 mm., giving an average of 6·2 mm., or an average of 3·5 per cent. of the total length.

(*b*) Dolichocephali.—The average glabello-occipital length of the twelve skulls was 187·4 mm.

(1) The præ-bregmatic segment varied from 67–88 mm., giving an average of 74·9 mm., or an average of 39·8 per cent. of the total length.

(2) The parietal segment varied from 97–113 mm., giving an average of 104 mm., or an average of 55·5 per cent. of the total length.

(3) The post-lambdoidal segment varied from 3–18 mm., giving an average of 8·5 mm., or an average of 4·4 per cent. of the total length.

Several points of interest arise in connection with these figures. Thus, taking into account the thickness of an ordinary skull, it must be evident that with a post-lambdoidal segment averaging 6·2 mm. or 8·5 mm., very little, if any, of the cranial cavity can lie behind the lambda. The præ-bregmatic segment, subject to deduction for the thickness of the skull in the region of the glabellum, would represent the præ-bregmatic segment of the cranial cavity.*

The difference of an average of 1 per cent. between the præ-bregmatic segments of these two outstanding groups of skulls is equalised by a similar average difference of 1 per cent. between their post-lambdoidal segments. The interest, however, is largely associated with the parietal segment, which, notwithstanding the marked difference between the two groups of skulls in regard to the average length of the segment, viz., 97·9 mm. and 104 mm., yet presents an average percentage of the total length, which is practically identical in the two groups of skulls, viz., 55·3 per cent. and 55·5 per cent.

From these figures it would appear probable that the greater length of a dolichocephalic skull, as compared with a brachycephalic skull, is due to different rates of growth at the coronal and lambdoidal sutures, so that while the parietal bone maintains its percentage proportion of the total length in each group, the gain of 1 per cent. in the præ-bregmatic segment and the loss of a similar amount in the post-lambdoidal segment determines the brachycephalic skull, and the reversal of the gain and loss characterises the dolichocephalic skull. Stated in another way we might say that in brachycephalic skulls growth at the coronal suture is in excess of, or more active than, growth at the lambdoidal suture, while in dolichocephalic skulls the opposite condition prevails. Further, the increased activity in the plane of the coronal suture is associated with a similar condition at the spheno-parietal and squamoso-parietal sutures to enable the bregma to attain that elevation above the zygoma which characterises the brachycephalic skull.

Following up the same line of inquiry it seemed advisable to segment the diameter of greatest length, in terms of the occipital condyles, for the purpose of determining the relative position of the condyles to the glabello-occipital diameter, since it is upon the condyles that the weight of the head is transmitted to the spinal column, and more or less balanced in the erect attitude of man's progression.

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* For information with regard to the depth of the frontal sinuses, I await the appearance of an extremely elaborate paper which is now in course of preparation by Dr Logan Turner.

(a) *Brachycephali* :—

(1) The præ-condyloid segment varied from 87–100 mm., giving an average of 92·5 mm., or an average of 52·7 per cent. of the total length.

(2) The post-condyloid segment varied from 70–97 mm., giving an average of 82·7 mm., or an average of 47·1 per cent. of the total length.

(b) *Dolichocephali* :—

(1) The præ-condyloid segment varied from 92–108 mm., giving an average of 99·9 mm., or an average of 53·2 per cent. of the total length.

(2) The post-condyloid segment varied from 81–94 mm., giving an average of 87·5 mm., or an average of 46·6 per cent. of the total length.

Among these brachycephalic skulls, therefore, the præ-condyloid segment predominated over the post-condyloid segment by 9·8 mm., or 5·6 per cent. of the total length, whereas in the group of dolichocephalic skulls the præ-condyloid segment exceeded the post-condyloid by 12·4 mm., or 6·6 per cent. Thus we get an average difference of 1 per cent. upon the total length. In the dolichocephalic skull this was placed in front of the condyles, while in the brachycephalic skulls it was placed behind the condyles.

Now, although this is apparently a small amount of difference, yet as an average percentage it has some importance, for it seems to indicate that there is a more even and equitable adjustment of the balanced weight of the brachycephalic skull than of the dolichocephalic one, which still has a certain preponderance of its weight falling in front of its occipital condyles, as is the case in the lower mammals.

In this connection the measurements even of a few anthropoid skulls are extremely interesting. Thus, while in the brachycephalic group of human skulls the præ-condyloid segment exceeded the post-condyloid segment by 5·6 per cent., and in the dolichocephalic series the difference was 6·6 per cent., in the skull of an orangutan the difference was 31·1 per cent., in a young chimpanzee 33·9 per cent., in a young gorilla 40 per cent., and in an adult chimpanzee 46·7 per cent. No doubt in all of these apes the development of the frontal sinuses, superciliary ridges, and occipital crests may account for a considerable amount of the difference between the two segments. Nevertheless it is quite evident that a very pronounced alteration occurs in the relative proportions of the præ- and post-condyloid segments of the glabello-occipital length as we ascend from the anthropoid ape to man. In other words, there is an adjustment of the balance of the skull upon the atlas in association with the assumption of the erect attitude.

Prof. Cleland* has directed attention to the balance of the skull upon the vertebral column, and believes that its position in man results from the increasing prominence of the anterior extremities of the occipital condyles, whereby the skull is tilted upwards and backwards, the tilting of the skull being complicated by the increase in cranial curvature which proceeds from growth. He also points out that the falling forward of the skull in old age is due to the reduction in the size of the front ends of the occipital condyles.

Without attempting to criticise or traverse this argument, and while admitting the value and accuracy of the observations, I nevertheless think that the measurements which I have recorded show proportions between the præ-condyloid and post-condyloid segments of the glabello-occipital length which could not result merely from increased prominence of the occipital condyles, nor be affected by their partial absorption. It appears probable that varying rates of growth for the different sections or bones which contribute to the formation of the antero-posterior arch of the skull result in an increase of the occipital section, and therefore the condyles are proportionately moved forwards in relation to the base of the skull and the glabello-occipital diameter.

An examination of a number of skulls will show that a line drawn transversely between the centres of the two external auditory meatuses occupies a variable position with regard to the plane of the anterior margins of the occipital condyles. This is true not only of human skulls, but also of the skulls of other mammals. In order to obtain facts for comparison I segmented the glabello-occipital length in terms of the centre of the external auditory meatus.

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* Cleland, *loc. cit.*

(a) *Brachycephali.*—Only one skull gave similar præ-auditory and præ-condyloid segments. In three, the centre of the meatus was anterior to the plane of the condyles; and in the remaining seven, the centre of the meatus was behind the plane of the anterior borders of the condyles by distances which varied from 2–12 mm., the average for the seven being 4·2 mm.

(b) *Dolichocephali.*—In one skull the præ-auditory and præ-condyloid segments were alike. In two skulls the centre of the meatus was 2 mm. in front of the plane of the condyles; in the remaining nine the centre of the meatus was behind the plane of the condylar anterior margin by distances which varied from 1–6 mm., the average being 3·7 mm.

(c) *Anthropoids.*—In the four anthropoids that were measured the centre of the meatus was in each case definitely in front of the plane of the condyles by distances which varied from 8–15 mm., the average for the four being 10 mm.

These measurements indicate, that relatively the external auditory meatus is farthest forward upon the glabello-occipital length in anthropoids, and that it is farthest back upon the same diameter in brachycephalic skulls. Further, it would appear that during the acquisition of the erect attitude, as the occipital condyles have apparently moved forwards in relation to the glabello-occipital length, so the external auditory meatus has moved backwards until in the majority of skulls the centre of the meatus really falls behind the plane of the condylar anterior borders.

Of course this is a small number of measurements on which to base any broad generalisation, and such an attempt is not intended. The object has been to illustrate the kind of information which is within reach by means of a particular instrument, an extended application of which may evolve additional new facts. At the same time I cannot refrain from mentioning, in conclusion, that it seems more than a coincidence to observe, that, in every particular where an average total or an average percentage has been possible, the dolichocephalic skulls have been placed nearer to the anthropoids than in the case of the brachycephalic skulls.

BRACHYCEPHALI.

Race,	Sandwich Islanders.										
Locality,	Waimea Plains, Hawaii.				Oahu.						
Collection and Museum Number,	U. of E. A.	U. of E. B.	U. of E. C.	U. of E. D.	U. of E. K.	U. of E. S.	U. of E. V.	U. of E. Aa.	U. of E. I.	U. of E. Ac.	U. of E. Ah.
Age,	Ad.	23	Ad.	Ad.	Ad.	Ad.	Ad.	21	Ad.	Ad.	8
Sex,	♂	♂	♂	♂	♂	♂	♂	♂	♂	♂	♂?
***Minimum** Frontal Diameter,*	*90*	*102*	*98*	*95*	*96*	*98*	*97*	*92*	*94*	*88*	*92*
Right Segment,	46	50	49	49	50	51	51	47	47	46	46
Left Segment,	44	52	49	46	46	47	46	45	47	42	46
Stephanic Diameter,	*112*	*118*	*123*	*110*	*121*	*126*	*115*	*117*	*112*	*115*	*117*
Right Segment,	57	61	60	56	59	65	61	58	59	62	53
Left Segment,	55	57	63	54	62	61	54	59	53	53	64
Maximum Parieto-squamous Breadth,	*142*	*148*	*146*	*148*	*146*	*155*	*148*	*150*	*146*	*146*	*152*
Right Segment,	75	71	71	73	67	78	77	77	75	72	77
Left Segment,	67	77	75	75	79	77	71	73	71	74	75
Asterionic Diameter,	*101*	*113*	*108*	*104*	*102*	*116*	*110*	*105*	*105*	*102*	*114*
Right Segment,	52	56	53	53	52	58	55	51	53	51	57
Left Segment,	49	57	55	51	50	58	55	54	52	51	57
Basi-bregmatic Height,	*142*	*143*	*142*	*138*	*140*	*144*	*142*	*141*	*136*	*140*	*132*
Supra-zygomatic Segment,	112	117	115	110	110	120	118	114	109	117	111
Infra-zygomatic Segment,	30	26	27	28	30	24	24	27	27	23	21
Glabello-occipital Length,	*168*	*178*	*176*	*173*	*176*	*184*	*182*	*174*	*175*	*178*	*164*
Præ-bregmatic Segment,	62	74	84	67	64	88	82	62	72	70	67
Post-bregmatic Segment,	106	104	92	106	112	96	100	112	103	108	97
Præ-lambdoidal Segment,	162	173	173	165	168	178	178	167	168	169	158
Post-lambdoidal Segment,	6	5	Ap. 3	8	8	Ap. 6	Ap. 4	7	Ap. 7	9	6
Sagittal or Parietal Segment,	100	99	89	98	104	90	96	105	96	99	91
Præ-condyloid Segment,	98	88	93	95	91	87	96	100	90	91	89
Post-condyloid Segment,	70	90	83	78	85	97	86	74	85	87	75
Præ-auditory Segment,	98	97	95	90	96	91	99	102	92	90	85
Post-auditory Segment,	70	81	81	83	80	93	83	72	83	80	79

DOLICHOCEPHALI.

),	ABORIGINAL AUSTRALIANS.											
lity,	Riverina, N.S.W.	Mudgee Tribe, N.S.W.	Cape Leeuwin, W. Aust.	Roebuck Bay, Lat. 18°.	Benalla, Victoria.	Eucla, W. Aust.	Perth, W. Aust.	Macquarie River, N.S.W.	Swan Hill (*b*). N.S.W.	Swan Hill (*a*), N.S.W.	Wannon River, Victoria.	L. Alexandrina, S. Aust.
ection,	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.
	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.	Ad.
	♂	♂	♂	♂	♂	♂	♂	♂	♂	♂	♂	♀
imum Frontal Diameter,	*100*	*93*	*102*	*95*	*96*	*104*	*104*	*90*	*92*	*99*	*90*	*94*
Right Segment,	52	48	52	49	47	55	52	46	45	48	47	48
Left Segment,	48	45	50	46	49	49	52	44	47	51	43	46
phanic Diameter,	*108*	*104*	*113**	*100*	*116*	*109*	*113*	*108*	*108*	*110*	*104*	*105*
Right Segment,	56	54	62	51	60	54	57	50	56	59	51 N.	51
											60 44 } Breg.	
Left Segment,	52	50	51	49	56	55	56	58	52	51	53 N.	54
ximum Parieto-squamous Breadth,	*132*	*129*	*134*	*122*	*131*	*132*	*135*	*128*	*133*	*133*	*124*	*131*
Right Segment,	66	64	68	58	65	65	66	61	68	67	59	67
Left Segment,	66	65	66	64	66	67	69	67	65	66	65	64
terionic Diameter,	*106*	*103*	*120*	*110*	*107*	*116*	*110*	*110*	*110*	*106*	*105*	*105*
Right Segment,	53	51	61	53	52	58	56	53	55	54	53	53
Left Segment,	53	52	59	57	55	58	54	57	55	52	52	52
si-bregmatic Height,	*146*	*135*	*136*	*130*	*135*	*136*	*134*	*135*	*134*	*144*	*134*	*128*
Supra-zygomatic Segment,	112	100	114	99	105	105	106	111	107	112	105	98
Infra-zygomatic Segment,	34	35	22	31	30	31	28	24	27	32	29	30
abello-occipital Length,	*200*	*182*	*192*	*178*	*182*	*199*	*187*	*185*	*183*	*190*	*191*	*180*
Præ-bregmatic Segment,	88	68	74	71	70	75	74	82	67	70	88	71
Post-bregmatic Segment,	112	114	118	107	112	124	113	103	116	120	103	109
Præ-lambdoidal Segment,	197	168	187	172	174	184	181	181	174	172	185	172
Post-lambdoidal Segment,	Ap. 3	14	5	6	8	15	6	Ap. 4	9	18	Ap. 6	8
Sagittal or Parietal Segment,	109	100	118	101	104	109	107	99	107	102	97	101
Præ-condyloid Segment,	108	101	108	97	96	105	95	94	99	100	104	92
Post-condyloid Segment,	92	81	84	81	86	94	92	91	84	90	87	88
Præ-auditory Segment,	114	105	110	95	98	108	96	100	100	106	102	92
Post-auditory Segment,	86	77	82	83	84	91	91	85	83	84	89	88

* Bregma for point.

Race,	Orang-utan.	Gorilla.	Troglodites.	Hylobata Sp.	Chimpanzee.
Locality,	...	...	Niger (?)	...	...
Collection,	U. of E.	U. of E.	U. of E.	U. of E.	U. of E.
Age,	Juv.	Juv.	Juv.	Juv.	Adult.
Sex,	?	? ♂	?	?	?
Minimum Frontal Diameter,	*66*	*67*	*68*	*45*	...
Right Segment,	33	32	34	23	...
Left Segment,	33	35	34	22	...
Stephanic Diameter,	*63*	*51*	*61*	*43*	...
Right Segment,	33	24	30	22	...
Left Segment,	30	27	31	21	...
Maximum Parieto-squamous Breadth,	*112*	*111*	*119*	*57*	...
Right Segment,	57	53	60	28	...
Left Segment,	55	58	59	29	...
Asterionic Diameter,	*91*	*95*	*81*	*29*	...
Right Segment,	47	48	41	18 (?)	...
Left Segment,	44	47	40	11 (?)	...
Basi-bregmatic Height,	*90*	*89*	*85*	*47*	*92*
Supra-zygomatic Segment,	66	64	59	Ap. 41	62
Infra-zygomatic Segment,	24	25	26	6	30
Glabello-occipital Length,	*116*	*130*	*130*	*68*	*135*
Præ-bregmatic Segment,	41	55	62	41	...
Post-bregmatic Segment,	75	75	68	27	...
Præ-lambdoidal Segment,	106	105	124	64	...
Post-lambdoidal Segment,	10	25	6	Ap. 4	...
Sagittal or Parietal Segment,	65	50	62	23	...
Præ-condyloid Segment,	76	91	87	?	99
Post-condyloid Segment,	40	Ap. 39	43	?	36
Præ-auditory Segment,	67	83	72	36 (?)	91
Post-auditory Segment,	49	47	58	32 (?)	44

	AVERAGES.															
	Basi-bregmatic Height.	Supra-zygomatic Segment.	Percentage.	Infra-zygomatic Segment.	Percentage.	Glabello-occipital Length.	Pre-bregmatic Segment.	Percentage.	Parietal Segment.	Percentage.	Post-lambdoidal Segment.	Percentage.	Pre-condyloid Segment.	Percentage.	Post-condyloid Segment.	Percentage.
Brachycephalic (11), .	140	118·9	81·3	20	18·6	175·2	72	40·9	97	55·3	6·2	3·5	92·5	52·7	82·7	47·1
Dolichocephalic (12), .	135·5	106	78·1	29·4	21·8	187·4	74·9	39·8	104	55·5	8·5	4·49	99·9	53·2	87·5	46·6
Orangutan, . .	...	...	73·3	...	26·6	...	...	35·3	...	56	...	8·6	...	65·5	...	34·4
Gorilla, . . .	...	...	71·9	...	28	...	...	42·3	...	38·4	...	19·2	...	70	...	30
Chimpanzee (Ad.), .	...	...	67·3	...	32·6	...	...	...	...	...	...	...	...	73·3	...	26·6
Chimpanzee (Juv.), .	...	...	69·4	...	30·5	...	...	47·7	...	47·7	...	4·5	...	66·9	...	33
Gibbon, . . .	...	...	87·2	...	12·7	...	...	60·2	...	33·8	...	5·8	...	...	...	...

Preliminary Note of Experiments showing Heat of Combination in the formation of Alloys of Zinc and Copper to be Negative when the Proportion of Copper is less than about 30 per cent. By Alexander Galt, D.Sc.

(Read May 1, 1899.)

In March of last year I communicated to the Society a paper giving a detailed account of the procedure adopted in an experimental investigation, which I had undertaken at the request of Lord Kelvin, on the heat of combination of pairs of solid metals in the formation of alloys, along with a statement of some results which had then been obtained.

The pairs of metals chosen were copper-zinc and copper-silver, and, briefly restated, the experiments were conducted as follows:—About half a gramme of alloy, reduced to powder by means of a file, was dissolved in dilute nitric acid, and an equal weight of a mixture of the same two metals in the same proportions was also dissolved under exactly similar conditions. The difference between the initial and final temperature in each case is an indication of the heat of solution. If the two results are the same then there is no heat of combination in the formation of the particular alloy tested. But if the heat of solution of the mixture exceeds that of the alloy, then there must be heat of combination in the formation of the alloy equal to that difference. The experimental results, then, afford a means of approximately determining in absolute measure the heat of combination of the metals in the formation of an alloy.

The investigation has been continued since my last communication, and I expect to have it finished and to communicate the results to the Society either towards the close of the present session or at the beginning of the next. Altogether twenty-one different copper-zinc alloys, varying in composition from 5 to 90 per cent. of copper, and also five copper-silver alloys, containing from 10 to 65 per cent. of copper, are under observation.

I may here note, however, that the copper-silver alloys show little or no heat of combination, that many of the copper-zinc alloys do indicate heat of combination, the highest value, 52 gramme-water heat units centigrade per gramme of alloy, being obtained from those alloys whose composition is at or near the ordinary chemical combining proportions of the metals considered as bi-valent elements, that is 49·1 per cent. copper and 50·9 per cent. zinc. With lower percentages of copper the heat of combination rapidly falls until, at about 30 per cent. copper, the alloy shows no heat of combination.

But a still more surprising result—and this is the chief cause of my present communication—was obtained when alloys containing, respectively, 20·5, 16·0, and 10·5 per cent. of copper were tested. Each of these gave a greater heat of solution than the corresponding mixture, proving that in all of them the heat of combination had a negative value. The amount of this negative value was fourteen gramme-water heat units centigrade per gramme for the alloy containing 20·5 per cent. copper and also for the alloy containing 16·0 per cent. of copper, while it was eight gramme-water heat units centigrade for the alloy containing 10·5 per cent. of copper. This result was then verified in another way.

Half a gramme of an alloy numbered "5," and containing 26·5 per cent. copper, and which had already shown no heat of combination positive or negative, was dissolved in the usual manner, and the result obtained compared with that given under similar conditions, by a mixture of ·23 gramme of free zinc with ·27 gramme of an alloy, numbered "C," and which contained the metals in the chemical combining proportions (49·1 per cent. copper and 50·9 per cent. zinc) previously referred to. As ·27 gramme of alloy No. C contained ·1325 gramme of copper, the quantitative composition of the two powders tested was the same. A mean of five tests gave 8·64° C. as the rise of temperature in the experiment with alloy No. 5, and 8·42° C. with the mixture. In one powder the quantity (·1325 gramme) of copper present was alloyed with its chemical equivalent of zinc; and in the other the same quantity of copper was also present, but it was alloyed with an excess of zinc above the chemical equivalent. We might have anticipated that the large heat of combination previously noticed for alloy No. C would also have been found for alloy No. 5, but the larger heat of solution for the latter indicates that the excess in it of combined zinc (·23 gramme), over the quantity (·1375 gramme) chemically equivalent to the copper (·1325 gramme) present, apparently exerted a cooling effect. The difference between the two heats of solution is 0·22° C., or 0·44° C. per gramme of metal dissolved. The quantity of nitric acid used was 70 cubic centimetres of density 1·360 at 15° C.; the water equivalent of the apparatus was 3·5 grammes. The result works out at 28·07 gramme-water heat units centigrade per gramme of metal; but one gramme of the mixture contained ·54 gramme of alloy No. C, so that for one gramme of this alloy the value would be $28{\cdot}07 \times \frac{100}{54} = 52$, which is near the value otherwise obtained as the heat of combination per gramme of this alloy. Thus again it is shown that for alloy No. 5 the heat of combination of the copper with its equivalent of zinc is exactly neutralised by the cooling effect produced by the excess of zinc present.

On Duplicitas Anterior in an Early Chick Embryo. By Thomas H. Bryce, M.A., M.B. (With Four Plates.)

(Read June 19, 1899.)

The literature of Duplicity in Birds affords, out of a total of about ninety-five recorded cases of multiple formations of all kinds on a single blastoderm, from the stage of the primitive streak to the fourth day of incubation, only a small proportion of instances of "duplicitas anterior." Dareste (i.) in his atlas figures three; Gerlach (ii.) adds representations of three others—one case of his own, a second originally described by Ahlfeld, and a third by Reichert; Klaussner (iii.) gives a seventh case; and Bianchi (iv.) describes a monstrous embryo at a later stage (1·5 cm. in length).

Most observers have been content with the partial information derived from the study of the whole object, and only three embryos of this class, which have been studied in serial sections, have been described:—

1st. Erich Hoffman's (v.) with three somites.
2nd. Mitrophanow's (vi.) with six somites.
3rd. Kaestner's (vii.) with seven somites.

The present case makes the fourth. It lies in point of development intermediate between Hoffman's embryo and Kaestner's, and in general features it closely resembles them, but differs from Mitrophanow's in the greater symmetry of the components, for in his case, one member was considerably further developed than the other.

Thus, anterior duplicity is comparatively rare in birds, although it is the universal form in fishes, and the general form in reptiles and amphibians. Among the domestic mammals it is not uncommon, but the only case of the condition at an early stage, which has been studied by the help of serial sections, is an early human derodyme of 18 mm. described recently by Laguesse and Bué (viii.).

The embryo which is the subject of this communication was obtained from an egg which had been incubated for thirty-four hours under normal conditions, and was the only abnormal embryo found in two dozen eggs which were in the incubator at the same time. The area pellucida was circular and regular; the sinus terminalis and the vessels of the vascular area were just beginning to appear. The blastoderm was removed and fixed in picro-sulphuric fluid, the specimen was lightly stained with borax-carmine, dehydrated, cleared in cedar oil, and then drawn with the camera lucida.

Figure A represents the embryo viewed from above and magnified 18 diam. Its length, measured from the head end to the extremity of the primitive streak, is 3·7 mm. The anterior end consists of two independent members, which diverge from one another at an angle of 80°. The right member in the figure is slightly smaller and less regular than the left; each has a complete somatopleure and a complete splanchnopleure, but the somatopleure becomes single, 0·3 mm., the splanchnopleure, 0·4 mm., behind the head end, and the neural tubes remain independent for a distance of 0·76 mm. From this point to the first somite the neural canal is single, with the medullary folds approaching one another, but not meeting. In the region where the mesoblast is segmenting, the medullary folds are very wide apart, and between them is seen a series of five large and regular cubes, forming an azygos series of primitive segments. The lateral series have six segments, which are smaller and irregular. Posteriorly, where the mesoblast has not segmented, but still within the compass of the widely apart neural folds, two grooves appear, which run back for some distance parallel to one another, then bend towards the middle line, but do not meet. The right groove seems to fade away, while the left is continued directly into a typical mesial primitive streak.

Comparison with a normal embryo of the same period of incubation shows that there has been considerable retardation in development.

The specimen was embedded in paraffin, and a complete series of sections obtained 1-200 mm. thick. The remaining figures represent certain typical members of the series.

Notochord.—There are two separate notochords throughout the whole length of the embryo, so far as it is differentiated. At their posterior ends they become enlarged and pass directly into the mass of cells underlying the primitive grooves. The notochords gradually come nearer together from before backwards, but never come into contact.

In the four cases cited above the notochords were separate throughout except in Kaestner's, where they were united at their posterior ends.

Mesoblast —In the region of the primitive groove proper (fig. 14) the mesoblast extends outwards from the axial mass of cells as two lateral sheets in a typical manner. Traced forwards, the groove gradually dies out, and we have a flat ectodermic plate (fig. 13), continuous with which is a broad axial mass of indifferent cells, somewhat loosely arranged, from which the lateral plates of mesoblast extend. A few sections further forward two slight grooves indent the ectodermic plate, and when traced forwards, it is found that one of these deepens, and from its apex there extends a keel-like mass of cells continuous with the mesoblast on each side, while the other is only a slight dip in the ectodermic plate (fig. 12). Further forward, however, this dip deepens into a well-marked groove, beneath which is a similar keel-like mass of cells, so that now there are two lateral sheets of mesoblast, and an axial mass between the two keel-like thickenings (fig. 11). These gradually become marked off from the mesoblast (fig. 10), and then from the epiblast (fig. 9) forming the enlarged posterior ends of the notochords, while the grooves become the two neural furrows to be presently described. Between the two notochords lie the azygos primitive segments. In some sections (fig. 7) they are simple and have a single cavity, while in others (fig. 8) they are double, and have two cavities.

Neural Canal.—Fig. 1 is the 58th of the series; from this to the 94th (fig. 3) the medullary canals of the components are quite independent, and the medullary folds meet one another along their whole length, but do not fuse. At the level of the 84th section the mesial folds become enlarged, and two processes grow from them towards the middle line, until in the 94th (fig. 3) they meet and fuse. These processes project above the level of the somatopleure, so that a space is left between that membrane and the fused mesial folds. The point where the separate neural plates collide lies in front of the heart anlage at the anterior end of the ventral cœlom, at a point where that cavity still consists of two separate portions as in a normal embryo—in fact, at the point where the head fold proper ceases, and where the lateral folds begin to unite. From the 94th section to the 121st, the neural plate is single, but there are two canals bounded by three folds, a mesial and two lateral (fig. 4). The mesial fold consists of two parts, which, as the neural plate becomes more and more limited, and the neural canals approach one another, come nearer together and finally fuse to form a solid septum, which in turn disappears, and a single large canal is formed with gaping medullary folds (fig. 5). Further back the medullary folds approach more closely again, but nowhere unite (fig. 6). Section 263 (fig. 7) shows that the floor of the now widely open neural canal is indented by two shallow grooves above the two notochords, which are separated by a mesial thickening of the common neural plate, and when followed backwards are seen to be continued into the two grooves described above.

The *Fore-gut* consists, at the anterior end, of two entirely separate tubes, but, in the 75th section, their mesial walls come in contact, and a few sections further back there is only one very broad and compressed cavity. The lateral parts of the ventral cœlom are here very far apart (fig. 2), and in a mass of mesoblastic cells between them are seen two spaces which unite in section 96 (fig. 3) to form a single cavity, again to separate further back, where the splanchnopleuric folds begin to part. In a normal embryo, the two anterior *cul-de-sacs* of the ventral cœlom do not unite until a later stage opposite, or rather under the heart anlage. The two halves of the primitive heart tube are still wide apart (fig. 4), for the splanchnopleuric folds have parted in front of the heart anlage. It is also to be noted that the primitive heart tubes are abnormally broad and flattened out.

In the three cases of double chick embryo belonging to this class, cited above, as well as in the monstrous human embryo of Laguesse and Bué, there were two notochords, though in Kaestner's embryo they came in contact at their posterior ends. In Kaestner's case and in Hoffman's, there was only a single neural plate throughout, folded in the latter into two canals at the anterior end, while in the former the canal was single along its whole length. On the other hand, in my own case, and in Mitrophanow's, the anterior divergence of the axis was sufficient to allow of two separate neural plates and two neural canals being formed, and for the same reason two fore-guts, with complete somatopleuric and splanchnopleuric folds, while in the other cases these structures were single.

As regards the condition of the primitive streak, the four cases present certain differences; in Kaestner's the compound chorda passed into a simple streak, which forked at its posterior end; in Hoffman's the streak was forked at its anterior end, and the two limbs were continuous with the mesial stem behind, but again at its posterior extremity there were traces of what seemed to be accessory streaks. In Mitrophanow's case the two streaks fused behind, but the primitive groove of the smaller component was continued further back than that of the larger, from which he concludes: "Dass das Zwillingspaar . . . aus zwei ganz selbstandigen Keimen entstanden ist, welche in Form von Primitivistreifen noch selbstandig waren, wobei der Primitivistreifen des kleineren etwas hinter dem des grosseren lag, und letzterer seinerseits zu dem ersteren in einem Winkel gelegen war." In my own example, the same from a surface view is apparently the case, only the union of the streaks is further forward, and there is a single mesial primitive groove behind. A careful study of the sections has convinced me that such an interpretation cannot apply, but that there has been from the first a Y-shaped primitive streak. The limbs of the Y unite to form the broad flat plate described (fig. 13), from which the stem passes backwards, narrowing as it goes, till it has the typical form of a normal streak (fig. 14). On this Y-shaped streak, however, the primitive furrow has not developed as a continuous groove, but independently on the stem and limbs of the Y. On the left limb the furrow has differentiated further back than on the right, till it has almost met the mesial furrow. There was no trace of accessory rudimentary primitive streaks as in Kaestner's and Hoffman's cases.

In considering the nature of a double embryo, such as the one above described, one must distinguish between the question of the ultimate origin of the duplicity and that of the determination of its anatomical characters. It is in regard to the latter question that the study of a series of sections of an early stage of the condition can provide a definite answer.

There has been much discussion over the meaning and fate of the primitive streak. Many authorities, notably Balfour, have allowed it no part in the formation of the embryo, others have regarded it as gradually converted from before backwards into the primitive axis, but considerable divergence of opinion has prevailed as to the exact share it takes in its formation. Recently, however, Kopsch (x.), by direct experiment, has demonstrated that the whole streak in birds is used up in the formation of the embryo, only the anterior part of the head being laid down in front of it. Thus variations of the primitive streak acquire increased significance. Multiple primitive streaks on a single blastoderm have been described by Allen Thomson (xi.), Bruckhardt (xii.), and Mitrophanow (xiii.)—but no case has yet been described, so far as I know, of an undoubted simple single streak bifurcated in front. This is the condition, I believe, in the present case, the only alternative explanation being that there are three radial streaks, which I can hardly accept as possible in view of the appearances presented by the series of sections through this region.

If one attempted to interpret the condition in the light of Hertwig's "Urmund" theory (xiv.) or Kopsch's (x.) modification thereof, and regarded the primitive streak as homologous with the drawn-out gastrula mouth, the lips of which come together to form the axis of the embryo, such a Y-shaped streak would form an interesting parallel to the Y-shaped figures which O. Schultze (xv.) describes as resulting from abnormal gastrulation in frog's eggs treated by compression and rotation, and which resulted in tadpoles showing duplicitas anterior.

But, theory apart, the primitive streak being the anlage of the embryonal axis, the final constitution of the double form will be determined, in its variety and degree, by the relations of the streaks (or the limbs of the streak) to one another, and by the measure in which the growth of parts is interfered with, owing to the approximation of the cell masses constituting their anlagen, as shown by Dareste. This is not "fusion" strictly, however, for the anlagen of intermediate organs are composite, fused, from their first laying down, at least in cases such as the present, where the germinal disc is regular and single. The series of sections shows clearly how the whole constitution depends on the relation of the axes to one another, and the form and disposition of the future organs can be definitely forecast.

I do not purpose to enter in detail on the question of the ultimate origin of the duplicity. Professor Cleland (xvi.), in his notable contributions to the subject, regarded (1889) the cause as lying in a fission of the germinal mass occurring as the result of the application of some excitant, probably from without, the ultimate result depending on the date at which the fission takes place. It seems to be now practically universally admitted that a fission in some sense of the formative material does take place; but recent research all tends to prove that by the time the germinal layers are laid down, the cells have acquired their specific characters, and are incapable of dividing to form two individuals of the same potentiality.* This is supported by the striking fact that in birds duplicity has not yet been produced artificially. Gerlach (ii.) sought by varnishing eggs, except over a Y-shaped area above the blastoderm, to produce a fission of the germinal mass, but his results are universally regarded as proving nothing. Moreover, considerable light has, within recent years, been thrown on the possible nature of this fission by the experiments of the Entwickelungs-mechanik school, associated with the names of Roux, Driesch, Morgan, Wilson, Loeb, Herlitzka, O. Schultze, and others mentioned in the literature list appended. By various methods double and monstrous forms have been artificially produced from a single egg.

All these experiments show that while in normal circumstances each blastomere has its fixed share in the developmental process, under conditions of separation, of altered mutual relationship, or of disturbed equilibrium, they may revert to the condition of the entire ovum, segment as it does, and produce double, or if separation is incomplete, monstrous, forms.

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* Assheton (xxix.) thinks it conceivable that irregular splitting may sometimes arise in the morula stage, and during the formation of the blastodermic vesicle, in such a way that the embryonic mass becomes divided into two portions.

It is therefore in the early stages of development that the origin of duplicity is to be sought. It is one of the problems which modern biological thought has hunted back to the ovum itself, and there the secret must lie hidden, until we are in a position to explain the conditions determining the orderly rhythm of normal development, from which it is a departure.

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EXPLANATION OF PLATES.

Plate I., fig. A.—Whole embryo, × 18 D.

Plates I., II., and III., figs. 1–8, represent typical members of the series, arranged in serial order, × 100 D.

Plates III. and IV., figs. 9–14, represent sections through the posterior end of the embryo, serially arranged, × 350 D.

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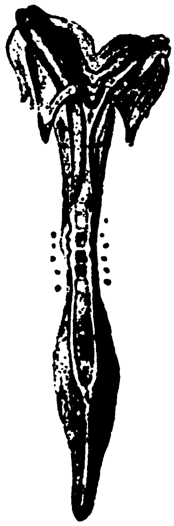

FIG. A.

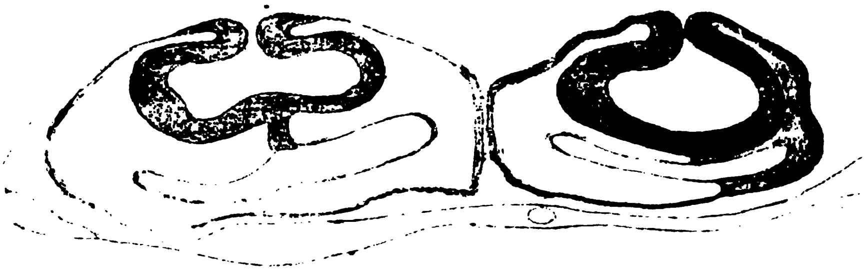

FIG. 1.

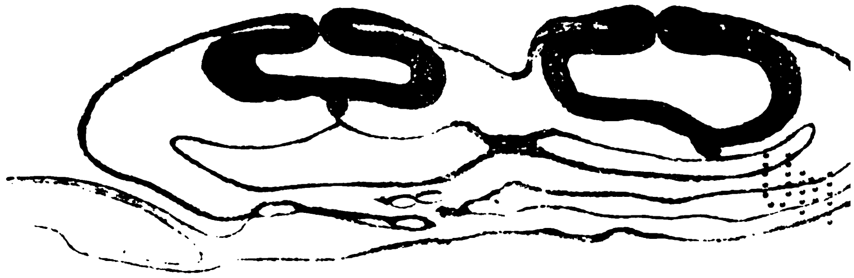

FIG. 2.

T. H. BRYCE.—Plate I.

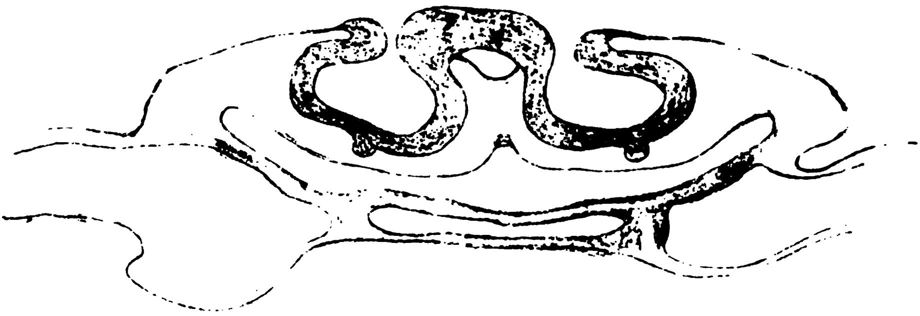

Fig. 3.

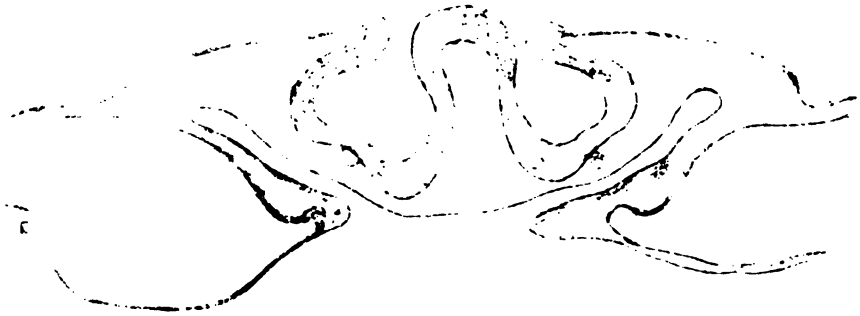

Fig. 4.

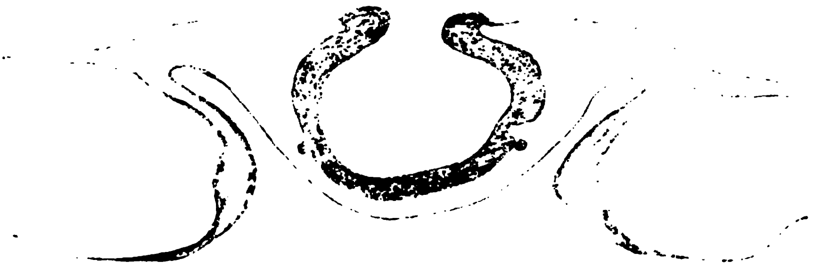

Fig. 5.

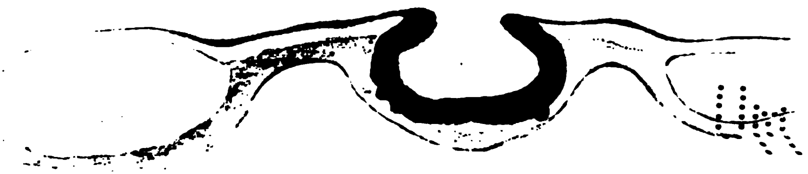

Fig. 6.

Plate II.

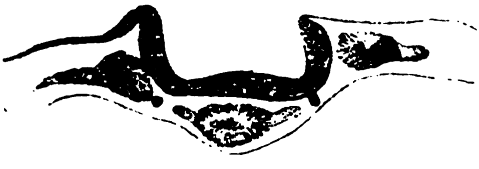

FIG. 7.

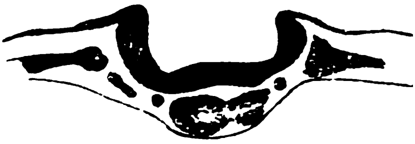

FIG. 8.

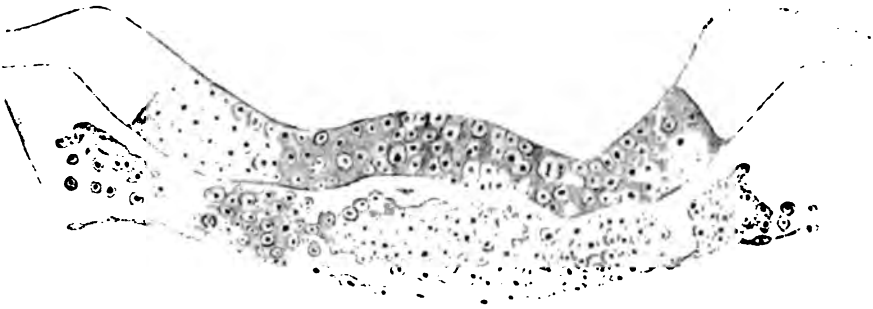

FIG. 9.

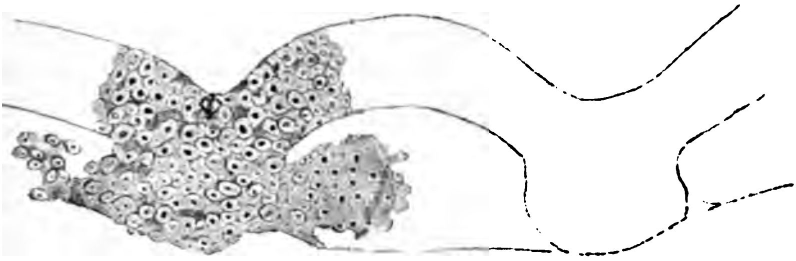

FIG. 10.

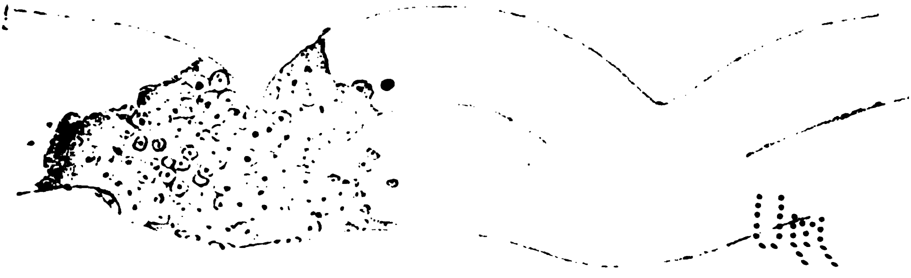

Plate III.

FIG. 11.

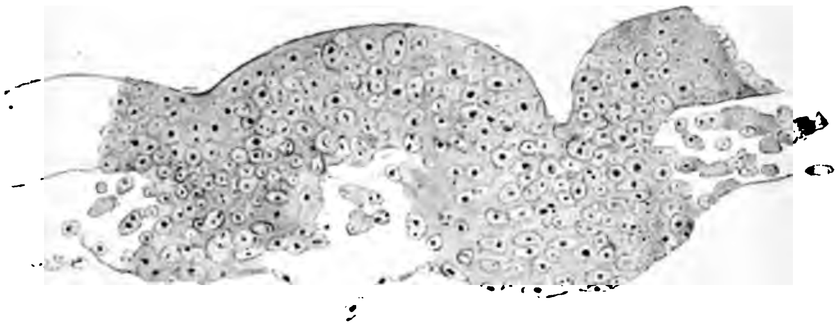

FIG. 12.

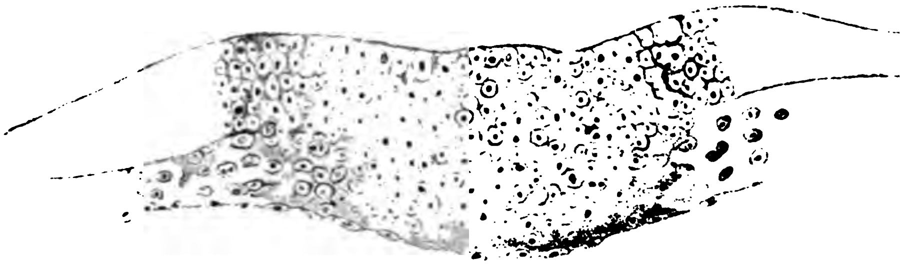

FIG. 13.

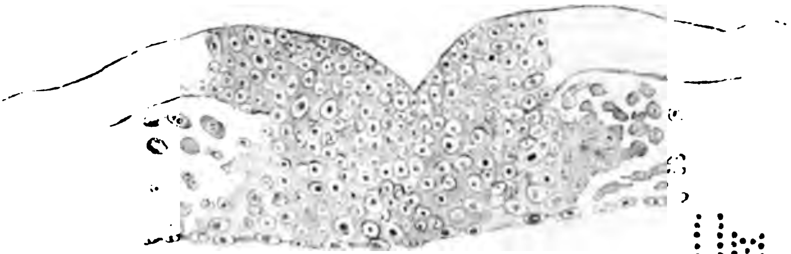

FIG. 14.

Plate IV.

Magnetism and Molecular Rotation.

By Lord Kelvin, *P.*R.S.E.

(Read July 17, 1899.)

§ 1. Consider the induction of an electric current in an endless wire when a magnetic field is generated around it. For simplicity, let the wire be circular and the diameter of its section very small in comparison with that of the ring. The time-integral of the electromotive force in the circuit is 2AM, if A denote the area of the ring and M the component perpendicular to its plane, of the magnetic force coming into existence. This is true whatever be the shape of the ring, provided it is all in one plane. Now, adopting the idea of two electricities, vitreous and resinous, we must imagine an electric current of strength C to consist of currents of vitreous and resinous electricities in opposite directions, each of strength $\frac{1}{2}$C. Hence the time-integrals of the opposite electromotive forces on units of the equal vitreous and resinous electricities are each equal to AM.

§ 2. Substitute now for our metal wire an endless tube of non-conducting matter, vitreously electrified, and filled with an incompressible non-conducting fluid, electrified with an equal quantity, e, of resinous electricity. The fluid and the containing tube will experience equal and opposite tangential forces, of each of which the time-integral of the line-integral round the whole circumference is eAM, if the ring be a circle of radius r; and the effect of the generation of the magnetic field will be to cause the fluid and the ring to rotate in opposite directions with moments of momentum each equal to $e\mathrm{AM}r$, if neither fluid nor ring is acted on by any other force than that of the electromagnetic induction. Their angular velocities are therefore $e\mathrm{AM}/rw$, $e\mathrm{AM}/rw'$, and their kinetic energies are $\frac{1}{2}e^2\mathrm{A}^2\mathrm{M}^2/w$, $\frac{1}{2}e^2\mathrm{A}^2\mathrm{M}^2/w'$, where w, w' denote the masses of fluid and ring respectively.

§ 3. Suppose now for simplicity in the first place, the ring to be embedded in ether, viewed as an incompressible solid, and attached to the ether in contact with it firmly enough to prevent slipping. The circuital impulse on the ring by the generation of the magnetic field will give rise to a rapidly-subsiding train of waves of transverse vibration, of the kind which, in communications to Section A of the British Association* at its meeting in Bristol last September, I described as a solitary wave of transverse vibration of the simplest possible kind in an elastic solid, and again, for periodic motion, as a very simple and symmetrical case of a train of periodic waves of transverse vibration. The work done by the circuital force on the ring is spent on waves of this class travelling outwards through ether, and in a very short time the ring comes practically to rest. It does not come to perfect rest suddenly by the departure from it of waves carrying away all its energy; it subsides to absolute rest in an infinite time according to the law $\epsilon^{-pt}\sin qt$. The resinously-electrified fluid within the ring continues revolving with unaltered energy as long as the force of the magnetic field is maintained constant.

§ 4. The simple molecular arrangement thus imagined supplies the rotatory or revolutional motion and the "moment of momentum," which forty-three years ago I pointed out † as wanted to explain, "simply by inertia and pressure," the rotation of the plane of polarisation, then recently discovered by Faraday, for light transmitted through heavy glass in a powerful magnetic field along the lines of force. In my Baltimore Lectures I showed that embedded gyrostats would in fact produce exactly the rotation of the plane of polarisation in a magnetic field discovered by Faraday. The idea which forms the subject of the present communication shows how the fly-wheels of the gyrostats may be started into rotation in virtue of the generation of the magnetic field and stopped when the magnetic field is annulled.

§ 5. The simply embedded gyrostat has not, however, the vibrational quality which is the essential of the Stokes-Maxwell-Sellmier vibratory molecule. For this a gyrostatic vibrator, capable of originating from a single blow on itself a subsidential train of at least 200,000 waves of light, must be connected with the surrounding ether by springs, having sufficient resilience to store up in themselves the total energy thus radiated out. Taking now as gyrostat our electric doublet of vitreously-electrified rigid hollow ring filled with fluid resinously electrified, consider what must be the nature of the elastic communication between it and a rigid lining of a spherical hollow in ether around it, to fulfil some of the known conditions of radiant molecules.

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* "On the Simplest Possible," etc.: "On Continuity in Undulatory Theory," etc., B. A. Report, 1898.

† "Dynamical Illustrations of the Magnetic and the Helicoidal Rotatory Effects of Transparent Bodies on Polarised Light."—*Proc. Roy. Soc. Lond.*, vol. viii., June 1856; *Phil. Mag.*, March 1857.

§ 6. (*a.*) Let the spring connection be equivalent to a simple force between I, the centre of inertia of ring and fluid, and O, the centre of the spherical sheath, varying directly as the distance between those points. The gyrostatic influence will be inoperative, and the result will be precisely the same as if we had a single Maxwell-Sellmier material point at I, of mass equal to that of ring and fluid together.

(*b.*) Let points on the ring be connected by springs with points on the sheath. Supposing now the sheath to be held fixed, the stiffnesses and the tensions of these springs may be adjusted to give 21 arbitrary values for the co-efficients in the quadratic for the potential energy of any infinitesimal displacement, specified by three components of linear displacement of I, and three components of rotational displacement round axes through I. The well-known solution of the problem of infinitesimal vibrations about a position of equilibrium of a rigid body, modified in respect to moments of inertia to take into account the fluidity of the incompressible fluid in the ring, gives us immediately the periods and geometrical specifications of six fundamental modes of simple harmonic vibration. Hence our combination, serving as a radiant molecule, without magnetic force would give six bright lines (understood of course that each of the six periods is within the range of light-periods). Suppose now a vast number of such molecules, all equal and similar in every respect but with different orientations, to be scattered through a flame. Each molecule, whatever its orientation, will give six lines of the same periods, though of different intensities when seen in any particular direction, according to the chances of orientation and of impulses. Hence each of the six bright lines will be perfectly sharp.

§ 7. Now suppose a magnetic field to be suddenly instituted.

The moment of momentum generated in any one of the molecules is $r\mathrm{AM}\cos\theta$, where θ denotes the inclination of the axes of its ring to the lines of force. The gyrostatic influence will split each of our six fundamental modes of vibration into two, greater than it and less than it by equal very small differences. These differences will be different for different molecules, because of the different values of θ for their different orientations. Hence each bright line is not split into two sharp lines, but is broadened to an extreme breadth corresponding to the value $\theta = 0$. No simplifying suppositions as to the character of the molecule, such as symmetry of forces and moments of inertia round the axes of the ring, can possibly give Zeeman's normal results of the splitting of a bright line into two sharp lines circularly polarised in opposite directions, when the light is viewed from a direction parallel to the lines of magnetic force; and the dividing of each bright line into three, each plane polarised, when the light is viewed from a direction perpendicular to the lines of force. Hence, although from 1856 till quite lately I felt satisfied in knowing that it sufficed to explain Faraday's magneto-optic discovery, I now, in the light of Zeeman's recent discovery, discard my old tempting gyrostatic hypothesis for an irrefragable reason, which is virtually the same as that stated by Larmor* in the following words:—"Hence a principal oscillation which is thus magnetically tripled must be capable of being excited with reference to any axis in the molecule; otherwise there would be merely hazy broadening or duplication instead of definite triplication."

§ 8. It now seems to me that the theory of H. A. Lorentz (of Leyden), as expressed by equations (1) in Zeeman's first paper "On the Influence of Magnetism on the Nature of the Light emitted by a Substance,"† is essentially true.

§ 9. Though it cannot explain Zeeman's discovery, the molecular rotation caused by the institution of a magnetic field, which is the subject of the present communication, may, however, be considered as interesting, not only because the idea of it seems to be new in electromagnetic theory, but also because it may conceivably constitute the explanation of Faraday's diamagnetism. Go back to §§ 2, 3 above, and remark that if a body containing a vast number of the molecules there described is situated between the poles of a steel magnet, the total energy will be greater than if there were nothing but ether between the poles, by a difference equal to the kinetic energy of the motion of the resinously-electrified fluid. Hence, if a body containing the supposed congregation of molecules is movable, it must be repelled from the place of strong magnetic force between the poles to places of weaker force further from them.

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* *Phil. Mag.*, vol. xliv., 1897, "On the Theory of the Magnetic Influence on Spectra," p. 507, line 3.

† *Phil. Mag.*, vol. xliii., 1897, p. 226.

On the Restoration of Co-ordinated Movements after Nerve Section. By Robert Kennedy, M.A., D.Sc., M.D., Glasgow. *Communicated by* Professor M'Kendrick.

(*Abstract.*) *

It is now well known that after reunion of a divided nerve by means of suture, function may return sooner or later, and that not only does this restoration of function imply simple return of sensation and of the capacity of making voluntary movements, but the re-establishment of complete localisation of sensation and of co-ordination of movements. When it is remembered that the individual nerve fibres are supposed to be paths between well-defined centres and peripheral endings, it is clear that so perfect a restoration of function is in many respects remarkable. Thus, in coapting a divided nerve, it is not likely that this can be effected with so great accuracy, that the corresponding ends of the divided nerve fibres can be brought into apposition. It is more likely that in coapting the nerve, most of the ends of the nerve fibres which are brought into contact are ends which do not correspond; and thus it would be expected that in the reunited nerve new paths for the nervous impulses would be established. And yet, despite this complexity of structure and of function, the reunited nerve seems to be as capable as before the division of subserving its functions.

In suturing divided nerves, care is usually given to secure that the two segments are approximated as nearly as possible in their old relationship. Thus, the perfect recovery of localised and co-ordinated function might be due to a majority of the fibres being thus placed in a position for union as before the division to take place.

To investigate this subject, three experiments were performed on dogs, the sciatic nerve being chosen. In two experiments

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* The paper is published *in extenso* in the *Transactions.*

the peripheral segment, before reunion by means of suture, was twisted so as to bring the maximum number of non-corresponding nerve fibre ends into juxtaposition, while in the third experiment reunion was made as accurately in the old position as possible.

The object of this was to ascertain:—

1st. If the time taken for the first evidence of recovery of co-ordinated movements, and the course of development of the same, were identical or different in the two cases.

2nd. If the resulting cicatricial nerve segments showed microscopically any important differences in the arrangement of the nerve fibres.

In all three experiments, the animal being anæsthetised, the sciatic nerve was divided at the level of the trochanter; but in the first two, before coapting the two ends by suture, the peripheral segment was rotated to the extent of a semicircle. Thus, on coaptation, the fibres of one side of the central segment were brought into apposition with fibres with which formerly they did not connect, *i.e.*, the fibres of the opposite side of the nerve. In the third experiment the nerve was divided at the same point, and accurate coaptation in the old relationship effected by suture.

The following is the course which the experiments exhibited, with reference to recovery of function.

EXP. I.—*Division of sciatic nerve at level of trochanter: rotation of peripheral segment to extent of semicircle and suture.*

Until the 7th day after the operation, the animal showed no sign of returning function. It walked on the limb, which was supported by plaster of Paris bandages. The paw was left unsupported for examination purposes, and until the 7th day, while the animal walked on the limb, the paw was dragged along the ground dorsal surface down. But on the 7th day the paw was first used normally in walking, the plantar surface being placed correctly on the ground. This was taken as evidence of returning co-ordinated movements. Frequently, while walking, the paw turned over, and the animal rested on the dorsal surface, but on these occasions the normal position was voluntarily regained after a step or two.

Restoration of localised sensation could not be ascertained, but undoubted evidence of sensation was obtained on the 10th day.

On the 14th day the supporting plaster of Paris splint was removed, and the wound was found healed. The animal then used the leg perfectly, almost always placing the plantar surface of the paw on the ground correctly.

By the 19th day the dog had practically completely recovered from the effects of the operation.

On the 54th day the nerve was again exposed, and was found united in the position in which it had been sutured. Stimulation above, below, and on the seat of reunion gave the normal contractions in the muscles of the leg.

EXP. II.—*Division of sciatic nerve at level of trochanter: rotation of peripheral segment to extent of semicircle and suture.*

In a few days the animal was walking about on its splinted leg, but dragging the paw along the ground dorsal surface down.

On the 7th day it was first noticed that the plantar surface of the paw was correctly placed, and, although occasionally turning over on the dorsal surface, was on these occasions voluntarily replaced in position after two or three steps.

By the 21st day distinct evidence of sensation was exhibited, and the recovery of the normal use of the limb was practically complete.

The physiological examination of the nerve on the 30th day showed it to be united in the position in which it was sutured, and to have regained its normal irritability and conductivity.

EXP. III.—*Division of sciatic nerve at level of trochanter: accurate coaptation by suture in normal position.*

On the 7th day the animal first showed signs of recovery of function, as, for the first time, it was then noticed that it walked placing the plantar surface of the paw on the ground. It was not, however, able to voluntarily readjust the paw when it happened to turn over with the dorsal surface down. This power was, however, exhibited on the following day.

By the 11th day evidence of sensation was obtained, and by the 14th day the recovery of function was complete.

On the 49th day the physiological examination showed that reunion of the nerve had taken place both anatomically and physiologically.

As regards recovery of function, therefore, these three experiments followed a practically identical course.

The microscopic examination of the three sciatic nerves showed the following characters.

The central segments showed the characters of the normal sciatic nerve, except in so far that the lymphatic spaces were somewhat distended, thus accounting for the increased thickness of the nerve which was macroscopically evident. This character was maintained to within 5 mm. of the seat of section.

The peripheral segments agreed in showing no old nerve fibres in any part examined, either close to the seat of reunion, or in the terminal divisions of the nerve. In their place were abundant young nerve fibres with well-defined characters. Lying between the young nerve fibres were degeneration remains of the old fibres, showing that Wallerian degeneration had taken place.

The cicatricial segments were also all three practically identical in structure. In no case could continuity of nerve fibres from the central to the peripheral segment be traced in a single section. The arrangement of the young fibres was very irregular, the fibres running a very tortuous course, and showing, therefore, in the section cut in all directions. The sections closely resembled the appearance of a neuroma.

The microscopic appearances of the seat of reunion leave it, therefore, doubtful whether the restoration of function in the two first cases was due to the re-establishment of the old paths for the nervous impulses by decussation in the cicatrix, or to the production of new paths by connections made between non-corresponding ends of nerve fibres.

The conclusions from the physiological and histological results of the experiments are :—

1st. That after section and immediate coaptation of a nerve, restoration of conductivity and of voluntary function may be effected in a few days.

2nd. That this early restoration of conductivity need not be the result of reunion of the old nerve fibres, *i.e.*, reunion by so-called first intention, or without Wallerian degeneration, but may be the result of regeneration of young nerve fibres in the peripheral segment.

3rd. That voluntary co-ordinated movements are regained equally soon, whether the two ends of the divided nerve are united as accurately as possible, so as to bring the corresponding ends of the nerve fibres into contact as nearly as possible, or whether, previous to reunion, the peripheral segment is twisted so that, when united to the central segment, non-corresponding ends of the nerve fibres are brought into contact.

4th. That in the latter case the microscopic examination of the seat of reunion leaves it doubtful whether the restoration of function is due to the re-establishment of the old paths by decussation in the nerve cicatrix, or to the reunion of ends of nerve fibres which do not correspond, but which happen to be brought into apposition.

5th. That in suturing a divided nerve no trouble need be taken to secure that coaptation of the two segments is effected in the old relationship, the simple approximation of the two ends, no matter in what relationship, being all that is required.

On the Anatomy of *Prosthecocotyle torulosa* (Linstow) and *Prosthecocotyle heteroclita* (Dies.). By Dr O. Fuhrmann, University of Geneva. *Communicated by* Sir JOHN MURRAY, K.C.B. (With a Plate.)

(Read May 1, 1899.)

In the Report on the Entozoa collected by H.M.S. "Challenger" Dr O. von Linstow[*] described two new tapeworms, which he named *Tetrabothrium torulosum* and *T. auriculatum.* The resemblance of the head of *T. auriculatum* with that of *Prosthecocotyle Forsteri* described by Monticelli[†] seemed to me to indicate that the *Tetrabothrium* described by Linstow probably belonged to the genus *Prosthecocotyle.*

Through the kindness of Sir John Murray, I received from the British Museum the original specimens for inspection, for which I wish to express here my warmest thanks. The following lines give the results of my examination; but before entering upon the description, let me say something on the systematic position of these two animals. As I have already remarked, Prof. Monticelli has rightly erected a new genus named *Prosthecocotyle* for the tapeworm *Tænia Forsteri*, Krefft. To this genus belongs, besides *P. Forsteri*, other parasites of birds which have been placed in the genera *Tænia, Tetrabothrium, Amphoterocotyle, Bothridiotænia, Bothriocephalus.* According to my researches the following species belong to the genus *Prosthecocotyle*:—

1. *P. Forsteri* (Krefft), (syn. *Tænia Forsteri*, Krefft), from *Delphinus Forsteri* (Gray) and *Delphinus delphinus*, L.

2. *P. Monticellii*, Fuhrmann (syn. *Tænia erostris*, *ex parte*, *Both-*

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[*] Linstow, O. von, Report on the Entozoa collected by H.M.S. "Challenger,"—Challenger Reports, Zoology, vol. xxiii., 1888, pl. ii. figs. 16–20.

[†] Monticelli, S., "Nota intorno a due forme di Cestodi,"—*Bull. dei Musei di Zoologia ed. Anat. comp. della R. Universita di Torina*, vol. vii., 1892, figs. 4–13.

ridiotænia erostris, var. *minor*, Lönnberg), from *Fulmarus glacialis*, L.

3. *P. umbrella*, Fuhrmann, from *Diomedea* sp. (?).

4. *P. torulosa* (Linstow), (syn. *Tetrabothrium torulosum*, Linstow), from *Diomedea brachyura*, Temm.

5. *P. macrocephala* (Rud.), (syn. *Tænia immerina*, Abildgard (?), *Rhytis immerina*, Zeder (?), *Bothriocephalus macrocephalus*, Rud., *Tetrabothrium macrocephalum*, Rud., *T. Zedri*, Baird, *T. Colymbi*, Troiles Viborg (?)), from *Colymbus glacialis*, L., *C. septentrionalis*, L., *C. arcticus*, L., *Podiceps cristatus*, L., *P. cornutus*, Gm., *Uria troile*, L., *Aptenodytes* sp. (?), *Totanus glareola*, L. (?).

6. *P. juncea* (Baird), (syn. *Bothriocephalus junceus*, Baird, *Tetrabothrium junceum*, Baird), from *Sarcoramphus papa*, L.

7. *P. cylindracea* (Rud.), (syn. *Bothriocephalus cylindraceus*, Rud., *Tetrabothrium cylindraceum*, Rud., *Bothridiotænia cylindracea*, Rud.), from *Larus glaucus*, Brünn, *L. atricilla*, L., *L. ridibundus*, L., *L. canus*, Brünn, *L. marinus*, L., *Rissa tridactyla*, L.

8. *P. erostris* (Lönnberg), (syn. *Tænia erostris*, Lönnberg, *Bothridiotænia erostris*, Lönnberg), from *Larus marinus*, L., *L. canus*, Brünn, *L. fuscus*, L., *L. argentatus*, Brünn, *Rissa tridactyla*, L., *Sterna* sp. (?).

9. *P. eudyptidis* (Lönnberg), Fuhrmann, (syn. *Bothridiotænia erostris*, var. *eudyptidis*, Lönnberg), from *Eudyptes catarractes*, Gm.

10. *P. heteroclita* (Dies.), (syn. *Tetrabothrium heteroclitum*, Dies., *Amphoterocotyle elegans*, Dies., *Tetrabothrium auriculatum*, Linstow), from *Daption capensis*, L., *Thalassœca glacialoides* (Smith).

11. *P. intermedia*, Fuhrmann, from *Procellaria* sp. (?).

12. *P. campanulata*, Fuhrmann, from *Procellaria* sp. (?).

13. *P. pelecani aquilæ* (Rud.), (syn. *Tænia pelecani aquilæ*, Rud., *Tænia heterosoma*, Baird, *Tænia sulæ fuscæ*, Baird), from *Atagen aquilus*, L., *Sula fusca*, Viellot.

14. *P. sulciceps* (Baird), (syn. *Tænia sulciceps*, Baird), from *Diomedea exulans*, L.

15. *P. porrigens* (Molin), (syn. *Tetrabothrium porrigens*, Molin), from *Nystiardea nycticorax*, L., *Larus melanocephalus*, Natt. (?).

16. *P. triangulare* (Dies.), (syn. *Tetrabothrium triangulare*, Dies.), from *Delphinorhynchus rostratus*, Cuv.

The genus *Prosthecocotyle* may be characterised as follows:—The head, without rostellum and hooks, has a quadrangular shape, which is caused by the singular structure of the suckers. Each sucker shows the peculiarity of having on the outmost end a lateral protrusion, which has the same histological structure as the oval and powerful suckers themselves. The segmentation begins not very far behind the head, and the segments are always shorter than broad, except the last segments on the posterior end. The morphology of the genital organs also shows some typical characters. *Male Organs.*—The cirrus-pouch is always small, and has a globular shape. It lies at some distance from the margin, and is joined with the genital cloaca by a canal which I call 'male cloacal canal.' It is rarely armed. The vas deferens is very long and without vesicula seminalis. The testicles, to the number of 8 to 60 in each segment, are confined to the dorsal portion of the median field. *Female Organs.*—The small vitelligenous gland lies always *before* the great ovary. The vagina passes directly from the ventral side of the cirrus to the ovary, showing a receptaculum seminis varying in form and structure in the different species. The genital cloaca is always on the left margin of the strobila; it is very deep and possesses a very complicated musculature. The eggs are enveloped by three shells.

Prosthecocotyle torulosa (Linstow). (*Tetrabothrium torulosum*, Linstow.) Figs. 1–3.

This Tænia was found in *Diomedea brachyura.* The length is, according to O. von Linstow, 175 mm. The width behind the head is 1 mm., from there increasing gradually, and 9 cm. behind the scolex it attains 5 mm., which is the maximum width. The segmentation of the body begins immediately behind the head, but the segments are at first very short (0·022 mm.). The segments, with the genital glands well developed, are very thick; they are 0·126 mm. long by 5 mm. wide, and have a vertical diameter of about 2 mm. The measurements are almost the same for the segments crowded with eggs. The posterior border of each segment overlaps—with a prominent fold—the succeeding one. All the specimens that I had occasion to study were strongly contracted. The measurements given above have therefore no absolute value, for they vary according to the degree of the contraction of the animal. The scolex is unarmed, without rostellum and hooks; it has the typical shape of the head of the *Prosthecocotyle* species. The head differs from the drawing published by O. Linstow, as may be seen by comparing fig. 17 of Linstow with my fig. 1 drawn with Abbé's drawing apparatus. The scolex is 1 mm. wide and about 2 mm. long. Its shape is quadrangular. This singular form is produced by the protrusions of the suckers characteristic of all *Prosthecocotylæ.* These are the suckers that exhibit anteriorly and externally ear-shaped protrusions, which have the same structure as the suckers. The suckers with their small openings lie on the dorsal and ventral side of the scolex; they are oval, with a long diameter of 0·34 mm. and a transverse diameter of 0·23 mm. They touch one another in the median line of the head, and are very powerful and deep. Unhappily, I cannot give information on the histological structure of this interesting scolex, because I could not make sections of it. Neither can I state whether the posterior part behind the head is normal.

The segments, which are very short and thick, and separated by a deep incision, are covered by the cuticula and the subcuticular layer of cells. The cellular structure of the parenchyma is distinct everywhere; it is not crossed over by muscular fibres. We find it so in the lateral extremities of the internal parenchyma and in the cirrus-pouch. The calcareous corpuscles are found specially in the external parenchyma, but also between the muscles and in the intermediate layer of parenchyma situated between two segments. In the ripe segments they are particularly numerous in the lateral parts of the internal parenchyma, where one can study their development out of the cells of the parenchyma.

We find directly under the cuticula a simple layer of circular and longitudinal fibres. In the parenchyma we discover two layers of longitudinal muscles. The external stratum is formed by small bundles of spindular fibres (*ca.* 10 fibres). This thickly laid stratum of muscles envelops completely the central part of the segment, and is only interrupted at the point where the genital canals pass out. They go through the strobila without diminishing in thickness at the limits of the segments. This muscular layer, which has a thickness of 0·038 mm., touches the cuticula only at the limits of the segments, and it follows from this disposition that the external parenchyma of each segment is completely separated. The external muscular layer is much more powerful. It consists of bundles (diameter 0·114 mm.) formed by fifty fibres at the utmost. This stratum of longitudinal muscles is separated from the external layer by a small zone of parenchyma, and only interrupted for a short space on both sides of the strobila. Inside of the longitudinal muscles we find a layer of transverse muscles. In the lateral part of the strobila these muscles radiate only in the external parenchyma, and fix themselves on the cuticula of the anterior side of the segments. The dorso-ventral fibres are specially situated where they are not disarranged by the development of the genital glands, and are therefore mostly found in the lateral sides of the proglottis and between the different segments. These fibres can, without doubt, modify the depth of the incision between the segments. As to the form of the fibres, we find it differs in the different systems. The dorso-ventral fibres are very long and fine. The transverse fibres are also very long. The longitudinal fibres are, on the contrary, short and thick.

We could only study the water-vascular system in the strobila. The two pairs of longitudinal vessels are placed one above the other; the dorsal one has a diameter of about 0·01 mm.; the ventral one is much better developed, and is 0·06 mm. wide. The vessel that joins the ventrals has a diameter of 0·032 mm. The water-vascular system has a very peculiar structure, the vessels having a powerful musculature which transforms this system into a contractile organ. The dorsal vessel is especially muscular, and thus varies much in size. On the ventral vessel the circular and longitudinal fibres are neatly developed, but each one is represented by a simple stratum. Such a complex musculature has never been noticed in the water-vascular system of the cestodes. Riehm * has described circular muscles on the vessels of *Dipy-*

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* Riehm, G., "Studien an Cestoden," *Zeitschrift f. d. ges. Naturwiss.*, Bd. 54, pp. 545–610. 1881.

lidium Leuckarti, Roboz * found on the vessels of *Solenophorus* a feeble stratum of circular and longitudinal fibres. The vessels which bind the ventral vessels are without muscles. I only saw the two powerful longitudinal nerves which are situated on the outside of the water-vascular system.

As the segments are separated by a deep incision, the opening of the genital cloaca is situated on the left and anterior side of the proglottis. The genital cloaca itself is wrinkled, and lined by the continuation of the cuticula of the body. On the walls of the cloaca are fixed numerous muscles descending from the transverse musculature. These muscles have the function of flattening out the cloaca; they are therefore expanders of the genital cloaca. At the bottom of the cloaca a long canal leads to the cirrus-pouch, which may be called the 'male cloacal canal.' This canal has the same structure as the 'hermaphrodite canal' described by me in *Tænia depressa.*† This ductus has therefore a powerful circular and longitudinal musculature, surrounded by numerous nuclei pertaining to myoblasts, and parenchyma cells. Through this canal the cirrus passes to the outside. The cirrus-pouch, being spherical, has the typical shape of the *Prosthecocotyle* genus. The wall of this organ is 0·018 mm. thick. The internal part consists of circular, the external of longitudinal muscles. Inside this organ is the short vas deferens, which is enveloped by a feeble musculature. The penis is unarmed. The vas deferens passes nearly in a straight line under the nerve and between the two vessels of the water-vascular system. A series of circumvolutions, becoming always closer, and passing on the dorsal side of the median field to the middle of the segments, commences here. The testicles are placed on the dorsal side and are in number *ca.* 50. Their diameter is in the dorso-ventral direction *ca.* 0·17 mm., the transverse diameter being 0·08 mm.

The female glands are, except the uterus, situated on the ventral side. The ovary is well developed and deeply lobed. The egg-cells are very large (0·057 mm.). A protoplasmic mass containing

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* Roboz, F. von, "Beiträge zur Kenntnis der Cestoden," *Zeitschrift f. wiss. Zoologie*, Bd. 37, 1882, pp. 263–285.

† Fuhrmann, O., "Beitrag zur Kenntnis der Vogeltaenien," *Revue Suisse de Zoologie*, t. iii., 1895, pp. 454–458.

the nuclei of young egg-cells occurs at the bottom of the ovarian tubes (slightly swelled at their extremity). It is here that during a short time new eggs are produced. The vagina begins on the ventral side of the cirrus-pouch, and runs nearly in a straight line in the middle of the median field to join the oviduct. This canal dilates on two points of its course. The first widening is longish but not very considerable; it is probably the receptaculum seminis, in which I could not find spermatoids,* also the structure is not the same that one generally finds in the other cestodes for the same organ. The second widening, very short, is not far from the point where the vagina rejoins the oviduct. The structure is the same as that of the vagina, but with a much more powerful longitudinal and circular musculature. The oviduct begins with a very well developed muscular funnel. This funnel is carpeted inside by an epithelium. From this organ the oviduct goes to meet the vagina, forming distinct circumvolutions (fig. 3). The oviduct is very wide, and carpeted by an epithelium, formed partly by columnar cells, out of which issue long cilia. From the point where the oviduct meets with the vagina, it becomes very narrow. It descends towards the shell-glands, and it is there that the vitelloduct throws itself into the oviduct. The small and slightly vitelligenous gland is the sexual gland which is situated before the ovary, as in all the *Prosthecocotylæ*. From the shell-glands the oviduct leads in a straight line towards the dorsal side of the proglottis, where it opens into the uterus. The uterus is therefore situated on the dorsal side of the ovary. Its most considerable diameter is in the median line of the strobila; it diminishes promptly laterally, where it passes between the longitudinal vessels of the excretory system. This conformation of the uterus is found in the segments where the sexual glands are still well developed. It is in this state that the uterus is carpeted by very high cells. When the uterus is distended with eggs, these cells suffer a granular degeneration of which we find the remains between the eggs. Are these granules used to nourish the eggs which are for a long time devoid of an envelope, even when they have already begun their segmentation? Or do they help the

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* We must say that our material consisted only of a few well-preserved fragments.

formation of the envelopes of the eggs? As I have just said, the eggs remain for a long time without shells, and are very poor in vitellus; it is only in the last segments that one finds the embryo with its six hooks enveloped by three shells (diameter of the embryo and the first envelope 0·036 mm.; hooks, 0·014 mm.; diameter of the second envelope 0·039 mm.; diameter of the third envelope 0·054 mm.). In the segments which are quite ripe, the sexual glands have completely disappeared and the uterus fills up the internal parenchyma.

Prosthecocotyle heteroclita (Dies.).
Syn. *Tetrabothrium heteroclitum* (Dies.), *Amphoteracotyle elegans* (Dies.), *Tetrabothrium auriculatum* (Linstow). Figs. 4–7.

A serious comparative study of the originals of *T. auriculatum* and of Diesing's examples of *T. heteroclitum* has shown me that the two species are identical. This interesting Tænia was found in the intestines of *Thalassœca glacialoïdes*, Smith, and *Daption capensis* (Linn.). According to Linstow, the animal attains a length of 112 mm.; the scolex is 0·48 mm. in breadth and 0·34 mm. in length. According to my measurement the breadth of the head is of 0·38 mm. The powerful suckers occupy almost the whole of the scolex, and each one exhibits anteriorly and externally an ear-shaped protrusion. "At a distance of 0·6 mm. behind the scolex the segmentation begins. The first proglottides are 0·012 mm. long and 0·41 mm. broad; those in the middle are 0·29 mm. by 1·64 mm., while those furthest back measure 0·42 mm. in length by 2·5 mm. in breadth." This worm has not such a great thickness as *P. torulosa*, but is, on the contrary, very flat. Also the proglottides are a little longer than in that species, but the separation of the proglottides is equally distinct, and the separation between each segment reaches the longitudinal muscles. In the external parenchyma numerous calcareous corpuscles are found. The internal parenchyma is reduced by the development of the sexual glands.

Besides the subcuticular musculature, which shows nothing very remarkable, we find, as in *P. torulosa*, a double zone of bundles of longitudinal muscles, the external ones of which are less strong.

Dʀ FUHRMANN ON PROSTHECOCOTYLE.

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Fig. 1.

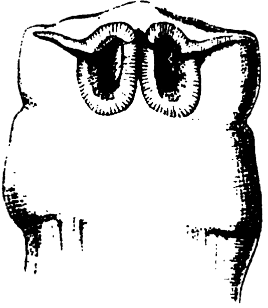


Fig. 2.

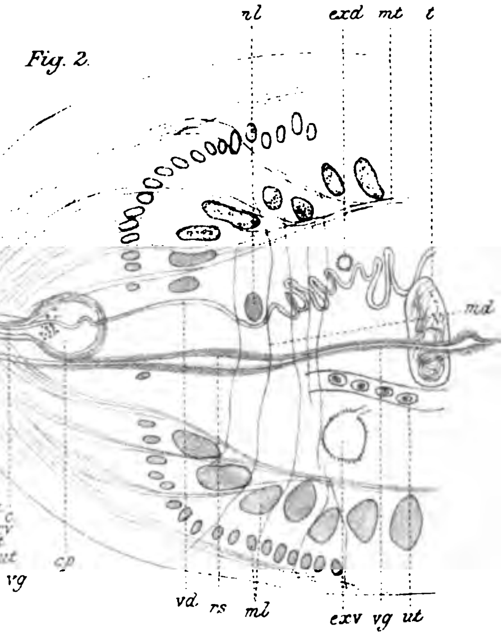


Fig. 5.

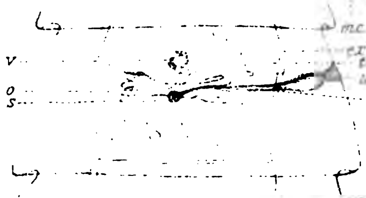


Fig 4.

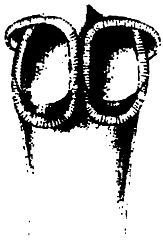


Fig. 3.

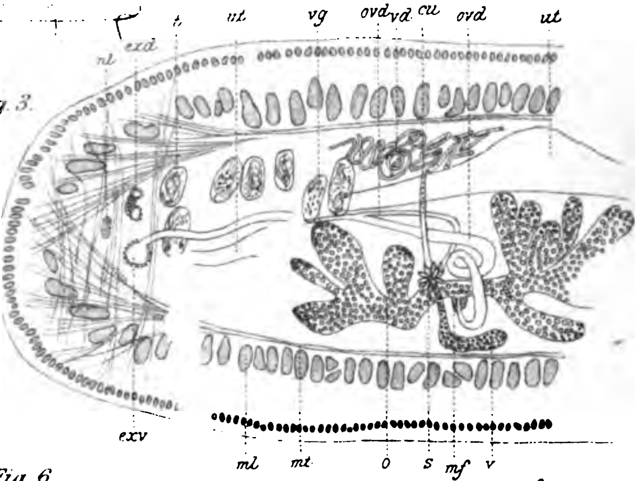


Fig. 6

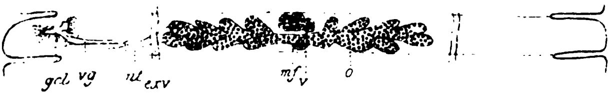


Fig. 7.

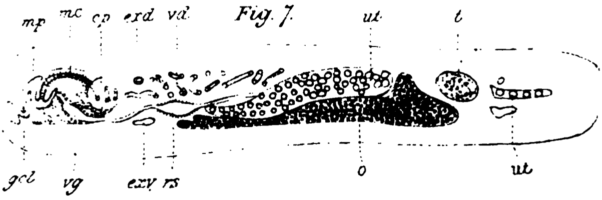
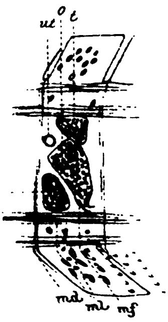


Fig. 8.



F Huth Lithʳ

The external zone is continuous, which means that it is only interrupted at the point where the sexual organs pass out. The internal zone is confined to the ventral and dorsal surfaces. On the internal side of the longitudinal musculature lies a feeble layer of transverse muscles, which assists the formation of the complicated musculature surrounding the terminal part of the sexual canals. The dorso-ventral musculature is feeble.

The nervous system consists in the strobila of two longitudinal nerves which are situated on the outside of the water-vascular system.

The vascular system, which I could only study in the segments, is situated rather far from the lateral side (in a proglottides 1·65 mm. in diameter the vascular system is 0·4 mm. distant from the side). It consists of two ventral vessels and two dorsal vessels placed above the first. These two pairs of vessels are surrounded, as in *P. torulosa*, by a musculature. The ventral vessels are joined in every segment by a large vessel.

The male sexual glands are composed of 28 testicles situated on the dorsal side of the internal parenchyma. The vas deferens is very long, and presents numerous convolutions converging towards the left side, that is to say, the side on which issues the sexual canals in all the *Prosthecocotylæ*. The interpretations of the male sexual apparatus given by Linstow are inexact, which is due probably from the fact that this author did not make sections. The cirrus-pouch, into which enters the vas deferens, presents a form and a structure which is, so to speak, identical in all the species of *Prosthecocotyle*. It is spherical, and contains a vas deferens with thick walls. The cirrus, which is very long, passes through the canal that I called in the description of *P. torulosa*, 'male cloacal canal.' This canal, having the shape of a very prominent papilla, has, as well as the genital cloaca, into which it enters, a complex structure. The papilla and cloaca are carpeted by the continuation of the cuticula of the body. Their muscular system is composed of internal circular fibres and of others with a radial disposition. The whole is surrounded by a system of fibres proceeding from the transverse musculature of the parenchyma. So the whole presents, in transverse sections, the structure of a sucker (figs. 6, 7). Into the deep genital cloaca, with wrinkled walls, the vagina passes out, on the ventral side of the male papilla. The vagina is large at the beginning, and surrounded by a strong musculature, essentially formed by circular fibres, which cross each other. At the entrance into the internal parenchyma, the diameter of the female canal diminishes, to swell again as soon as it has passed the water-vascular system. The second extension, spindle-shaped, is equally muscular; but here it is the longitudinal fibres which prevail. It is a receptaculum seminis similar to that of *P. torulosa.* From this point the vagina goes towards the middle of the proglottis to meet with the canal that comes from the ovary. Before joining, the canal narrows suddenly, presenting at this point a stronger musculature. It is probably a sphincter which prevents the spermatoids returning. The sexual glands are composed of an ovary, lobed, situated ventrally, and occupying all the length and breadth of the parenchyma placed between the water-vascular system. At the origin of its canal is situated a muscular funnel: an egg-aspirator. The vitelligenous gland, situated in the middle, lies ventrally, and is small; it is placed before the ovary. Where the vitelloduct meets the oviduct is to be found a shell-gland from which the canal runs towards the dorsal side to open into the uterus, which has the same shape as in *P. torulosa* (fig. 7). In the last proglottides the uterus fills the whole internal parenchyma. I could not study the hooks of the embryo, the eggs not being in a sufficiently advanced state. As I had not the last proglottides, I cannot say if there are three shells surrounding the eggs.

It is very probable that in this species the fecundation of the eggs is often produced by the male organ of the same proglottid; but the very complex disposition of the musculature of the cloaca, and the peculiar development of the male papilla, lead me to believe that fecundation may also take place between different proglottides.

EXPLANATION OF THE PLATE.

ml.	longitudinal muscles.
mt.	transverse muscles.
md.	dorso-ventral muscles.
nl.	longitudinal nerve.
ex.v.	ventral canal.
ex.d.	dorsal canal.
gcl.	genital cloaca.
mc.	male dorsal canal.
mp.	male papilla.
cp.	cirrus pouch.
c.	cirrus.
v.d.	vas deferens.
t.	testicle.
v.	vagina.
rs.	receptaculum seminis.
ovd.	oviduct.
cu.	canal of the uterus.
s.	shell-gland.
vitd.	vitelloduct.
o.	ovary.
mf.	muscular funnel.
v.	vitelligenous gland.
ut.	uterus.

All the figures are drawn with Abbé's drawing apparatus.

Prosthecocotyle torulosa (Linstow).

Fig. 1. Head of *P. torulosa.*

Figs. 2, 3. Transverse sections of half of segment.

Prosthecocotyle heteroclita (Dies.).

Fig. 4. Head of *P. heteroclita.*

Fig. 5. Young segment.

Fig. 6. Horizontal section of segment.

Fig. 7. Transverse section of segment.

Fig. 8. Sagittal section in the median plane of segment.

On the Vascular System of the Hypocotyl and Embryo of Ricinus Communis, L. By Edith Chick, B.Sc., Quain Student in Botany, University College, London. (With Three Plates.)

(Read June 19, 1899.)

Sections of the hypocotyl of *Ricinus communis* are frequently examined as typical of the young stem, in which widely separated vascular bundles are ultimately united by a band of interfascicular cambium, and this by giving rise to secondary tissues connects the xylems and phloems of the primary bundles to form concentric continuous rings.

In the young unthickened hypocotyl of Ricinus there are eight such bundles connected by a continuous starch sheath and one layer of smaller cells immediately inside it. In some sections, however, confusion arises owing to the increase of the number of vascular bundles, and in the case of older hypocotyls these extra bundles are embedded in the cambium ring and have rather the appearance of secondarily formed tissues.

It was in the first instance to clear up the origin of these bundles that, at the suggestion of Professor F. W. Oliver, the following work was begun. Afterwards, as there seemed some advantage in working out in full the arrangement of the vascular system in this plant, which is so frequently used as a dicotyledonous type, it has been followed from the root, through the hypocotyl, to the 1st and 2nd internodes of the epicotyl.

The well-developed primary root of the seedling of *Ricinis communis* averages about 2 mm. in diameter. About 5 mm. below the junction with the stem it enlarges rapidly to attain the diameter of the hypocotyl, which varies between 4 and 5 mm. It is from this swollen portion of the root that most of the first developed lateral rootlets take their origin. They appear at the surface at points corresponding to the position of the proto-xylems within, and form four vertical rows corresponding with the tetrarch symmetry of the root cylinder. The hypocotyl tapers slightly from its junction with the root to the level at which the cotyledons arise. The first pair of leaves are opposite and decussate with the cotyledons; the subsequent leaves are alternate.

As in the following descriptions references will frequently be necessary to the plane passing through the junction of the hypocotyl and root, it seemed well to use a definite term to designate this.

The term 'collet' has been used by some writers* to mean the whole region in which the transition from root to stem structure (internal as well as external) takes place. Later writers† use the term to designate the geometrical plane passing through the region at which the piliferous layer of the root gives place to the epidermis of the hypocotyl.

In Ricinus it is immediately below the 'collet,' using the word in this latter sense, that lateral roots also are given off; so that in this case the term denotes a plane separating all the external stem characteristics from those of the root.

THE PRIMARY ROOT.

Gerard‡ states that the bundles of the root vary from four to eight, and that out of ten specimens examined five were tetrarch, two were pentarch, two had seven and one had eight xylems and phloems. However, in twenty-one specimens examined I found only one varying from the tetrarch type; this one was clearly abnormal, and, since it presented several points of interest, will be described later.

Near the root tip the centre of the cylinder is occupied by the fused bases of the four xylem bundles. Approaching the collet this position is taken by the pith.

Passing outwards the xylem and phloem are quite normal in their characters and call for no remark.

The pericycle forms wedge-shaped masses, eight to nine cells deep from apex to base, opposite the protoxylems; and narrows down until it consists of one or sometimes two layers of cells to the outside of the phloem arc.

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* M. Clos, "Du collet dans les plantes," *Ann. Sci. Nat. Bot.*, 1849.

† Dangeard, "Le Botaniste," 1889, Van Tieghem, *Traité de Botanique.*

‡ *Annales des Sciences Naturelles Bot.*, 1881.

The endodermis, of cells smaller than those of the cortex, is fairly well marked, and the thickenings on the radial walls are well stained with aniline safranin.

When secondary thickening begins the cambium appears first in the small celled tissue opposite the primary phloems, and secondary xylem is formed centrifugally in this position. The cambium thus forms a ring broken by the four xylems. At a later stage the pericycle cells outside the protoxylems divide tangentially to form a cambium which bridges across the gap. These divisions do not take place in the cells *immediately* outside the protoxylem, and there are usually a few crushed pericycle cells to be seen between it and the secondarily formed tissues. The cambium ring is now complete, and forms secondary xylem and phloem normally.

Following the root upwards, near the collet certain cells of the pericycle opposite the phloems divide to form a cork cambium.

PASSAGE FROM ROOT TO HYPOCOTYL.

The passage from the root structure to that of the stem takes place in the space of a few millimetres in the swollen root portion mentioned above. A section at the collet shows eight typical stem bundles with their protoxylems directed towards the centre; and considering everything above the collet as stem and that below as root; the transition from root to stem takes place entirely in the root.

The plant from which the following observations were made was young, the cotyledons being still in the seed; the root was rather more than 4 cm. in length, and the passage occupied about 9 mm.

Passing upwards to this region, the four xylem bundles, which in the unthickened root are narrow and conical (fig. 1), become gradually broader at the base and of less radial depth (fig. 2).

At length the four bases form a nearly continuous ring round the pith, and the protoxylems are drawn down to the middle of each bundle. The appearance at this stage is that of eight xylem bundles lying with their long axes directed tangentially and joined two and two by their protoxylems (fig. 3). The four phloem bundles have not changed their position, nor do they do so throughout the passage.

The next stage is the separation of the eight xylem bundles from each other, and at the same time a rotation as on a pivot through 90° from a tangential to a radial direction, the protoxylems coming to point towards the pith (fig. 4). Thus the four original xylem bundles have each split into two, and the halves separating come into contact with the halves from the original xylem bundle on either side; the union taking place in front of the phloems (figs. 4 and 5). There are now four typical stem bundles, each, however, with two distinct protoxylems (fig. 5). The last stage is the splitting of these four bundles, half of the original phloem going to each (fig. 6). In this way the eight bundles of the hypocotyl appear.

Exactly similar stages could be made out in a microtome series of transverse sections of an embryo dissected from an ungerminated seed of Ricinus.

GAMOSTELY IN AN ABNORMAL ROOT.

In external appearance the abnormal root mentioned above was irregularly oval instead of being cylindrical.

Near the tip, sections showed that, corresponding to the oval form, the vascular system was composed of two typical root steles, each with a normal pith, but the two inclosed by one common endodermis.

A little higher up the piths connected across, and some of the xylems and phloems were, in consequence, pushed to the outside. Fresh bundles also entered from the lateral roots until the number of xylem bundles was increased from eight to ten, and later, by the division of one to two, the number was further increased to eleven. The phloems formed irregular shaped patches between the xylem bundles.

The vascular cylinder now had roughly the appearance of a crescent, on the concave side of which the endodermis had become less distinct (fig. 7 (*a*)).

Higher up, the horns of the crescent fused, and a portion of the cortex was inclosed. By this time the concave arc of endodermis had quite lost its distinctive characters, and the imprisoned cortex had all the appearance of a normal pith. At this stage the xylem bundles, which were of very different size, consisted of mingled larger and smaller tracheal elements, in which it was impossible to determine the positions of the protoxylems; it was consequently impossible to say when the root arrangement ended, and that of the stem began. The positions, and also the numbers, of the bundles were always changing, due to fusions and divisions.

Just before the 'collet' was reached, the bundles, after much shifting, arranged themselves symmetrically, and the section of the false stele was roughly a square, each side of which was occupied by a typical stem bundle with two protoxylems pointing to the centre, and the four angles each by a group of small xylem vessels (possibly protoxylem), which went off to lateral roots, and were lost (fig. 7 (*b*)).

Above this plane the structure conformed to the type normal for the base of the hypocotyl, and the splitting of the four stem bundles gave the ordinary eight hypocotyledonary bundles.

ARRANGEMENT OF BUNDLES IN THE HYPOCOTYL.

From the 'collet' the eight hypocotyledonary bundles pass upwards unbranched till within a few centimetres of the point of origin of the cotyledon petioles. The distance from the root at which the apparent branching takes place depends on the length of the hypocotyl.

In a well-grown specimen the hypocotyl varies between 12 and 25 cm. in length.

In the case of a hypocotyl 15 cm. long, the original eight bundles remained unbranched to a height of 10·5 cm.; in one, 14·5 cm. long, the first appearance of branching took place at about 10 cm. from the collet; and in the case described in detail below, the hypocotyl was 19·5 cm., and branching first appeared at 13 cm. above the 'collet.'

In every case the branching is perfectly regular up to within some few millimetres of the cotyledons; higher up and in the epicotyl, although, in the main, the arrangement of bundles in various specimens is similar; differences occur in some minor details.

In the specimen chosen for description, while the hypocotyl was fully elongated, the epicotyl had reached a length of less than 2 cm., two internodes only being visible.

The dimensions were as follows:—tap root, 7 cm.; hypocotyl, 19·5 cm.; first internode, 1·5 cm.; second internode, ·2 cm.

At the base of the hypocotyl there was a broad active band of interfascicular cambium, giving rise to secondary xylem and phloem elements at various points in its circumference. At a distance of two or three centimetres from the 'collet' these secondary structures were no longer met with, although the cambium ring was still broad. From this level upwards, the cambium ring became gradually narrower, and at about 18 cm. from the collet it had almost disappeared.

In this specimen, also, it was possible to follow the change of the same layer of cells from the endodermis of the root to the starch sheath of the hypocotyl.

The layer retained its endodermal characters, *i.e.*, its thickened radial walls, for nearly 2 cm. in the hypocotyl, and then, with the appearance of starch, at first in small and higher up in larger quantities, took on the characteristics of a typical starch sheath.

BRANCHING OF THE EIGHT BUNDLES.

In this specimen, at 13 cm. from the collet, the first signs of branching appeared. In four out of the eight bundles (fig. 10—1, 4, 5, 8, *i.e.*, those next the longitudinal plane perpendicular to the plane passing through the cotyledons), the phloem becomes divided to three; one central large mass, and one smaller one on either side.

The lateral phloem masses leave the central one; immediately afterwards lateral portions of the xylem also break away, and two lateral branch bundles are completely separated from the central one (fig. 11).

In the remaining four bundles (fig. 10—2, 3, 6, 7), a similar branching takes place on one side only of each; there is, therefore, a gap left between bundles 2 and 3 and 6 and 7. Thus there are, at this level, eight hypocotyledon bundles and twelve others, the result of branching (fig. 11). The subsequent history of these bundles is as follows:—At about 15 cm. from the collet, the twelve bundles unite two and two, and the single bundles formed from these fusions take up an intermediate position between adjacent main hypocotyledonary bundles.

Just above this level, however, there is given off from each of the bundles 2 and 6 one small branch which takes up a position midway, *i.e.*, in the plane of the cotyledons, between bundles 2 and 3 and 6 and 7 (*α*, *β*, fig. 12). In some other specimens bundles 3 and 7 also contributed branches to *α* and *β*, but these were much less important than those from 2 and 6. The bundles *α* and *β* are much smaller than the other similarly situated six, and continue unbranched until just below the point of insertion of the cotyledon petioles; there they each break up to three or more reduced bundles (fig. 14 and fig. 8), which leave the central ring and form the vascular system of an axillary bud found in the axil of the cotyledon, and sometimes attached to the axis of the plant, sometimes to the cotyledon petiole.

To return to the other six united branch bundles, at about 15 cm. from the collet (fig. 8, X, Y), these divide again, each giving off two lateral branches (fig. 13). About 2 cm. higher up the original eight hypocotyledonary bundles begin to move out from the central ring, four supplying each cotyledon petiole. These eight bundles take with them to the petioles eight arcs of starchy endodermis, together with the small-celled conjunctive surrounding each ("external conjunctive" of Flot).

The four bundles of each cotyledon petiole branch on either side. These branches run obliquely up the petiole and anastomose as indicated in fig. 15. At the lateral margins of the petiole two longitudinally running bundles are formed as a result of the anastomosis. Fig. 15 represents the branching and anastomosis as taking place in one plane, while in reality it occupies a length of several millimetres, and occurs irregularly so that a symmetrical arrangement is never seen in any given section.

Precisely the same branching was made out in the cotyledon petioles of a very young germinating plant (of which a microtome series was cut), whose cotyledons were still within the testa.

ARRANGEMENT OF BUNDLES IN THE EPICOTYL.

Even before the cotyledon traces leave for the petioles, the groups of three bundles between them, of which the central one is always the largest, increase their number by divisions. After the bundles α and β and the branches arising from them have moved out from the stele, there is a gap left corresponding to their positions. To fill this up, there is a branching in sometimes one and sometimes both of the groups adjacent to the gap at a much lower level than in the case of the other groups. In the present specimen branching took place in the groups between cotyledon traces 1 and 2 and 5 and 6 at about 16·5 cm. from the collet, and the median members of the group divided to two. These two still being larger than the lateral members of the same group. In the other groups division usually takes place in the lateral members, these remaining much smaller than the central one (fig. 14).

The divisions do not take place regularly, nor at the same level (fig. 8 and fig. 16), but there is a certain uniformity in the arrangement of the bundles in every specimen examined. By means of the more active divisions in the groups between 1 and 2 and 5 and 6, enough bundles are formed, so that by a general shifting the gaps in the plane of the cotyledons are filled up; one of the larger bundles of each of these groups moving round to take up the central position. As a result, the larger bundles are now arranged more or less regularly round the circle.

These large bundles leave the stele simultaneously for the first pair of leaves which are decussate with the cotyledons, approximately half going to each leaf. Again lateral shifting takes place to fill the spaces left by these bundles. Of the smaller bundles remaining, all are not of the same size, some being larger and more fully developed than others. These larger bundles are again arranged at approximately regular intervals round the stele.

Subsequent leaves are alternate, but each is supplied by these larger bundles situated all round the stem. These starting from the side opposite the leaf progressively leave the cylinder and anastomose to form a nodal plexus which runs completely round the stem, and gives off branches to the petiole (figs. 17 and 17*a*). (The arrows in fig. 17 (*a*) mark the original position of these larger bundles.)

In every case the bundle reaches its greatest development just as it leaves the stele for its respective leaf.

Lestiboudois,* in a paper on vascular anatomy, figures and describes among others the hypocotyl of *Ricinus communis.* He mentions the eight cotyledon traces of the hypocotyl, and refers to the bundles *a*, *b*, *c*, etc. (fig. 8), as belonging to the first pair of leaves, but does not mention their branching, and he apparently considers continuations of the cotyledon traces to pass on upwards to the epicotyl. The rest of his remarks, which are almost impossible to follow and understand, is concerned with the arrangement of the epicotyledonary bundles, to allow of a phyllotaxy of $\frac{2}{5}$ above the first pair of leaves.

ARRANGEMENT OF BUNDLES IN THE EMBRYO.

To determine at what stage in the development of the plant these foliar bundles made their appearance, much younger specimens, and finally embryos, were examined.

The whole of the branching bundle system, described above in the fully elongated hypocotyl, was already completed in the youngest seedlings examined. In one case (fig. 22) the cotyledons were still inside the testa; the length of hypocotyl outside was 1·9 cm., and inside ·2 cm. (A. B. fig. 22). Into this ·2 cm. the whole course of the branching of the cotyledon bundles was compressed, the details being exactly the same as those described above.

Finally, transverse and longitudinal microtome series were cut of embryos dissected from the Ricinus seed.

In the root region of the embryo the tetrarch arrangement of the adult plant could be distinguished, and the arrangement of vascular elements in the passage from root to shoot, as described above, was already laid down, though neither xylem nor phloem elements were fully differentiated. In the hypocotyl the eight cotyledon traces were distinctly visible, but everywhere the xylem and phloem were only distinguishable by the relative size of their elements; it was impossible to make out sieve tubes, and even the protoxylem elements were unthickened (fig. 24).

* Lestiboudois, *Ann. des Sci. Nat. Bot.*, vol. x. ser. 3, 1848.

The only bundles to be seen besides the primary eight were in general those marked *a* and *b* in fig. 12, *i.e.*, the median members of those going off to the first pair of leaves after the cotyledons in the fully-grown plant.

These 'bundles' were simply strands of smaller and more actively dividing cells than the rest of the tissue, and which had not even acquired the characteristic shape of the fully-formed bundle. On following the hypocotyl upwards, these bundles *a* and *b* appeared suddenly between cotyledon traces 1 and 8 and 4 and 5 (fig. 19), and in only one series out of five examined could anything approaching branching from these traces be made out. In the others the bundles were quite unconnected with the cotyledon traces.

In the series alluded to, however, a branch from 8 and one from 4 took up positions midway between traces 1 and 8 and 4 and 5, and became bundles *a* and *b* respectively. Following these bundles up towards the growing point, they are seen to continue on when the cotyledon traces leave the central ring, and afterwards pass into the first pair of leaves, whose rudiments are always present (figs. 18 and 21).

The embryos differed, however, in the amount of development of their tissues, and in some cases the procambial beginnings of some or all of the bundles *c*, *d*, *e*, *f* could be distinguished. These in a series cut from below upwards appear, however, later than the bundles *a* and *b*; in fact, they are seen only just before the cotyledon traces leave for the petioles.

Higher up still, at the level of fig. 21, these extra procambial strands had quite disappeared, and only *a* and *b* were to be seen in the first pair of leaves. In this series the bundle *c* could be traced through eight sections, each 7·5 μ in thickness, *i.e.*, through a distance of ·06 mm.

In an embryo of this stage the bundles *a* and *b* attain their greatest development just below the level of the growing point, and such of the bundles *c*, *d*, *e*, *f* as may occur are only found at this same level.

In a transverse section of this region they appear cut very obliquely, and are evidently in the act of passing outwards from the cone of apical meristem to the circumference of the vascular ring; *a* and *b* in the same section (fig. 20) are cut obliquely as they pass to the first pair of leaves.

A longitudinal radial section (fig. 18) in the vertical plane containing D E (fig. 21) shows bundles *a* and *b* in longitudinal section, while one slightly tangential shows *c* and *d* or *e* and *f* cut transversely on their way out to the vascular cylinder. Moreover, in the longitudinal radial section through some other diameter of the embryo than D E, procambial strands, giving rise later to *c*, *d*, *e*, and *f*, could be seen in the act of development in the region just below the growing point (fig. 23).

Hence, in the embryo the six bundles *a*, *b*, *c*, *d*, *e*, and *f* are developed just below the growing point, and extend downwards to the hypocotyl and upwards to the first pair of leaves; the bundles *a* and *b* being ahead of the others in their development in point of time. It seemed also probable that the branches between these bundles and the adjacent cotyledon traces were a later downward development of these six.

This idea was confirmed by an examination of the first branches (figs. 10 and 11) in the very young germinated specimen mentioned before (fig. 22). Of the six bundles *a*, *b*, *c*, *d*, *e*, and *f*, it was only in *a* and *b* that thickened xylem elements were present; at no level were they to be seen in the later developed bundles *c*, *d*, *e*, and *f*.

Following the branches which fuse to form the bundle *a* up from their connection with the cotyledon trace, they are seen at first to have no thickened elements at all. Higher up, thickening on the xylem vessels occurs, the number of such vessels and the amount of thickening increases till the branches anastomose and *a* is formed.

Hence the portion of the branch in connection with the cotyledon trace is the youngest, and the branch itself is a downward development from *a*, and not a branching in development from a cotyledon trace.

We conclude, therefore, that the foliar bundles *a*, *b*, *c*, *d*, *e*, and *f* going to the first pair of leaves are developed in a ring corresponding to the first node after that of the cotyledons, at a time when the position of the ring in question was just below the growing point, and in the same way the bundles going off to the leaf next above these were developed at a later date in a ring corresponding to the second node, when in turn this was just below the growing point, and so on. These, on their downward course, anastomosed with the already existing bundles *a*, *b*, *c*, *d*, *e*, *f*, at the level X Y (fig. 8).

THE EXTERNAL CONJUNCTIVE TISSUE.

In 1893, Flot* made an important contribution to the delimitation of the regions of the stele typical of dicotyledonous and coniferous stems. He distinguished an 'internal' from an 'external' conjunctive tissue, the former name being applied to the larger celled parenchyma commonly present in the centre of the stele, the latter to the tissue immediately investing the vascular ring. The external conjunctive may be divided into (1) pericycle, the peripheral layer of the stele; (2) rays, the tissue between the bundles; and (3) perimedullary zone, the layer bordering the pith. The internal conjunctive is equivalent to pith, if the latter term is taken to represent a naturally distinct tissue system, and is not a mere topographical designation for the tissue included within an imaginary circle, whose circumference touches the internal points of the primary xylems. Flot's scheme corresponds perfectly to the facts in those cases where the primary bundles of the cylinder are separated from one another laterally by narrow bands of tissue (rays). He rightly insists that one cannot draw a sharp line between perimedullary zone and ray, or between ray and pericycle, when the whole of the external conjunctive is parenchymatous. The three zones are in that case simply topographical names for parts (whose limits are marked out by the bundles themselves) of a single tissue system investing the bundles. Histological distinctions between them, however, frequently exist. The commonest of these is the partial or complete conversion of the pericycle into a sclerenchymatous band—a conversion often shared by the perimedullary zone.

If, now, we follow out the arrangement and structure of the external conjunctive in the root, transition region, hypocotyl and epicotyl of our seedling, we seem to get some light on the important topic of the relation of the bundle to the stele.

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* Flot, "Récherches sur la zone perimedullaire," *Ann des Sci. Nat. Bot.*, ser. 7, 1893.

ROOT AND TRANSITION REGION.

The pericycle in the root proper is represented by a band of varying thickness, consisting of layers of thin walled cells a little smaller than those of the endodermis. Since the outline of the cylinder is circular, and the xylems are placed nearer the centre than the phloems, these cells form wedge-shaped masses eight to twelve cells deep opposite the protoxylems, but opposite the primary phloems the pericycle narrows down to two, or in some cases one layer between the outermost point of the phloem arc and the endodermis. The innermost vessels of the primary xylem abut directly on the large cells of the pith; the phloems are separated from it by several layers of external conjunctive (fig. 1).

In the phloem itself, even in quite young specimens, fibres are to be found. These are in many cases lying side by side with sieve tubes and companion cells, and there can be no doubt as to their origin from the phloem, and not from the pericycle. The line of demarcation between the two is quite clear (fig. 25). This occurrence of fibres in the primary phloem of the root is rare, according to Van Tieghem,* who records them only in Leguminoseæ and Malvales including Malvaceæ, Sterculiaceæ, and Tiliaceæ, in certain of the Cycadeæ, and in Anona and Celtis.

In the transition region, after the split protoxylems have turned towards the centre of the cylinder, and the stele consistsof four double stem bundles (figs. 5, 26), the interfascicular pericycle consists of a band of seven to eight, or even more, layers of small thin walled cells connecting these bundles, but not stretching downwards to their protoxylems. This band is in the position of the wedge-shaped mass of pericycle opposite the protoxylem mentioned above, and it is from it that the lateral roots are seen to arise.

In the root arrangement the protoxylems are surrounded by small celled tissue (the pericycle). Some of this remains in connection with the protoxylem during its rotation, so that when the stages (shown in figs. 4, 5, and 6) are reached, the protoxylem now directed towards the centre is invested with this small celled tissue, which from its position is analogous to what Flot considers as perimedullary zone in the stem. It is interesting to note that these cells are continuous on the one hand with the pericycle below, and on the other with the perimedullary zone above. As the root is traced upwards to the "collet," the many layered pericycle is seen to get thinner, and by the time the "collet" is reached the interfascicular pericycle consists of two, or at most three, layers of small cells inside the endodermis which now contains starch.

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* Van Tieghem, *Ann. Sci. Nat. Bot.*, ser. 7, 1888.

On the other hand, the pericycle outside the phloem has not changed, and still consists of one or two layers of thin-walled cells external to the outermost point of the phloem arc, but is thicker on the flanks of the bundles. In somewhat older roots active cell division is to be seen in the pericycle marking the beginning of periderm formation.

HYPOCOTYL.

In the lower portion of the young unthickened hypocotyl (*i.e.*, below the point of appearance of the foliar traces), the pericycle is visible between any two of the eight cotyledon bundles as a single layer of small cells immediately inside the starch sheath (fig. 27).

In longitudinal section the pericycle cells are seen to be elongated, but with horizontal end walls, while the pith cells are much shorter and broader (fig. 28). In both transverse and longitudinal sections of an embryo the pericycle and pith presented exactly similar relations. When the interfascicular cambium arises, it is by tangential divisions in this layer of pericycle cells.

The bulk of the 'primary ray' (in the old topographical sense) is occupied by a large celled tissue, on which the pericycle directly abuts, and which, following out Flot's idea, we must regard simply as an outward extension of the internal conjunctive or pith (fig. 27).

Opposite the bundle the pericycle is composed of tissue of two different kinds. Immediately beneath the endodermis is a layer of thin walled cells containing starch, similar to, and continuous with, the pericycle between the bundles; and between this and the sieve tubes of the phloem is a mass of cells, elongated and pointed as seen in longitudinal section, and with but a thin layer of protoplasm lining the wall of each. These are the young pericyclic fibres. The mass of fibres at a later stage is often not continuous, but is broken up to two or three divisions by the interposition of parenchyma cells.

Higher up in the hypocotyl the parenchymatous layer mentioned above dies out, and the pericycle fibres come to lie directly within the endodermis.

Pericycle fibres are also found to the outside of the foliar bundles *a*, *b*, *c*, etc., the number present and the amount of thickening depending on the age of the hypocotyl and the level at which the section is taken; but these are always less than in the case of the cotyledon traces. The number of pericycle fibres increases till the oldest part of the bundle is reached, *i.e.*, at the level at which it goes off to its respective leaf.

To the inside of each bundle, in transverse section, a conical cap of small celled tissue is to be seen. This surrounds the protoxylem, and extends up the flanks of the bundle till it meets the pericycle cells on either side (fig. 27). In radial section these cells form four or five layers of elongated cells to the inside of the innermost protoxylem elements (fig. 29). Higher up, masses of small cells are also found investing the foliar traces, but the cap is composed of fewer cells, and has not the same conical form as in the case of the cotyledon traces. Where the cotyledon and foliar bundles are nearly of the same size, as sometimes happens at the level represented in fig. 12, it is always possible to distinguish between them by means of this comparison.

When the cotyledon traces go off to the petioles, they take with them their investments of small cells, consisting of the inner caps and their segments of pericycle (*i.e.*, peridesm of Van Tieghem) and corresponding arc of endodermis.

EPICOTYL.

In the young epicotyl the character of the transverse section has somewhat changed. The eight traces have left for the cotyledons, and there is now a ring of foliar bundles of varying ages, separated by narrow rays five or six cells broad (fig. 30). Those which will go off to the first pair of leaves are the oldest and most fully developed, while those passing downwards from the youngest leaves are only represented by a mass of procambial tissue, which will become phloem, the xylem vessels being indistinguishable so far from the node at which the development of the traces in question begins. The bundles are often so close together that the arcs of pericyclic fibres belonging to each coalesce, forming an almost continuous ring of fibres, only broken where there is a wider gap between two adjacent bundles. The most fully developed bundles have an arc of fibres three or four layers thick lying outside their phloem, while for the most part a single layer only is found outside the younger bundles. The fibres are never found except in connection with the phloem. If the bundle is so young as to have no xylem distinguishable, and only small cells to represent the future phloem, to the outside of this rudimentary bundle, fibres, or cells already having the form of fibres, though unthickened, will be found; but in the gaps between bundles an inner layer of cortex, which can often be distinguished as a starch sheath, bends down a little, and comes to lie on parenchymatous pericycle cells. But the distinction between the cortex and pericycle, when the latter is parenchymatous, cannot always be made.

A further result of the close juxtaposition of the bundles is to bring the small cells which cover their flanks into contact, so that we get rays of small celled tissue formed (in the outer parts of which the interfascicular cambium arises), and even sometimes a continuous perimedullary zone, the pith being, as it were, pushed back by the closing up of the bundles (fig. 30). (At *a* a small celled ray is partially formed, a large pith cell occupying the position midway between the protoxylems, owing to the sloping away of the small celled tissue on the flanks of the adjacent bundles.)

These relations illustrate (1) the primary importance of the bundle as an anatomical unit in the stem of typical angiosperms, —an importance which has, of course, been recognised ever since modern plant anatomy had an existence, but has perhaps been somewhat obscured of late by the stelar point of view; and (2) the dependence of the stele on the close lateral approximation of the bundles for continuity of the tissues of external conjunctive, since the latter are primarily arranged in relation to the separate bundles.

Flot has not recognised with sufficient explicitness the existence of cases such as this.

The 'rays' do not exist here as parts of the external conjunctive, nor is there a continuous perimedullary zone. We have simply a small celled investment of each bundle, on the outer side forming a broad band, and giving rise to masses of fibres (the "fascicular pericycle"), and internal to the apex of the protoxylem consisting of a considerable mass of parenchyma, while on the flanks of the bundle it is two or three layers only. This tissue is continuous from bundle to bundle by the single layer of small cells above described (the "interfascicular pericycle"). This disposition is common enough in hypocotyls, where the bundles are widely separated. The individuality of the bundle as a unit of vascular tissue here comes out very markedly. It is the *bundle* which is surrounded by small celled tissue. The continuous parenchymatous pericycle of the root is still present at the base of the hypocotyl, but soon loses its continuity across the phloem as we pass upwards, and is interrupted by the masses of fibres abutting on the endodermis and developed very obviously from the same mass of procambial tissue as the phloem, as can be well seen in the embryo and very young seedling. About the same level the cells of the endodermis lose their characteristic thickenings, and are sometimes only distinguishable from the adjacent cortex by the possession of starch—a character which is itself lost later on. The object of the starch in the endodermis is obviously to supply the pericyclic fibres with material for thickening their walls, and in the portions between the bundles to feed the interfascicular cambium. As these two processes proceed it gradually disappears. The interfascicular pericycle itself is simply the birthplace of the interfascicular cambium, the cortex dipping between the bundles just far enough to bring the pericycle opposite the bands of small celled tissue between the xylem and phloem in which the fascicular cambium arises. Thus we may almost say that the stele of the root is preserved in the hypocotyl only as a matter of convenience; the bundle is the important unit.

This view is strengthened by a consideration and comparison of the vascular arrangements of (1) the root, (2) the hypocotyl, (3) the epicotyl of the present plant. In the root the cylinder or stele is the natural form in which to describe the condensed vascular structure. The compact arrangement is required to withstand pulling strains; and since the root, unlike the stem, directly performs an ultimate nutritive function (that of absorption), and does not bear organs of *unlike* morphological nature, its vascular structure is also, so to speak, *ultimate*, and is not made up of units belonging to another morphological category, as is the case in the stem.

In the young hypocotyl there are distinct, widely separated cotyledon traces, and it is only the presence of two single layers of cells—the interfascicular pericycle and the endodermis—which allows of the term stele being applied to the vascular arrangement here at all; these two layers are present in response to definite demands on the part of the plant; the interfascicular pericycle has to give rise to cambium, and the endodermis provides starch to be used up in its development and subsequent activity; also in the thickening of the 'bast fibres.'

In the epicotyl, the stele has even less individuality of its own. It is the result of close lateral approximation of separate leaf traces, each with its own cap of bast fibres; and the slight approach to a perimedullary zone which is found is due to the contact of the small celled external conjunctive tissue which surrounds the xylem portion of each single bundle.

In this region the leaf is the important thing (since the only function of the stem is to bear leaf structures), and the stele arises as a secondary phenomenon brought about by the association in the most obvious way of the important units the leaf traces, and it is the structure and course of *these* which provide the principal facts of vascular morphology.

The fact that the cylinder so formed is continuous with that of the root justifies, however, the use of the term *stele* as applied to the vascular cylinder of the axis as a whole.

There seems good reason to suppose that in the flowering plants, at least, the formation of a stele is generally determined by mechanical considerations.

It occurs in the root as a well-defined cylindrical axial strand, apparently for the purpose of resisting pulling strains. In the stem it is required for support and the resistance to bending strains, and hence the formation of fibrous pericycle and sometimes the sclerization of the rays and perimedullary zone; when this takes place we have the stele at its highest point of development.

On the other hand, the primary function of the vascular system is conduction, and this is carried on by the leaf traces whether a stele is present or not.

In conclusion, I wish to express my extreme indebtedness to Mr A. G. Tansley for the valuable advice and assistance he has given me in the preparation of this paper.

EXPLANATION OF PLATES.

Plate I.

Ext. c., external conjunctive; X., xylem; P., phloem.

Figs. 1-6. Diagrams, drawn to scale, representing the transition from root to hypocotyl; fig. 1 showing cortex; figs. 2-6 limited by endodermis.

Figs. 7, 7 (*a*), 7 (*b*). Diagrams illustrating gamostely in an abnormal root. All three figures are limited by the endodermis.

Fig. 7. Showing two normal root steles inclosed by a common endodermis (*end.*).

Fig. 7 (*a*). Vascular cylinder, roughly crescent-shaped in transverse section, internal endodermis becoming less distinct; inclosed cortex (*cor.*).

Fig. 7 (*b*). After much shifting the xylems and phloems are arranged symmetrically.

Also figs. 16*a*-21.

Plate II.

Fig. 8. Diagram illustrating the course of bundles in the Hypocotyl from 12 cm. to 19·2 cm. above the 'collet' in a given seedling. The vascular cylinder unrolled, and looked at from the inside. 1, 2, 3, 4, 5, 6, 7, 8, cotyledon traces; *a*, *c*, *d*, *b*, *f*, *e*, trunks by the union of epicotyledonary leaf traces, and each connecting with adjacent cotyledon traces by two divergent shanks; *α*, *β*, leaf traces in vertical plane passing through the cotyledons, which join on to the cotyledon traces at a higher level than the shanks of *a*, *b*, *c*, etc.; at their upper limit they pass off to axillary buds in the axils of the cotyledons.

Figs. 9-14. Diagrams of transverse sections of hypocotyl at different levels.

Fig. 9. At 12·5 cm., from 'collet' showing 8 cotyledon traces.

Fig. 10. At 13 cm., from 'collet' showing 8 cotyledon traces, and the junction of the shanks of *a*, *b*, *c*, *d*, etc., with them.

Fig. 11. At 14·25 cm., showing 8 cotyledon traces and the shanks of *a*, *b*, *c*, *d*, *e*, *f*.

Fig. 12. At 15·5 cm., showing the first appearance of *α*, *β*.

Fig. 13. At 16·5 cm., showing, between the cotyledon traces, the 6 groups of 3 epicotyledonary leaf traces each, which at a lower level unite to form the bundles *a*, *b*, *c*, *d*, etc.

Fig. 14. At 18·5 cm., the cotyledon traces are passing off to the cotyledons, and semicircular groups of bundles, resulting from the division of *α*, *β*, are passing off to the axillary bud on either side. The stele is now formed entirely of epicotyledonary leaf traces. The groups of 3 bundles between the cotyledon traces have divided more or less regularly, and the larger bundles are those which belong to the first pair of leaves above the cotyledons.

Fig. 15. Diagram representing the branching and anastomosing of the cotyledon traces in the petiole as taking place in one plane, whereas the process occupies a length of several millimetres.

Figs. 16–17 (*a*). Epicotyl.

Fig. 16. Diagram of section at 19·2 cm. above the 'collet.' The larger bundles are the traces of the next pair of leaves above the cotyledons.

See plate I. for figs. 16*a*–21.

Fig. 16 (*a*). Diagram of a slightly higher level. The traces belonging to the first pair of leaves have left the epicotyledonary stele. Those above the dotted line belong to one leaf, those below to the other.

Figs. 17 and 17 (*a*). Diagrams of sections of the epicotyl about the level at which the first isolated leaf is given off. The traces leave the stele first on the side remote from the leaf, and form a girdle round the stem. The arrows in 17 (*a*) mark the position in the stele of leaf traces belonging to this leaf.

Figs. 18–21. Diagrams of embryo.

Fig. 18. Diagram of radial longitudinal section through growing point of embryo in the plane passing between the cotyledons. *a.b.*, median leaf traces of first pair of leaves; *l.l.*, first pair of leaves; *c.p.*, confluent bases of cotyledon petioles.

Fig. 19. Diagram of transverse section of embryo at level C (fig. 18), showing *a* and *b* dying out in the hypocotyl.

Fig. 20. Transverse section at level B (fig. 18), showing *a* and *b* cut obliquely and first appearance of *c* and *d*.

Fig. 21. Transverse section at level A (fig. 18); the cotyledon traces are now in the petioles; *a* and *b* appear as the only bundles as yet developed at this level of the first pair of leaves.

PLATE III.

Fig. 22. Germinating Ricinus seed in which transverse sections of hypocotyl show 8 bundles only up to A. From A to B (2 mm.) branching of bundles, as in fig. 8, takes place.

The following sections are drawn with Abbe's Camera under Zeis's $\frac{1}{8}$ in.:

Fig. 23. Tangential longitudinal section through growing point of an embryo, showing at A the first appearance of one of the bundles, *c*, *d*, etc.; *l.*, first leaf; *c.p.*, cotyledon petiole base.

Fig. 24. Transverse section of embryo hypocotyl in region where 8 cotyledon traces only are found, showing one of the 8 traces; *cor.*, cortex; X., xylem; P., phloem; *per.*, pericycle.

Fig. 25. Transverse section root, showing fibres (*f.*) in the phloem in close connection with sieve tubes *s.t.*; *c.c.*, companion cells; *end.*, endodermis; *per.*, pericycle.

Fig. 26. Transverse section. Transition region between root and hypocotyl, showing (*per.*) many layered interfascicular pericycle; and *ext. c.*, small celled external conjunctive tissue investing the bundle; *end.*, endodermis; *cb.*, cambium.

Fig. 27. Transverse section. Cotyledon trace at base of young hypocotyl, showing *ext. c.*, external conjunctive, forming cap to inside of bundle, and all there is to represent the perimedullary zone of Flot; *end.*, endodermis; *per.*, pericycle; P., phloem; X., xylem; *cb.*, cambium; *f.*, fascicular pericycle cell, which will become a fibre.

Fig. 28. Longitudinal radial section through interfascicular region of hypocotyl, showing endodermis with starch (*end.*), and a pericycle (*per.*), consisting of a single layer of elongated cells; *cor.*, cortex.

Fig. 29. Longitudinal radial section passing through the protoxylem (*p.x.*) of a cotyledon trace; *ext. c.*, four layers of elongated external conjunctive cells, which form the investing cap of transverse section, and represent perimedullary zone.

Fig. 30. Transverse section, epicotyl. The leaf traces are here so closely packed as to bring their external conjunctive investments into lateral contact, and to form a perimedullary zone; *a.*, a large cell of the pith forcing itself between the protoxylems of two adjacent leaf traces; *f.f.*, almost continuous sheath of pericycle fibres.

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CHICK ON VASCULAR SYSTEM OF RICINUS COMMUNIS, L.– Plate I.

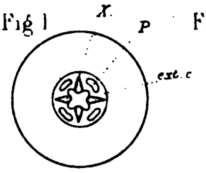


Fig. 1

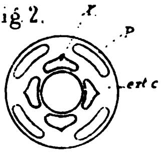
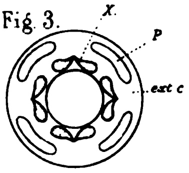


Fig. 2.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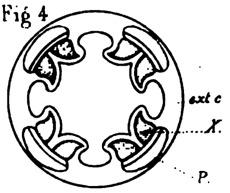
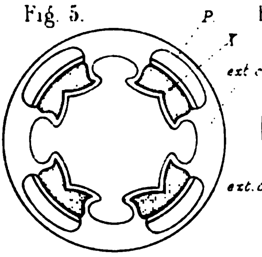


Fig. 4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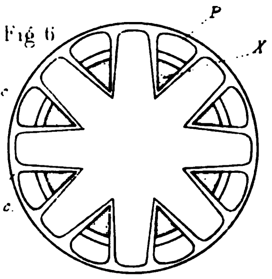


Fig. 6

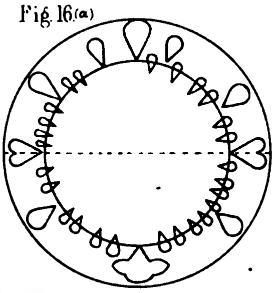


Fig. 16(a)

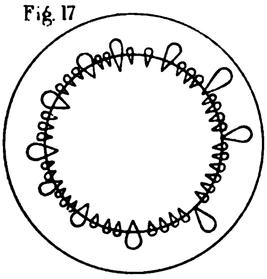


Fig. 17

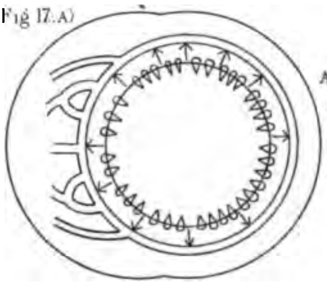
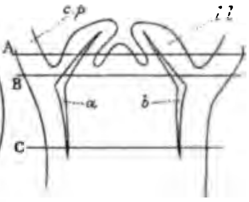


Fig. 17(A)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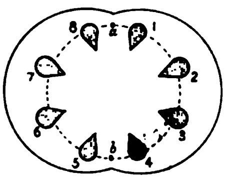


Fig. 19.

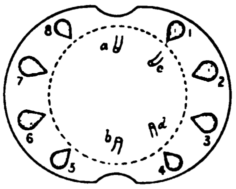
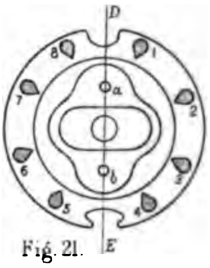


Fig. 20.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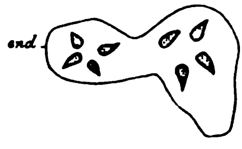
Fig. 7.

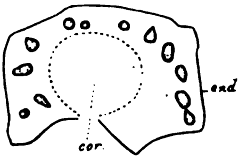
Fig. 7(a)

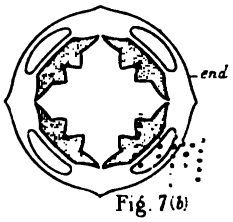
Fig. 7(b)

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19·2

18·5

18

17·5

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X-

15·1

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13

12

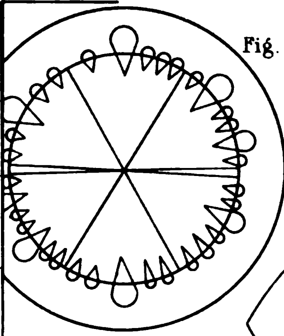


Fig. 16.

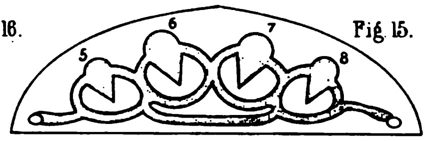


Fig. 15.

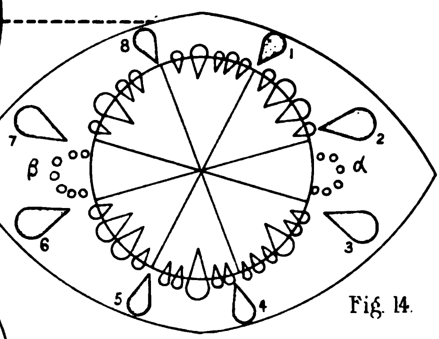


Fig. 14.

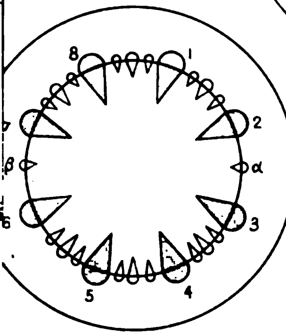


Fig. 13.

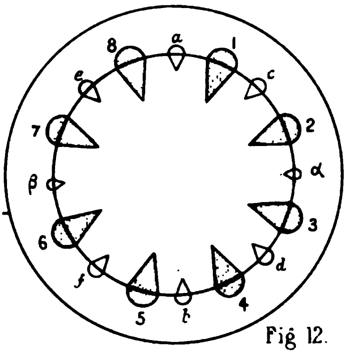


Fig 12.

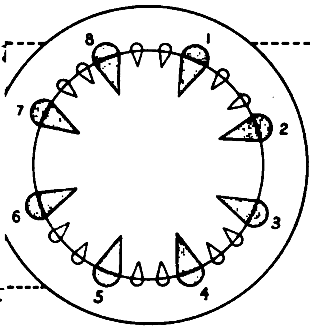


Fig. 11.

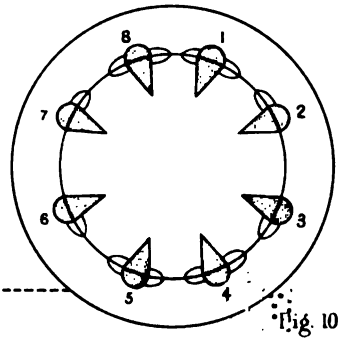


Fig. 10.

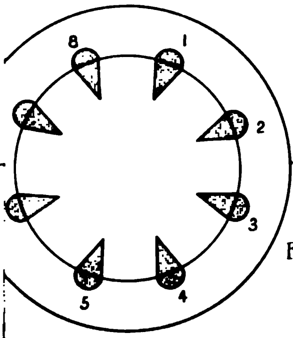
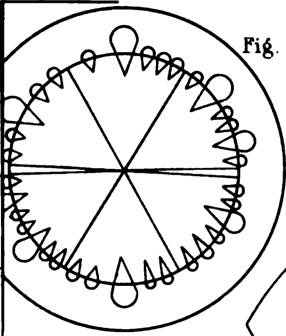


Fig. 9.

M'Farlane & Erskine, Lith. Edin^r



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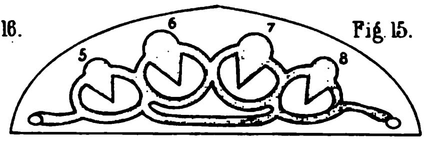

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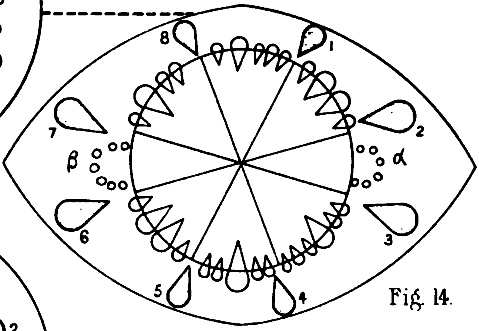

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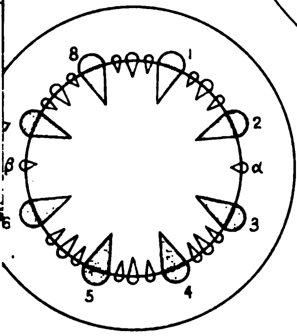
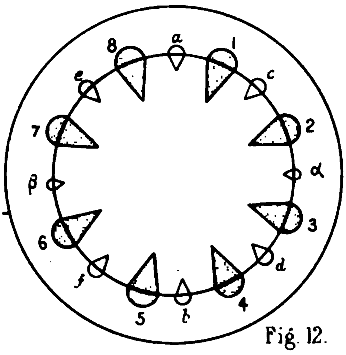


Fig. 13.



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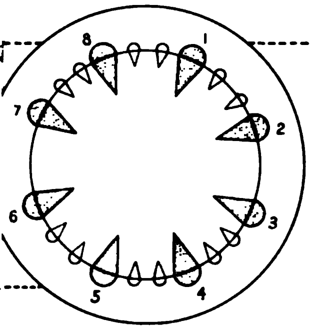

Fig. 11.

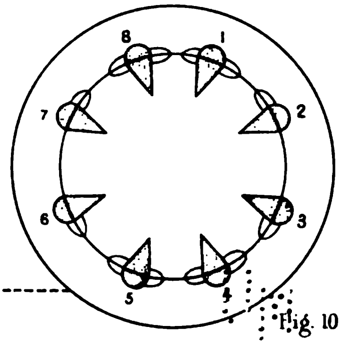

Fig. 10.

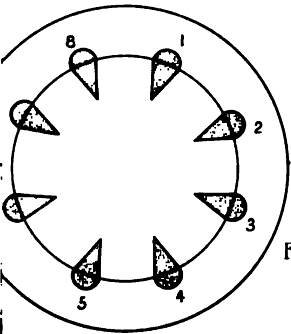

Fig. 9.

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CHICK ON VASCULAR SYSTEM OF RICINUS COMMUNIS. I.—Plate 3

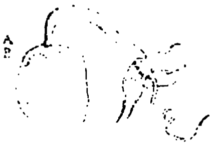

Fig. 22.

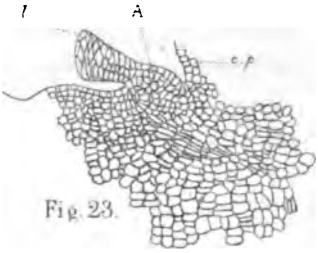

Fig. 23.

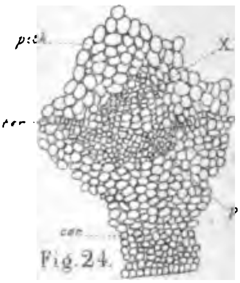

Fig. 24.

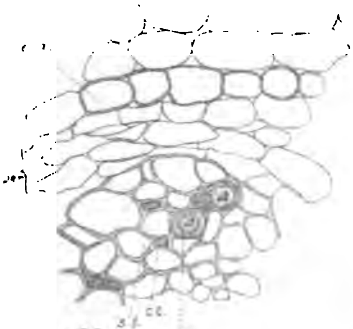

Fig. 25.

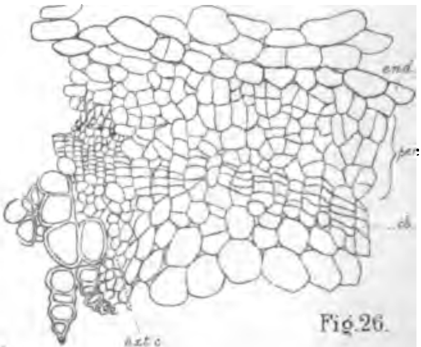

Fig. 26.

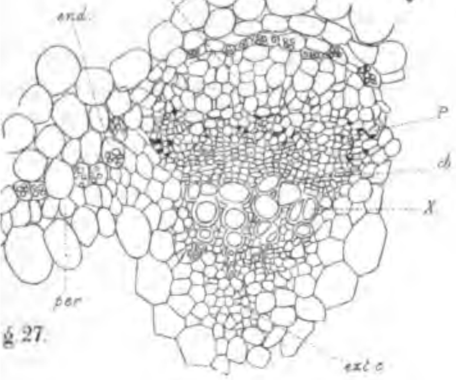

Fig. 27.

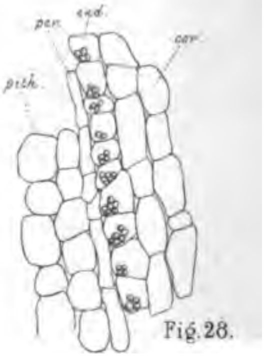

Fig. 28.

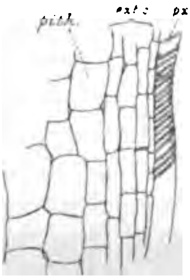

Fig. 29.

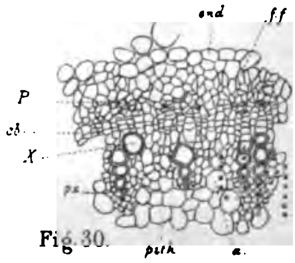

Fig. 30.

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M'Farlane & Erskine, Lith. Edin.

Changes that occur in some Cells of the Newt's Stomach during Digestion. By E. Wace Carlier, M.D., B.Sc., etc., Professor of Physiology, Mason University College, Birmingham. (With Five Plates.)

(Read 1st May 1899.)

With a view to ascertain as far as possible the minute changes that occur in the cells of the mucous membrane of the stomach during, and after, activity, I procured from England towards the end of winter a number of newts (*Triton cristatus*).

When they arrived they were very emaciated, and, indeed, in a starved condition, having no doubt but recently awakened from their winter sleep; they appeared, however, quite healthy, and swam well when placed in water, and took and retained food when given to them in the form of lively worms.

These English newts were fed at the same time with the same weight of worm, and killed at intervals after the meal, one being sacrificed every twenty-four hours, to the end of the tenth day.

Such long intervals were allowed to elapse between each sample taken, because Langley (13) states that during winter in thin and emaciated animals that have been fasting for a considerable time the changes take place very slowly.

These constituted the first set of experiments.

In June of the same year I procured a large number of fine *Triton cristatus* from the Eft Pond in the Braid Hills near Edinburgh. They were in good condition and very lively.

When worms were given them, they ate them readily, and were hungry again in less than twenty hours. However, to make sure that the stomach was quite empty and in a resting condition, I allowed an interval of one hundred and fourteen hours to elapse after the first feeding, and then, selecting newts of about the same size and weight, fed them a second time, each receiving the same amount of healthy worm. From the first snap to the disappearance of the worm finally within the mouth a period varying from five to ten minutes elapsed and was carefully noted in each case, time being counted subsequently from the moment of disappearance of the worm.

Of these newts one was killed every half-hour for the first few hours, and subsequently at intervals of an hour. In this way, as far as was possible, every variant except time was eliminated, though not with complete success, for in a few cases it was afterwards found that some parasitic disease of the mucous membrane of the stomach was present, and those so affected were removed from the series.

Method of Preparation.—At the prescribed time each newt was pithed, opened, its stomach, duodenum, and gullet removed without injury, and placed upon a piece of thin paper. The viscera were then opened by a longitudinal incision through their entire length; their contents, if any, removed, and the organs slightly spread out flat without stretching upon the paper, with the mucous surface uppermost.

They were then immersed, paper and all, in Mann's picro-corrosive fixing solution (modified formula, sp. gr. 1020), in which they remained till next day, when saturated corrosive sublimate (Heidenhain's formula) was substituted for it. At the end of another twenty-four hours this was removed, and the tissue taken up the alcohol series into chloroform, and finally embedded in paraffin (58° C., Grübler). Longitudinal sections were then cut in series with the Cambridge rocking microtome set to four teeth, and fixed upon albuminised slides after spreading on warm water.

To ensure equality in staining, and, therefore, comparability, one newt was used as a test object and all the slides stained up to it; the most efficient way of doing this is to place one section of the test newt upon each slide, and to see that in all cases that particular section is stained to exactly the same tint. This test section may be placed at one end of the slide and subsequently wiped off before the other sections are finally covered.

The sections were stained by immersion in various dyes, preference being given to Mann's (16) methyl-blue eosine long method, and M. Heidenhain's (7) iron alum, hæmatoxylin, both long and subtractive methods.

The photographs were all taken with a magnification of 600 diameters.

The cells in three regions of the stomach were examined, namely:—

1. The glands of the œsophageal end.
2. The glands of the pyloric end.
3. The glands of the narrow zone between 1 and 2.

The shape and arrangement of the cells in these different glands have been sufficiently described by Langley and others, and require no repetition here.

In the present communication I shall confine my remarks chiefly to the cells of the oxyntic glands, and when colour is mentioned, it refers to specimens stained with methyl-blue eosine.

RESULTS OF EXPERIMENTS OF NO. 1 SERIES.

(1) *Fasting newt ten days after food* (fig. 1, coloured plate).—The oxyntic cells are large, but do not occlude the lumen; they contain numerous zymogen granules, that stain of a vivid red colour, and vary in size from 2·8 μ to 1 μ in diameter, the majority measuring from 1·5 μ to 2 μ. Their great affinity for eosine points to their albuminous nature. The granules are not crowded together, there being plenty of room for more. Each granule is lodged in a tiny compartment in the pale blue protoplasm apparently containing fluid, as maintained for those of the lachrymal and other glands by A. Nicolas (20). It is quite impossible to distinguish any arrangement of these cells into zones as is the case with many gland cells. The nuclei are large, 10·6 μ × 12·8 μ, rather poor in chromatin and angular in outline, as described by Langley (13). The nuclear juice is coagulated by the action of the corrosive sublimate, and precipitated in the form of small granules of equal size, named *œdematine* by F. Reinke (22) or *lanthanin* by M. Heidenhain (7). These granules are not crowded together, and stain of a pale sky-blue colour, *i.e.* they are cyanophile (Krasser, 11).

The nucleoli are multiple, are not surrounded by a ring of chromatin, and vary in size from minute dots not larger than lanthanin granules to bodies measuring 2·7 × 1·8 μ. In shape some are rounded, others irregularly oval; they acquire a venous red tint with eosine, and are therefore albuminous in nature, but not so strongly so as the zymogen granules.

The chromatin stains of an ultramarine blue, due to its acid nature, and is arranged in part on the inner surface of the nuclear envelope, and in part in more or less isolated rounded or irregular karyosomes.

(2) *Twenty hours after food.*—(Worm partly digested.) (Fig. 2, coloured plate, photo No. 2.)—The cells are small, in some cases almost flattened, and the gland lumen is correspondingly large and stellate.

The zymogen granules have nearly disappeared from the protoplasm, those remaining being of large size, 2·16 μ in diameter, with an average of 1·6 μ. They are situated mostly near the free ends of the cells, leaving the base and sides clear. This gives rise to the appearance of two zones in the cells, the one basal, containing the nucleus, and the other apical, containing the granules. Only a few cells, however, show this division into zones; most of them have a few granules scattered throughout the protoplasm. No zymogen granules are visible in the lumen, from which I conclude that the granules dissolve in the protoplasm before the secretion leaves the cell, which it does in a liquid and unstainable form, the protoplasm appears condensed and stains pale blue, the condensation being due rather to the disappearance of the granules and discharge of the fluid in the vacuoles than the reconstruction of the protoplasm. In this I agree rather with Nicolas (20) than with Langley (13).

The nuclei are larger at this time than in the newt that was killed ten days after food, the average of a number of measurements being 11·85 × 13·36 μ. The nuclear membrane appears thickened, owing probably to the chromatin spreading itself out upon the inner surface. There is usually a great dearth of chromatin deeper in, and that which is present is pale and has a washed-out appearance.

The lanthanin granules have in the majority of cases undergone no change, but in a few nuclei they stain of a deeper tint. The nucleoli vary a good deal in size, but are not numerous. They stain of the same venous tint, and the larger ones may often be seen lying quite close to the nuclear membrane, and some of them may even be seen in the act of passing through the nuclear wall into the protoplasm, where they ultimately break down. There is no evidence of their transformation into zymogen granules as ver Eecke (5) would have us believe in the case of the pancreas.

(3) *Newt forty-four hours after food.*—(Remains of the worm still in the stomach.) (Fig. 3, coloured plate, photo 3.)

At this stage the cells are much larger, and the gland lumen is correspondingly reduced in size. The zymogen granules are numerous, though not crowded together; they are usually of medium or small size, the larger ones being most numerous near the apices of the cells, though in very many cells it is impossible to locate granules of any particular size in any definite part. They average 1·2 μ in diameter. The cell protoplasm hardly stains at all, or exhibits a pale reddish-grey tint, showing that it has taken up alkaline material from the blood.

The nuclei appear extremely wrinkled, and contain very little chromatin that has a washed-out appearance, and stains feebly blue. Average size of nuclei $8{\cdot}87 \times 11{\cdot}88\ \mu$—*i.e.* they are considerably smaller than in the preceding specimens. The lanthanin granules are very numerous, of small size, and densely crowded together, to such an extent that it is difficult to distinguish other nuclear structures. They stain of a deep, dull grey-blue, and impart by their number and deep coloration a misty appearance to the nuclei. Their reaction is changing. The nucleoli are frequently of immense size, and may be seen passing through the nuclear wall, which closes behind them. When in the nucleus they lie in a tiny vacuole free from lanthanin granules.

At this stage, therefore, the cells have already recovered considerably from their previous exhausted condition, many new zymogen granules have made their appearance, and the nuclei exhibit every sign of great exhaustion. Nevertheless, mitotic figures may be seen in a few of them.

(4) *Sixty-eight hours after food.*—(Worm dissolved, though debris and mucus are still found in quantity in the stomach.) (Fig. 4, coloured plate, photo 4.)

The gland lumen is now small, the cells large and crowded with big zymogen granules nearly all of one size, averaging 1·5 to 1·7 μ in diameter. The protoplasm has recovered its pale blue tint, but does not stain so vividly as in the fasting animal. The nuclei are very conspicuous, owing to their deep blue colour, due not to chromatin, which is practically invisible, but to the multitude of fine deeply-staining lanthanin granules that completely fill the nucleus. When a view of the chromatin is obtainable, it is seen to be scanty, but of a deeper blue colour than in the previous section.

The average size of the nucleus is 10·57 × 12·54 μ. It is therefore becoming larger, probably swelling up by absorption of material from the protoplasm, and this agrees well with the disappearance of the wrinkles from its surface, so obvious in the foregoing preparation. The nucleoli are sometimes large, and often situated near the margin of the nucleus, and a few may still be seen in process of ejection.

Near the pyloric end the nuclei are rather lilac than blue in tint, owing to the lanthanin granules exhibiting an affinity for the acid as well as for the basic dye, which indicates that their nature is becoming modified. Nucleolar expulsion is going on apace.

We have here a still further stage of nuclear repair.

(5) *One hundred and fourteen hours after food.*—(Stomach quite empty, intestines filled with digested worm.) (Photo 5.)

The gland lumen is small; the cells lining it are large and filled with brightly-stained zymogen granules that vary considerably in size, the larger ones being usually situated in the neighbourhood of the nuclei. The granules average 1·6 to 1·4 μ in diameter. The cell-protoplasm, which is hardly visible owing to the multitude of granules it contains, stains of a pale blue colour of the same tint as in the fasting animal.

The nuclei are plump-looking, and measure on an average 12·57 × 13·18 μ; they are therefore bigger than those of the previous section. They also exceed in size those of the newt that had long fasted, and are less angular. Many of these nuclei contain great numbers of tiny lanthanin granules that stain of a beautiful lilac tint, but though very numerous they no longer obscure the other elements. They are in a similar but more advanced condition than those of the pyloric end in the last preparation.

The chromatin is not very abundant, though more so than in the preceding case, but it is arranged in elongated masses and streaks that are more or less united together by fine threads of chromatin. In other cases, and this applies to almost every nucleus near the pyloric end, the lanthanin granules are less numerous, larger, and stain of the same pale sky-blue tint as described in the case of the fasting animal. The chromatin is still more abundant in these nuclei, and stains deep blue. The nucleoli are large and often numerous, but their extrusion appears to have ceased.

The cells seem now to have entirely recovered from their exertions, and are ready again, as soon as called upon, to recommence secreting. The newts themselves exhibited every sign of hunger, hunting about the vessels in which they were kept for food, which they seized and devoured the moment it was presented to them.

A further proof that the cells are now resting is that mitosis begins to appear frequently in them. (Photo 8.) Many beautiful mitotic figures in all stages are present in the oxyntic cells of this preparation, showing that Stintzing (25) was in error when he asserted that oxyntic cells never divide by mitosis. Cell-division goes on somewhat slowly in these animals, and continues to manifest itself, even though no food be given, to the tenth day, though with less and less energy.

SECOND SET OF EXPERIMENTS.

Newts fed a second time 114 *hours after the first meal.*

Half hour after food.—(No. 5, coloured plate.)—Many cells are already secreting, and the gland lumen is wide; the cells are diminished in size, and contain fewer granules, those present being mostly of large size. The protoplasm stains pale blue. The nuclei are irregular in shape, and measure on an average 10·66 × 14·03 μ, which is about the same as during rest. The chromatin is arranged in somewhat spiny karyosomes, is in fair amount, and stains blue. The lanthanin granules are not abundant, and present a blue-grey coloration. The nucleoli are large, and are being expelled in a few cases. Many cells may be seen in various stages of mitosis.

One hour after food.—The lumen has now become of considerable size, owing to further diminution in the size of the cells, which also contain fewer zymogen granules. The protoplasm is somewhat ragged-looking, owing to the vacuoles present in it, and blue-grey in colour. The nuclei appear somewhat wrinkled, and measure 10·19 × 12·14 μ on the average. They do not contain very much chromatin, but that present is arranged in somewhat large karyosomes, which have a washed-out or water-logged appearance, and stain of a watery-blue colour. The lanthanin granules are not numerous, being often greatly reduced in number, and stain of a slaty-violet colour. Nucleoli are being expelled, but no mitosis was observed.

Some cells in this preparation were just beginning to secrete, and their nuclei when measured were found to be of considerable size, namely, 11·29 × 15·83 μ, showing that at a very early stage the nuclei increase somewhat in size.

One and a half hours after food.—(Fig. 6, coloured plate.)—The lumen is still larger and the cells further reduced in size, and in some cases the zymogen granules have disappeared from the bases of the cells. Those present are mostly of small size. The protoplasm is in small amount, vacuolated, and of a pale blue-grey colour. The nuclei are very irregular in shape, with thick envelopes, and measure 10·75 × 10·09 μ. The chromatin, arranged in isolated rounded karyosomes, is not abundant, and stains of a blue colour of moderate intensity. The lanthanin granules are further diminished in number, and pale blue-grey. Nucleoli may be seen in process of extrusion. Mitotic figures are still visible here and there.

Two and a half hours after food.—The lumen is of about the same size as in the last specimen. The cells are shrunken, and contain fewer granules, which are of medium size; the protoplasm is pale grey. The nuclei are small and very irregular, measuring 9·75 × 10·95 μ. The chromatin is not abundant. The small rounded karyosomes stain blue, and are often very scanty. The lanthanin granules are scanty and pale blue-grey. Extrusion of nucleoli is still progressing.

Three hours after food.—(Fig. 7, coloured plate, photo 6.)—Lumen very wide, cells often reduced to narrow strips, containing few zymogen granules that are mostly confined to their free extremities. The cytoplasm is of a very pale grey, or even colourless, perhaps tinted with pink. The nuclei are much wrinkled, measuring $9 \cdot 2 \times 10 \cdot 82\ \mu$, and contain very little chromatin, which is stained of a decided red tint, rendering it difficult to distinguish from the nucleoli. The lanthanin granules are very pale and almost invisible. Some nucleoli are being extruded, and mitosis appears arrested.

Four hours after food.—The lumen is large and the cells very small, with only a few granules of zymogen in them. The protoplasm often appears condensed, but hardly stains at all. The nuclei, which are very irregular, measure $8 \cdot 8 \times 11 \cdot 8\ \mu$, and contain but very little chromatin, which seems concentrated. Nucleoli are not numerous, but some are still apparently being extruded, and mitosis was not observed.

Five hours after food.—(Fig. 8, coloured plate.)—The lumen is now somewhat smaller owing to increase in the size of the cells, which contain more zymogen granules of medium or small size that do not stain at all vividly. The protoplasm is pale blue, and the nuclei are less wrinkled and somewhat dusky in appearance, owing to increase in the number of lanthanin granules. These are of small size and stain deeply blue. Extrusion of nucleoli appears to have ceased, as has also cell division.

Six hours after food.—(Fig. 9, coloured plate.)—The lumen is still narrower, and the cells more protruding and filled with large zymogen granules; the cytoplasm stains pale blue. The nuclei are less angular, stain deeply, and have increased in size to $11 \cdot 44 \times 14 \cdot 55\ \mu$. Chromatin is more abundant in them, and the karyosomes show a tendency to unite together; they stain deep blue. Lanthanin granules are abundant, small in size, but do not produce clouding of the nucleus; they stain of a deep slate-blue colour, with a decided lilac tint. The nucleoli are not numerous, are usually of small size, and surrounded by a clear space free from lanthanin granules. No cells are dividing.

Eight hours after food (pyloric end).—(Fig. 10, coloured plate.)—The lumen is small, the cells large and filled, though not crowded, with zymogen granules of large size. The protoplasm stains pale blue. The nuclei are somewhat plump, measure $11 \cdot 21 \times 13 \cdot 97\ \mu$, and contain a fair amount of blue staining chromatin, arranged in karyosomes united by threads. Lanthanin granules are still abundant, of small size, but do not produce clouding; they stain of a lilac colour, tempered with grey. The nucleoli are mostly small, but a few large ones are seen in process of extrusion, and mitosis has again become visible.

Nine hours after food.—(Fig. 11, coloured plate.)—The lumen is now very small owing to the cells almost filling it up. They are crowded with medium-sized zymogen granules, a few being larger. The cytoplasm stains pale blue, and appears only as a fine network between the granules. The nuclei, however, have again diminished somewhat in size to 10 × 12·57 μ, which is almost the size they were to begin with. Their chromatin is abundant and blue in colour, and the lanthanin granules large and of a pale blue tint. Many cells are in process of mitotic division.

We may now say that the cells have completed the cycle of their changes due to secretion. They appear to secrete for about three to four hours, and then to require about the same time to recuperate; but the worm is still in the stomach at this period, and not completely digested.

Thirteen hours after food.—Lumen very wide and cells much shrunken, being almost reduced to thin strips lining the gland walls; they contain only a few scattered zymogen granules situated mostly near the free ends of the cells. The cytoplasm is pale blue-grey and in small amount. The nuclei have a shrivelled and pale appearance, measuring only 8·1 × 10·2 μ. Their chromatin has a pale spread-out look and watery-blue tint. The lanthanin granules are very scanty and of a pale blue colour. The nucleoli are small as a rule, and some are in process of extrusion. Mitosis is quite absent.

Therefore, after recovering from their first exhaustion, the cells again commenced to secrete, and have now again reached a stage bordering on complete exhaustion.

Fifteen hours after food.—(Photo. 7.)—Lumen again smaller owing to increase in size of cells, which contain a few more granules of large size. The nuclei are still small and wrinkled, measuring 8·2 × 12·55 μ. They contain a moderate amount of chromatin, which has a spread-out, watery look. Lanthanin granules are fairly numerous and of a dark-grey slate colour. The nucleoli are mostly large, and are being freely extruded. There is no mitosis present.

Seventeen hours after food.—The lumen is now small, and the cells are filled with large zymogen granules, those near the free extremities of the cells being somewhat smaller than the others. The cytoplasm is pale grey. The nuclei are large and deeply stained; they measure 11·82 × 14·88 μ, and contain chromatin of a deep blue colour. Lanthanin granules are abundant and stain deep blue, with a decided lilac tint. Nucleoli are often large, and some are being extruded.

By the nineteenth or twentieth hour the cells have again come to rest, and some mitotic figures make their appearance in them.

The cells lining the surface of the stomach, and those of the pyloric glands and necks of the oxyntic glands, have also been carefully studied. Although their secretion is of a different nature from that of the oxyntic cells, they exhibit during activity and repair precisely the same nuclear changes as the oxyntic cells. Therefore, without wearying you with further details, I may pass at once to the conclusions I have drawn from this investigation.

Résumé and Conclusions.

As soon as the food is swallowed secretion commences in the oxyntic glands of the stomach, and sweeps in a slow wave along the whole organ, commencing at the œsophageal opening and passing off at the pyloric end at such a rate that the cells near the pyloric extremity reach their maximum activity some one and a half to two hours after those near the œsophagus.

For any one cell, the wave reaches its maximum between the third and fourth hour or hour and a half, and is followed by a period of rest and recuperation, which lasts for another four or five hours; after which, if food is still present in the stomach, the cells again enter upon a second secretory period that follows the same course as the first.

According to Langley, and Langley and Sewall (13), the cells continue to secrete actively during the period of repair, statements which they base mainly on their experimental researches on the quantity of digestive substances present in the stomach, and on the observation that in all the specimens examined by them many small zymogen granules are present in the cells.

I have not been able to convince myself of the accuracy of this statement except in a modified form, because of the fact that the cells exhibit a second period of secretion and exhaustion after they have once regained their resting condition. One would have expected complete recovery to be delayed until the stomach had emptied itself of its contents, were secretion and reparation going hand in hand. Further, I believe that the small granules are in process of growth, rather than in process of solution. I have also been unable to see any increase of protoplasm during secretion, though it does undoubtedly increase very rapidly as soon as the maximum of the secretory wave is passed, and this is indicated by the staining reaction which changes from feeble affinity for the blue dye to an affinity for the acid eosine, whereby the protoplasm stains somewhat reddish or reddish-grey; indicating, I believe, that an absorption of albumin is taking place from the lymph into the protoplasm.

I think it would be more correctly stated if one said that repair goes on during secretion, *i.e.* prozymogen is produced at the same time that the zymogen already formed is being secreted by the cell.

With regard to the nuclei, they increase slightly in size at the very commencement of secretion, but soon become smaller and smaller, and finally show great wrinkling and puckering of the nuclear wall [Heidenhain (8), Hermann (10), Schieferdecker (24), Mann (16), etc.], owing to diminution of their contents. When repair sets in, which it does soon after it has begun in the protoplasm, the nuclei rapidly lose their wrinkled appearance and swell up to a considerable size, becoming much bigger than at the beginning of secretion, to again diminish somewhat as the resting stage is reached.

During nuclear activity the chromatin spreads itself out upon the inner surface of the nuclear membrane and becomes diminished in amount, losing some of its constituents, which are again replaceable during recuperation; it becomes spread out and pale-looking as if filled with a fluid, as observed by Bataillon (2), 1891, and further, it undoubtedly moves about during the process.

Of still more interest is, I think, the fact that when the cell is nearly exhausted the affinity of the chromatin for the blue dye diminishes, while its affinity for the acid eosine increases to such an extent that the spread-out karyosomes tend to stain rather red than blue. This can only be due to alteration occurring in the constituents of the karyosomes. This red stage does not, however, last long, the blue reaction coming quickly back again soon after repair has set in. It is interesting to note in this respect that the chromatin of the head of the spermatozoon shows the same affinity for acid dyes, though chemically rich in nucleic acid.

Of no less interest in connection with this, is the growth and extrusion of nucleoli during the period of nuclear activity, which goes on for some time also during the process of repair, getting less and less as the nucleus is nearing the resting condition. By their reaction the nucleoli are albuminous bodies. I believe this extrusion of nucleoli to be a constant phenomenon of nuclear activity, as I have observed it in many different kinds of cells. Henneguy (9) has also seen chromatic granules extruded through the nuclear wall, which he believes to be elastic, as it closes up again immediately after their passage. Further, A. Michel (17) has stated recently that nucleoli consist of two parts, a main substance and an accessory substance. More recently still, A. Pizon (21) from his studies on Ascidia (*M. socialis* and *M. simplex*) concludes that the nucleolus during the whole ripening of the ovum throws off big spherical masses of effete substance that leave the nucleus by passing through its wall, and passing into the vitellus are moved through it to its periphery in a peculiar manner; *i.e.*, the nucleolus, in part at least, is effete material.

I believe that in this animal the nucleoli consist entirely of effete material, which is produced during nuclear activity; that the nucleus is capable of tolerating this material in certain amount, but that when it becomes excessive, it is passed out into the protoplasm, where it disappears. It certainly does not become converted directly into zymogen granules, as some would have us believe.

I have shown elsewhere (4) that the nucleolus of the mammalian ovum should also be considered as mainly composed of effete material.

I cannot agree with Balbiani (1), who considers the nucleolus to be a sort of cell-heart, or with those who believe that chromatin is manufactured in it [Flemming (6), Malaquin (15), etc.], or even with those who maintain that chromatin elaborated elsewhere is stored up in the nucleolus [Sabotta (23), Labbé (12), Mingazzini (18), etc.]. I therefore agree with Pizon (21), and those other observers who hold that the nucleolus is entirely, or at least in great part, composed of effete material.

The changes occurring in the nucleo-hyaloplasm are also interesting. When precipitated with mercuric chloride, during the resting condition, the precipitate lanthanin of M. Heidenhain (7) stains pale blue. As activity proceeds, these granules become more and more scanty, but just after the nucleus reaches its maximum of shrinkage, the lanthanin becomes very abundant, and stains of a deep blue colour, which gradually gives place to a beautiful lilac tint as repair goes on, finally returning to pale blue when repair is complete. This denotes that the nuclear juice undergoes chemical change during nuclear activity. Strasburger (26) in 1892 stated that the reaction of nuclei depended upon their condition of nutrition; well nourished nuclei being erythrophile, and poorly nourished ones cyanophile. This corresponds well with what has just been stated.

The great fatigue exhibited by the nuclei of secreting cells during the conversion of zymogen into zymin by the action of very dilute acids has for a long time been a puzzle to me, for one would suppose the manufacture of zymogen would be far more exhausting than its mere passage out of the cell into the gland lumen.

The researches of MacCallum (14) of Toronto throw light upon this subject. He maintains that zymogen is preceded by a substance rich in phosphorus and iron produced at the expense of the chromatin of the nucleus. This substance he terms prozymogen, which becomes united with a constituent of the cell protoplasm to form zymogen. Mouret (19) also takes a similar view of the production of zymogen, and he named the antecedent substance "prezymogen," and Bensley (3) has corroborated these researches in his recent paper on the stomach.

I believe that the nuclear exhaustion so evident in my specimens is indeed brought about by the manufacture of prozymogen.

Almost directly after the cell had begun to pour out its secretion, the nucleus commences to form a new supply of prozymogen at the expense of its store of chromatin, and that the movement of the chromatin towards the nuclear membrane and its application to its inner surface is of service in facilitating this process; but I further believe that the prozymogen is not elaborated into zymogen until the cells have practically emptied themselves of the zymogen already produced, and have begun to take up substances from the blood. Once this has occurred, the cell quickly transforms the prozymogen into new zymogen, which is the meaning of the rapid reappearance of the zymogen granules.

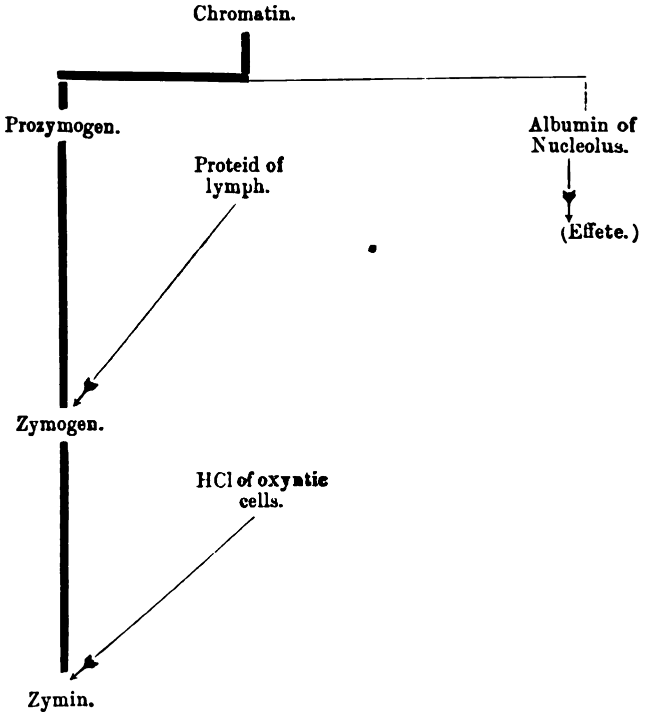
On the other hand, the power of the nucleus to manufacture prozymogen is limited, and once its store of chromatin is reduced to a certain point, it must renew the stock before it can again produce prozymogen, and that this renewal of chromatin is brought about by the entrance into the nuclear juice of an easily coagulable substance (probably a proteid material amongst other things), which is the meaning of the great increase in the lanthanin granules soon after complete exhaustion of the nucleus has occurred, and I believe that this necessity for repair sufficiently accounts for the phenomenon of the double secretory wave, the cell, though full of zymogen granules, not being in a fit state to secrete again until the nucleus is completely recuperated; but that when once this point is reached, there being no longer any further need of rest, the cell again starts secreting should any call be made upon its energies.

I have, therefore, been able to supply the link in the chain that seems to have been missing in the work of previous investigators, namely, the observation above recorded of an absolute diminution of chromatin occurring as a result of the production of prozymogen.

Chromatin contains nucleic acid and an albuminous substance; it is very stable, and acts as an acid to bodies less acid than itself; therefore it stains with methyl-blue. In order that this stable body may pass into the cytoplasm it must be rendered soluble (there being no evidence of its passage bodily out of the nucleus); in this process some of the albumin is removed as effete material that goes to form the nucleolus. This albumin acts as a base in presence of bodies more acid than itself, and therefore exhibits marked affinity for the eosine. Being now useless in the nucleus it is extruded into the protoplasm, there to be disposed of. The more acid part of the chromatin passes into the cytoplasm, where it remains in an invisible state, unless the phosphorus and iron it contains are unmasked and revealed by dyes as directed by MacCallum.

When the cell begins to take up material from the blood (serum albumin, etc.) this prozymogen quickly links on some albumin to become zymogen in the form of granules, nucleo-albuminous in nature, which is probably a still less stable compound than the prozymogen; it now reacts as a feeble base to eosine, and hence is stained red with that dye. It is also readily soluble when brought into contact with a weak acid, either in the cytoplasm or elsewhere. In other words, both chromatin and zymogen have the same nuclein radicle, their difference in staining property and solubility depending on the amount of albumin linked on to it.

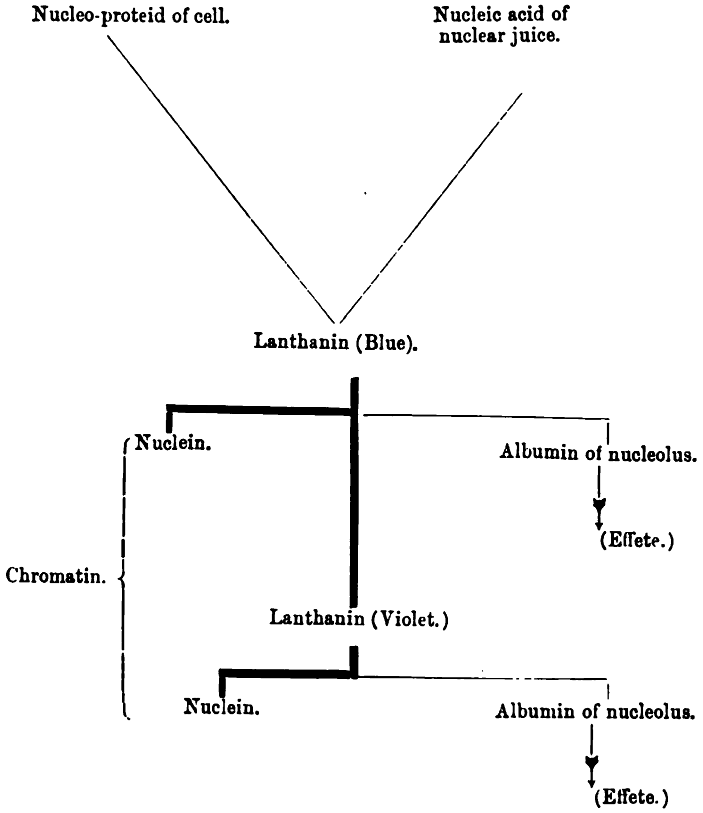
Formation of Zymin.



With regard to the question of repair of the nucleus, it can only be brought about by the passage into it of substances from the cytoplasm, which probably unite with nucleic acid in the nuclear juice to form a material analogous to chromatin, *i.e.* the blue staining lanthanin granules precipitated by mercuric chloride. This material seems gradually to give up its nucleic acid to increase the chromatin, leaving a less acid material in the juice, which when precipitated with corrosive sublimate stains lilac, a tint intermediate between blue and red. Finally, also, this material becomes all converted into chromatin on the one hand and into a red staining material on the other, which possibly passes to increase the size of the nucleolus, and may or may not be in sufficient quantity

Synthesis of Chromatin.

Proteid of lymph becomes in cell nucleo-proteid



to induce some nucleolar expulsion during the reparation stage. When this material is all disposed of, the nuclear juice, when precipitated, stains, as in the resting cell, *i.e.* the nucleus has become completely restored. I think, therefore, that one must look to the nuclear juice to find the earliest indication of the synthesis by which chromatin is built up from simpler compounds.

Another interesting point is the fact that so long as the nuclei are in an exhausted condition, no mitosis occurs, but that as soon as repair has reached a certain stage a few nuclei exhibit mitotic figures, which increase steadily in number until the cells are fully restored, when its maximum is reached, to again diminish and cease altogether when the nuclei again become exhausted.

Further, cells called upon to secrete during nuclear division do so exactly as if no mitosis were going on, and this secretion proceeds along with the division, both occurring at the same time, the chromatin of the mitotic figure showing exactly the same changes in appearance and colour reactions exhibited by the karyosomes of the cells that are not dividing, the only apparent difference being that, owing to the disappearance of the nuclear envelope no nucleoli are formed, the split-off albuminous material being disposed of directly by the cytoplasm.

Lastly, I would tender my thanks to the late Professor Rutherford for his courtesy and kindly support during the progress of the research, and to Dr T. H. Milroy for valuable assistance in working out the chemical side of the question.

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Proc. Roy. Soc. Edin. Vol. XXII.

Dr CARLIER ON OXYNTIC CELLS OF NEWTS STOMACH IN DIFFERENT STAGES OF ACTIVITY. SEMI-SCHEMATIC.

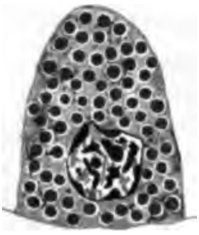
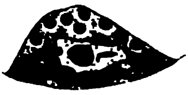


Fig. 1.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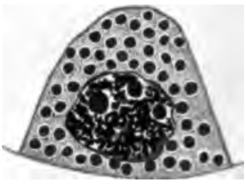
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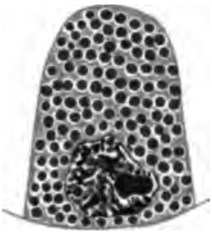
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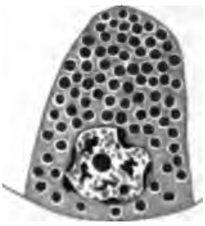
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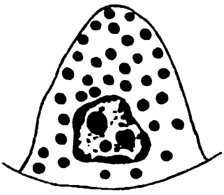
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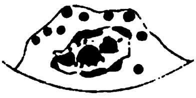
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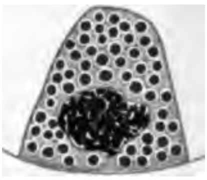
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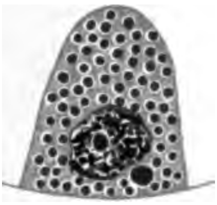
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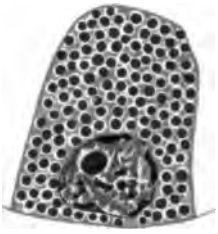
Fig. 10.

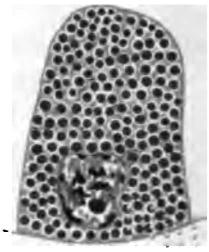
Fig. 11.

M'Farlane & Erskine, Lith.

Professor E. Wace Carlier.—Plate II.

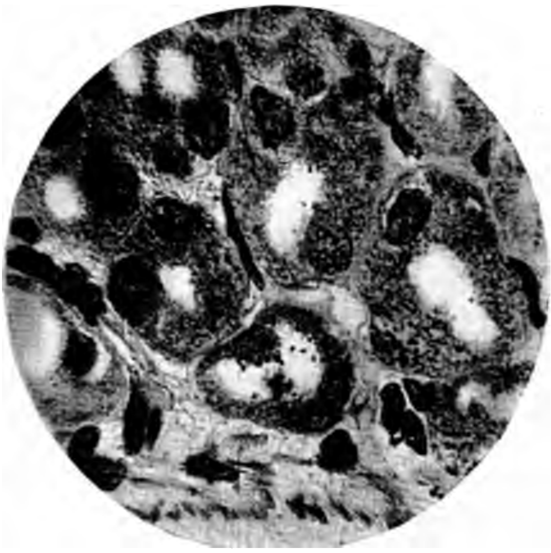

Fig. 3.

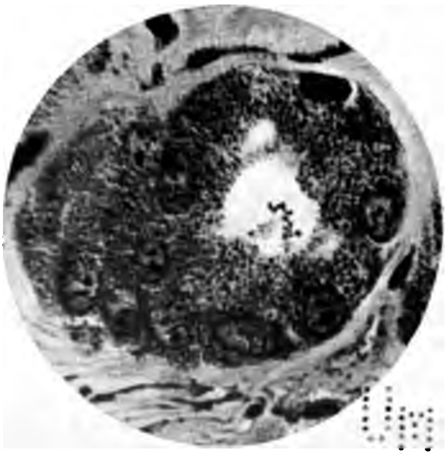

Fig. 4.

Professor E. Wace Carlier.—Plate III.

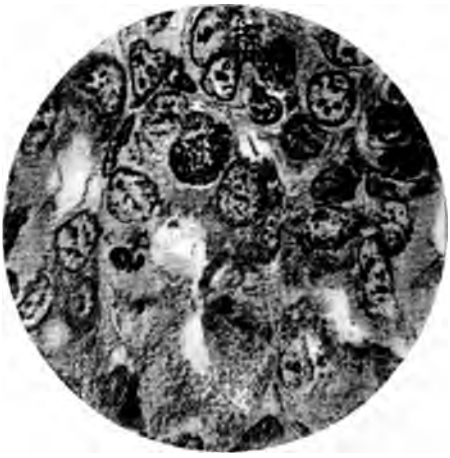

Fig. 5.

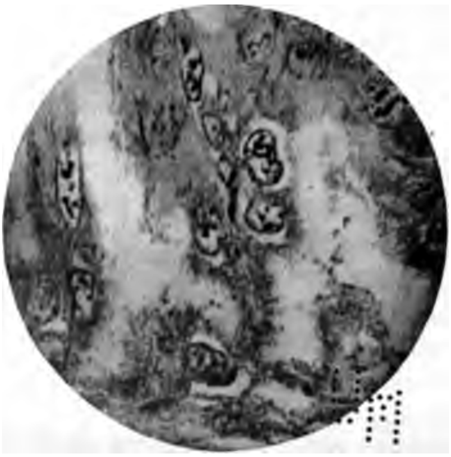

Fig. 6.

Professor E. Wace Carlier.—Plate IV.

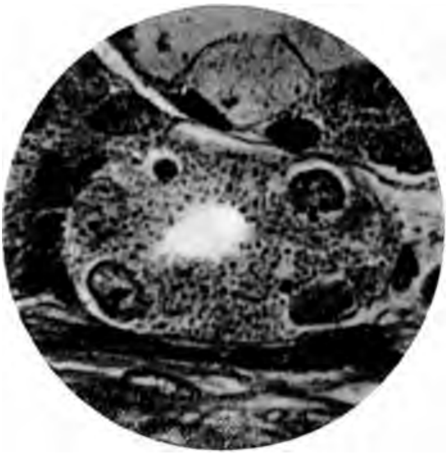

Fig. 7.

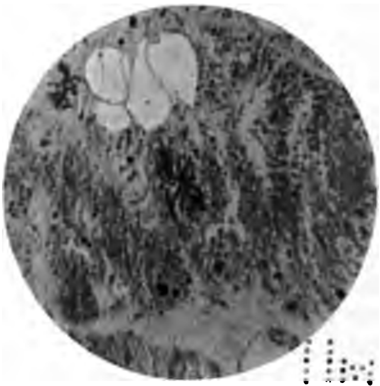

Fig. 8.

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EXPLANATION OF COLOURED PLATE.

The figures are semi-diagrammatic, and represented as stained with methyl-blue eosine.

I. Oxyntic cell of newt in a resting condition.

II. Similar cell after secreting for twenty hours. Taken from a winter newt.

III. Similar cell forty-four hours after the commencement of secretion; the nucleus is filled with deeply stained lanthanin granules.

IV. Similar cell sixty-eight hours after the commencement of secretion. The lanthanin granules exhibit an intermediate coloration, and the nucleolus is very large.

V. Oxyntic cell of summer newt half an hour after commencement of secretion.

VI. Similar cell one and a half hours after commencement of secretion. The chromatin stains of a pale colour, and the nucleoli are increasing in size.

VII. Similar cell three hours after commencement of secretion. The chromatin now stains red, and cell is practically exhausted.

VIII. Similar cell five hours after commencement of secretion. The nucleus is full of deeply staining lanthanin granules, and the cytoplasm stains of a reddish tint.

IX. Similar cell six hours after commencement of secretion, showing an extruded nucleolus lying in the protoplasm.

X. Similar cell eight hours after commencement of secretion. Increase in quantity of chromatin, lilac coloured lanthanin, and large nucleolus.

XI. Similar cell nine hours after commencement of secretion. The cell has almost returned to the resting condition.

EXPLANATION OF PHOTOGRAPHS.

These photo-micrographs were all taken with a magnification of 600 linear.

1. Oxyntic gland from near œsophagus in which the zymogen granules are of large size.
2. Oxyntic gland twenty-four hours after food.
3. T. S. oxyntic gland forty-four hours after food.
4. T. S. oxyntic gland sixty-eight hours after food.
5. T. S. oxyntic gland 114 hours after food, showing early stages of mitosis in the cells.
6. Oxyntic gland three hours after food, from near the pyloric end.
7. T. S. oxyntic gland fifteen hours after food, showing extrusion of nucleoli.
8. An oxyntic cell in process of mitosis.

On Some Remains of Scottish Early Post-Pliocene Mammals. By the Rev. Professor Duns. No. II.

(Read May 15, 1899.)

At the meeting of the Society on the 20th February 1893 I had the honour of reading a paper "On the Early History of some Scottish Mammals and Birds." Perhaps the title both of that paper and of the present should be "Remarks on the Literature associated with some Early Post-Pliocene Mammals."

The science of a Species means much more than the naming of its several parts by Latin and Greek derivatives. Even as regards these parts, one would like to know something of extinct forms at the time when they had a place among the living, just as, when dealing with recent forms, we find much to shed light on the structure, probable habits, and surroundings, especially of mammals which became extinct in quaternary time. Questions arise that are worth noting, were it for nothing more than to make it clear that the mere *obiter dicta* even of able and wide-minded experts may often be misleading. The tendency which prevails to determine species by characteristic bone-marks can never be perfectly satisfactory, because there are many instances in which close structural resemblance is associated with great dissimilarity of habit. It is not so easy as some seem to think to distinguish between several of the long bones of the wolf with the corresponding bones of the collie, or between those of the sheep and of the goat. Then, as to footprints on stiff, tenacious quaternary clays, is there any risk of a label?—"marks of the feet of a small ruminant"—the marks being those of the feet of *Sus scrofa* (*ferus*) and not of gentle *Ovis aries*, L.

Another word as to the title! When I intimated my first paper I was specially anxious to let it be known where certain specimens I had hoped to show are 'housed'—chiefly a noble skull of the urus and the mammoth tusk. They are often referred to in recent literature, sometimes as being in the university and sometimes as in the National Museum.

The subjects now referred to are of some interest both to the naturalist and the antiquary, especially when dealt with from wider than mere zoological points of view. Much of the information we have at present of recently extinct forms consists of details of structure couched in a terminology which only experts readily appreciate. No doubt, progress in palæo-zoology would be impossible without such details, but it would be greatly helped were they associated with other than specific features—with, for example, what might suggest habits and habitats, physical and vital surroundings, gradational relations, geographical and geological distribution. Extinct animals might thus come to have a place in recent history. Thus regarded, the following questions of general interest are raised:—Do the remains of extinct forms, looked at in the light of their environments, point to local climatal changes, or to alterations of surface in the areas in which they occur, or to links dropped out of gradational rank, or to the realisation of new conditions of life by the disappearance of an old or by the introduction of a new species? And suppose we assume the ever-present influence in species of an innate, ever-active element of structural variation, in what direction does it work? Is it towards new species, or is it only towards increase of variation in species? These questions are put in the belief that we have materials to warrant them. In Scottish literature, reaching back three or four hundred years, we have references to mammals and birds now extinct by men who manifestly had cultivated the habit of the eye—men who saw the forms around them and who could describe what they saw.

In the literature to which I refer we have information (1) on the physical and vital environments of mammals and birds; (2) on the physical features of the districts in which they had been, or in which they were, at the time of the observer's first record; (3) on climatal conditions; (4) on the prevalent fauna and flora of their time; (5) *occasionally*, on the nature of superficial deposits; (6) on important phenomena connected with migration; (7) on the limits, if not on the laws of variation; and (8) on outstanding facts in the ancestral history of several extinct forms. That the men to whom we are indebted for all this information often held views of things seen and unseen superstitious enough, none who know their works will deny. These, however, did not overshadow their great attainments. They lay alongside of them, and thus, by contrast, set them in greater relief. Many examples might be given of the deep indebtedness of present students of science to these literary records of the far past. Some of the sources of the information we are in quest of may be named. Leaving out of view what may be called the literature of the chartularies, we have Adamnan's *Vita Sancti Columbæ*, A.D. 697; Boece's *Scotorum Historia a Prima Gentis Origine*, A.D. 1526; Lesley's *De Origine Moribus et Rebus Gestis Scotorum*, A.D. 1578; Sibbald's *Scotia Illustrata, sive Prodromus Historia Naturalis*, A.D. 1684; Gordon's *Itinerary*, A.D. 1726; and later, Chalmers's *Caledonia*, Ure's *History of Rutherglen*, Pococke's *Travels in Scotland*, Pennant's *Travels in Scotland*, Fleming's *British Animals*, etc. In them and kindred works much light is shed on the mammals of those times by giving prominence to their environments, physical and vital. These sources of information were referred to in my last paper, and instances illustrative of their value were quoted. They deserve to be made more of than they are by recent writers, were it for no other end than making them a part of common culture. The power to utilise them would itself imply scientific attainments, because such records of the remote past owe much of their value to the fact that they shed reflex light on man's social, industrial, and often even artistic condition at the time; in a word, on stages of civilisation,—the outcome of men in touch with and able to control their environments.

In the geology of the surface—in superficial strata—we have another source of information regarding the natural history of pre-historic and early historic species. Here, however, very much depends on the qualifications of the student. If a specialist, say, in any one vertebrate family of organisms, his descriptive identification of species will be perfectly trustworthy in that family of fishes or reptiles, birds or mammals. But some of the most important points touching it, may, very often do, require more than a general knowledge of quaternary strata, whether the specimens are geological or archæological, or both in one. The occurrence of the bones of mammals and birds in quaternary beds alongside of bits of pottery, fragments of weapons, remains of ancient industrial implements, or of rude pre-historic art, introduce new elements of observation and inference. The association of objects of man's handiwork with the remains of extinct animals goes far to indicate the nature of the environments of the animals—so far, indeed, as to warrant generalisations in regard to the physical conditions, the fauna and flora in the midst of which both the lower animals and man lived and moved. Natural science may thus do for these extinct forms a service analogous to what the historian has done for the men of "the long ago," showing them as influenced by and influencing their surroundings.

Now, subjects of importance as to the distribution and associations of extinct forms are suggested here. In some parts of Britain, and, more widely, over the Continent, traces of the presence of man alongside of remains of extinct mammals are common, indeed, so common, as to warrant the inference of contemporaneity. But in Scotland, such cases occur without any traces of man. Does this imply that in such localities extinct species had looked their last before man appeared on the scene? Some argue that this was so. But, in reality, the question of time is not raised here, though the question of population is. Contemporary with the animals where man met them, there were wide areas which the people had not occupied, and in these animals had lived undisturbed by man, yet they had disappeared. I don't attempt to account for the disappearance, but can only theorise touching the setting in of physical conditions fatal to animal life. The terms pre-glacial, glacial, post-glacial, ultra-glacial, and inter-glacial come into mind at the bare mention of unfavourable physical conditions. But I wish to keep clear of these in this paper as much as possible. This does not mean, however, that we are to keep out of view the nature, order, and relations of the surface deposits in which the remains of extinct forms occur. On the contrary, it is of chief importance that we should try to collate the superficial beds of one district with those of another which geologically differs from them but contains the same remains. This indicates the importance of studying these deposits both from the biotic and the stratigraphical points of view. It will be evident from the foregoing remarks that the object I have in view under the title of the paper is mainly to indicate that the palæozoologist should not limit himself to the determination of species by the use of a terminology which only practised specialists can fully understand. No doubt the terminology is necessary for the progress of science, but if work in this department is to promote general culture in a natural, easy, and interesting way, the characterization of extinct species will include the physical and vital conditions of the localities in which they occur. Looking at the trend of present speculative science touching the upward march of species, I dare say most of us whose work lies in this or in closely-related branches, have heard friends whose training in other branches qualify them to deal with the rules of evidence, regretting that they know nothing of and cannot estimate the value of the alleged scientific facts held to warrant certain sweeping biological generalizations.

The mammal remains which I have had before me both in this and in my first paper may be stated thus:—ELEPHANTIDÆ. The Mammoth (*Elaphus primigenius*, Blum.) CERVIDÆ. 1. The so-called Greater Red-Deer (*Cervus elaphus*, L.). 2. The Reindeer (*Cervus tarandus*, L.). 3. The Great Irish Deer (*Cervus megaceros*, Owen). BOVIDÆ. 1. The Urus (*Bos primigenius*, Boj.), and 2. The Celtic Shorthorn or Longfronted Ox (*B. longifrons*, Owen). I had intended to place on the table examples of remains of each, but at present will limit my remarks to the Mammoth Tusk, the Great Red-Deer Antlers, and the skull of the Celtic Shorthorn.

I. The Mammoth. Traces of this form have been met with in Ayr, Berwick, Edinburgh, Lanark, and Perth shires. I notice only the Mid-Lothian tusk. Its history is soon told. It was found in 1820 on the Clifton Hall estate near Ratho, by labourers employed in making the Union Canal, at a point where the earth slopes and forms the valley through which the river Almond flows, and was taken by the finders to a cottage near the works. The find was reported to Sir Alexander Maitland, the proprietor of the estate, who, accompanied by the engineer of the works, visited the cottage. When the workmen learned through them that the tusk was ivory, they quietly sent two of their number with it to Edinburgh, with instructions to sell it to an ivory-turner. This being reported to Sir Alexander, he hastened to the city, where he found it in the hands of the ivory-turner, who had already sawn it across in three places, and had prepared one of the parts for the lathe, in order to form chessmen. Sir Alexander repaid the money which had been given for it, and gave it a place in Cliftonhall as a very highly-prized specimen. There is abundant evidence that the naturalists of the day were greatly interested in and put a high value on the tusk. At Sir Alexander's death it passed into possession of his son, John Maitland, Esq., late Accountant of the Court of Session, and was given by him to me in 1864. There has been a good deal of controversy touching both as to the nature of the deposit and the depth in it where it was found. I think it can be proved that it was not at a depth of from 15 to 20 feet, but only near the surface. It was never placed in the university or the National Museum, as Sir Charles Lyell and others have alleged.

II. The so-called Greater Red-Deer (*Cervus elaphus*, L.). This is the *Strongylocerus* of Owen. The antlers on the table are fine examples of the horns of this variety of red-deer. They were found at a point a little below the surface of a dried-up pond near Kingskettle, Fife. The term "greater" ascribed to this form is warranted by the fact that many of its remains seem to indicate an animal of larger growth than the present common red-deer. They occur deep down in peat bogs, marl beds, and in the sites of lochs long dried up. Deer-stalkers generally are of opinion that there is a slow but real declension going on among stags. If so, there must be a cause. Does it result from the ever-increasing narrowing of the areas of range, or from the growing scarcity of favourite food, or from the changeful elements of civilisation, with its railways and steamboats bearing in on their usual haunts, or from all these things working together begetting, as Owen once alleged, a restlessness not favourable to growth?

The horns seem to have been torn off, not shed. Their weight is 23 lbs. 14 ozs. Looking at them as right and left, the former is 11 lbs. 10 ozs., the latter 12 lbs. 4 ozs. This discrepancy between the two appears also when they are measured, thus:—

	Right Horn.	Left Horn.
Length of beam from the burr to the base of the sur-royal, . . .	27 inches.	27½ inches.
Length to highest point of same, .	35 ,,	35 ,,
Brow antler, ...	10½ ,,	11 ,,
Royal antler (outer edge), . .	17 ,,	16 ,,
Bez antler (outer edge), . . .	13½ ,,	15 ,,
Circumference at the burr, . .	10 ,,	11½ ,,

And so throughout. The brow antler of both horns is broken; but looking at its thickness near its junction with the beam and the gradual way it must have thinned to a point, it must have been as long again as it is now.

III. The Ancient Shorthorn Ox (*Bos longifrons*, Owen). Remains of this ox seem to indicate that it lived from early Post-Pliocene time down to the historic epoch. They occur in the same deposits as those in which the bones of the urus are found. That the urus preceded the shorthorn, but had not died out when the latter was introduced to Scotland, appears likely. There are records of the occurrence of its remains in most, if not in all, the counties of Scotland. I have a part of the skull of one which was found in the brick clay near Dunbar, to my surprise in the same layer as a perfectly preserved pretty little ophiuroid (*Ophiolepis gracilis*), seldom met with now in the same locality. The portion of the skull of this ox on the table was presented to New College Museum, and was for a time believed to be Scottish, but it was afterwards found to have been got in County Limerick, a little below the surface of a peat moss, along with a bit of skin with the hair attached, a piece of coarse cloth, and a fragment of earthenware. The horns are entire, but the bones have lost their mineral constituents altogether. Archæologically, this specimen is of much interest.

There are other forms which, in this connection, are worth noting at some length, but can only now be named:—The Beaver, Reindeer, Roedeer, Great Wild Ox, Pine-Martin, Wild Boar, Wolf, and Wild Cat.

In conclusion, reference may be made to "finds" which supply many instances illustrative of, at least, the general drift of this paper. One of these occurs in the description of an Ayrshire Crannog by Dr Robert Munro, to whom we are all indebted for many able and wide-minded contributions to anthropology and archæology. The following zoological and botanical remains were found in the crannog:—(*a*) Zoology: *Bos longifrons*, *Sus scrofa* (*ferus*), *Ovis aries*, *Cervus elaphus*, *C. capreolus*, *C. tarandus*, and *Equus caballus*. (*b*) Botany: *Alnus glutenosa*, *Betula alba*, *Corylus avellana*, *Ulmus montana*, and *Pteris aquilina*. In addition to all these, it was found that man had left marks of his presence among them. There were implements of stone, bone, wood, bronze, and iron, which had all been in use, and along with these ornaments were met with which shed some light on the art of the time. Now, were we to fix our attention on any one bone, say a bone of the reindeer (*Cervus tarandus*, L.), after determining the species, would there not be much to suggest interesting questions touching the physiography of the district, its climatal conditions, the geographical distribution of this species of deer, and even something worth knowing as to man and his art at a very early period?

Meetings of the Royal Society—Session 1897-98.

Monday, 22nd November 1897.

General Statutory Meeting. Election of Office-Bearers. p. 1.

FIRST ORDINARY MEETING.

Monday, 6th December 1897.

The Right Hon. Lord Kelvin, G.C.V.O., President, in the Chair.

Dr J. O. Affleck and Mr James Robert Erskine-Murray were admitted Fellows of the Society.

The President read a short Communication on events interesting to the Society during the last Session, and laid on the table short obituary notices of Ordinary and Honorary Fellows who had died since the opening of Session 1896-97. *See* pp. 2-10.

The following Communications were read:—

1. On the Food, Fuel, and Air of the World. By the Chairman.
2. Chapters on the Mineralogy of Scotland. Chapter viii., Silicates. By the late Professor Heddle. *Trans.*, xxxix. pp. 341-358.
3. Note on the Disturbance of the Magnetic and Meteorological Instruments at the Colaba Observatory during the Earthquake of 12th June 1897. By N. A. Moos, Director of the Observatory. pp. 11-21.
4. On a Problem of Sylvester's in Elimination. By Professor E. J. Nanson. Communicated by Professor Chrystal. pp. 150-157.
5. On the Velocity of Graded Actions. By Professor Walker, Ph.D., D.Sc., University College, Dundee. pp. 22-32.
6. Preliminary Note on a Characteristic of certain Chemical Reactions. By Professor J. Gibson, Ph.D. pp. 33-37.
7. On the Directions which are most altered by a Homogeneous Strain. By Professor Tait. pp. 162-164.

Mr James Currie, jun., Mr John Archibald Purves, and Mr Charles Tweedie were balloted for, and declared duly elected Fellows of the Society.

SECOND ORDINARY MEETING.

Monday, 20th December 1897.

T. B. Sprague, LL.D., in the Chair.

The following Communications were read :—

1. Obituary Notice of the Rev. John Wilson, M.A. By C. G. KNOTT, D.Sc. p. i.
2. Investigations on the Life-History of the Salmon in Fresh Water. By F. D. BOYD, J. C. DUNLOP, A. L. GILLESPIE, E. D. W. GREIG, G. L. GULLAND, S. C. MAHALANOBIS, M. J. NEWBIGIN, and D. NOËL PATON. Communicated by Dr NOËL PATON.
3. Note on the Passage of Water and other Substances through India-rubber Films. By Dr R. A. LUNDIE. Communicated by Professor TAIT. pp. 258–264.
4. A Relation between Permanents and Determinants. By THOMAS MUIR, LL.D. pp. 134–136.

THIRD ORDINARY MEETING.

Monday, 17th January 1898.

The Rev. Professor Flint, D.D., Vice-President, in the Chair.

The following Communications were read :—

1. Note on the Axes of Symmetry which are Crystallographically possible. By HUGH MARSHALL, D.Sc. pp. 62–65.
2. The Old Red Sandstone of the Orkneys. By JOHN S. FLETT, M.A., B.Sc. Communicated by Professor GEIKIE, F.R.S. *Trans.*, xxxix. pp. 383–424.
3. On the Fossil Fishes of the Orcadian Series of the Old Red Sandstone of Scotland. (*With Lime Light Illustrations.*) By R. H. TRAQUAIR, M.D., LL.D., F.R.S.

Mr ALEXANDER CULLEN, Principal W. WALLACE, and Professor G. ADAMI were balloted for, and declared duly elected Fellows of the Society.

FOURTH ORDINARY MEETING.

Monday, 31st January 1898.

Professor Chrystal, LL.D., Vice-President, in the Chair.

The following Communications were read :—

1. Obituary Notice of the late EDMUND CHISHOLM BATTEN, M.A. By Major CHISHOLM BATTEN. p. iii.

2. Report of the Seventh International Geological Congress (1897). By J. K. Talmage, Ph.D., F.G.S., one of the Society's Delegates to the Congress.

3. The Relations between the Coaxial Minors of a Determinant of the Fourth Order. By Thomas Muir, LL.D. *Trans.*, xxxix. pp. 323–339.

4. On Converging Prisms. By Charles A. Stevenson, B.Sc., M. Inst. C.E.

5. Observations on the Migratory Movements of Salmonidæ during the Spawning Season. By W. L. Calderwood, Esq. pp. 47–55.

6. On the Ellipse-Glissette Elimination Problem. By Professor Nanson. Communicated by Professor Chrystal. pp. 158–161.

FIFTH ORDINARY MEETING.

Monday, 7th February 1898.

Professor M'Kendrick, M.D., Vice-President, in the Chair.

The following Communications were read :—

1. Notes on some Specimens of Rocks from the Antarctic Regions. By Sir Archibald Geikie, F.R.S. ; with Petrographical Notes by J. J. H. Teall, F.R.S. pp. 66–70.

2. Notes on some Additional Fossils collected at Seymour Island, Graham's Land, by Dr Donald and Captain Larsen. By George Sharman, F.G.S., and E. T. Newton, F.R.S. Communicated by Sir A. Geikie. pp. 58–61.

3. Contributions to the Spectroscopy of Hæmoglobin and its Derivatives. By Dr David Fraser Harris. pp. 187–208.

4. On the Action of Hydroxylamine Hydrochlorate. By Dr W. Brodie-Brodie. Communicated by Dr M'Kendrick. pp. 56–57.

5. On the Derivation of the Curves of Vowel Sounds by the Microscopical Examination of their Phonograms. By Dr Boeke, of Alkmaar. Communicated by Dr M'Kendrick. pp. 88–96.

6. On the Fourierian Analysis of Phonographic Tracings of Vowels. By Dr R. J. Lloyd, Liverpool. pp. 97–117.

7. Observations on the Theories of Vowel Sounds. By Dr M'Kendrick. pp. 71–87.

A Paper by Dr John Murray on "Bipolarity" in the Distribution of Marine Organisms was postponed for want of time.

The following Candidates for Fellowship were balloted for, and declared duly elected :—Cecil Carus-Wilson, James Campbell Irons, A. T. Masterman, Benjamin Hall Blyth, Richard Vary Campbell, the Hon. John Abercromby.

SIXTH ORDINARY MEETING.

Monday, 21st February 1898.

The Right Hon. Lord Kelvin, President, in the Chair.

The Hon. JOHN ABERCROMBY and Mr JOHN A. PURVES were admitted Fellows of the Society.

The following Communications were read:—

1. On the Thermodynamics of Volta-contact-electricity of Metals. By the Right Hon. LORD KELVIN. pp. 118–125.
2. The Cones of Equal Angular Displacement in a Homogeneous Strain. By Professor TAIT. pp. 163, 164.
3. On Bipolarity in the Distribution of Marine Organisms. By Dr JOHN MURRAY, F.R.S. (*With Lantern Illustrations.*)

SEVENTH ORDINARY MEETING.

Monday, 7th March 1898.

Dr Burgess in the Chair.

The following Communications were read:—

1. Heat of Combination of Metals in the Formation of Alloys. By ALEXANDER GALT, B.Sc. pp. 137-149.
2. Note on a Modified Form of the Rutherford Microtome. By DAVID FRASER HARRIS, B.Sc., M.D.
3. On the Characteristics of Lighthouse Apparatus. By J. A. PURVES, B.Sc.
4. On the Nature of "Palladium Hydrogen." By JOHN SHIELDS, D.Sc., Ph.D. Communicated by Professor CRUM BROWN. pp. 169–186.

Dr JOHN GLAISTER, Mr FRANCIS CHALMERS CRAWFORD, Dr GEORGE NEWMAN, Mr DAVID BROWN, Mr W. ALLAN CARTER, Mr ALEXANDER VEITCH LOTHIAN, and Mr S. C. MAHALANOBIS were balloted for, and declared duly elected Fellows of the Society.

EIGHTH ORDINARY MEETING.

Monday, 21st March 1898.

The Right Hon. Lord Kelvin, President, in the Chair.

Mr JAMES CAMPBELL IRONS, Dr JOHN GLAISTER, Mr F. C. CRAWFORD, and Mr W. ALLAN CARTER were admitted Fellows of the Society.

The following Communications were read :—

1. On Thermodynamics, founded on Motivity and Energy. By the PRESIDENT. pp. 126–130.

2. An Investigation of the Microscopical Appearances of the Grains in the more commonly occurring Starches. (*With Lantern Illustrations.*) By HUGH GALT, Esq., M.A., M.D. Communicated by Professor M'KENDRICK.

3. On Methods of Mapping Rainfall. By A. J. HERBERTSON, Esq.

4. The Normal Rainfall of India and the Abnormalities in 1896. By A. J. HERBERTSON, Esq.

NINTH ORDINARY MEETING.

Monday, 4th April 1898.

Professor Copeland, Vice-President, in the Chair.

Mr CECIL CARUS WILSON, Mr ALEXANDER VEITCH LOTHIAN, and Mr S. C. MAHALANOBIS were admitted Fellows of the Society.

At the request of the Council, an Address on "Theories concerning the Structure and Origin of Coral Reefs and Islands" (*with Lantern Illustrations*), was given by Dr JOHN MURRAY.

Mr W. COSSAR MACKENZIE and Dr T. H. BRYCE were balloted for, and declared duly elected Fellows of the Society.

TENTH ORDINARY MEETING.

Monday, 2nd May 1898.

Dr Munro in the Chair.

The following Communications were read :—

1. On Consonant Sounds. By R. J. LLOYD, D.Lit., Hon. Reader in Phonetics at the University College, Liverpool. pp. 219–247.

2. On a Supposed Resemblance between the Marine Faunas of the Arctic and Antarctic Regions. By Professor D'ARCY W. THOMPSON. pp. 311–349.

3. The Distribution of Mean Monthly and Annual Rainfall over the Land Surface of the Globe, illustrated by Thirteen New Maps. By Mr ANDREW J. HERBERTSON.

Mr WILLIAM ARCHER TAIT, Dr ALBERT A. GRAY, and Mr ALEXANDER W. ROBERTS were balloted for, and declared duly elected Fellows of the Society.

ELEVENTH ORDINARY MEETING.

Monday, 16*th May* 1898.

The Hon. Lord M'Laren, Vice-President, in the Chair.

Mr W. Owen Williams and Dr Albert A. Gray were admitted Fellows of the Society.

The following Communications were read :—

1. Guesses as to the Origin of some of the Characters in the Phœnician Alphabet. By Professor Crum Brown.
2. On the Absolute Thermal Conductivity of Nickel. By Mr T. C. Baillie. Communicated by Professor Tait. *Trans.*, xxxix. pp. 361-382.
3. The Emblem of the Crab in relation to the Sign Cancer. By Professor D'Arcy Thompson. *Trans.*, xxxix. pp. 603-611.

TWELFTH ORDINARY MEETING.

Monday, 6*th June* 1898.

The Hon. Lord M'Laren, Vice-President, in the Chair.

The following Communications were read :—

1. The Strains produced in Iron, Steel, Nickel, and Cobalt Tubes in the Magnetic Field. By Dr Knott. *Trans.*, xxxix. pp. 457-490. (*Abstract*) pp. 216-218.
2. On the further Anatomy and Budding Processes of *Cephalodiscus dodecalophus*. By A. T. Masterman, D.Sc. *Trans.*, xxxix. pp. 507-527.
3. A new Silurian Scorpion, and some additional Eurypterid Remains, from the Pentland Hills. By Malcolm Laurie, B.Sc. *Trans.*, xxxix. pp. 579-590.
4. On the Theory of Archimeric Segmentation and its bearing upon the Phyletic Classification of the *Cœlomata*. By Dr Masterman. pp. 270-310.

THIRTEENTH ORDINARY MEETING.

Monday, 20*th June* 1898.

Sir William Turner, Vice-President, in the Chair.

The following Communications were read :—

1. On Steam and Brines. By J. Y. Buchanan, M.A., F.R.S. *Trans.*, xxxix. pp. 529-573.

2. On Torsional Oscillations of Wires—(*a*) Experimental, (*b*) Theoretical. By W. Peddie, D.Sc. (*Abstract*) pp. 212–215. *Trans.*, xxxix. pp. 425–455.

3. Crystalline Hydrates of Sodium Thiosulphate. By W. W. Taylor, M.A. Communicated by Professor Crum Brown. pp. 248–251.

4. The Metabolism of the Nucleins under Physiological and Pathological Conditions. By T. H. Milroy, M.D., and J. Malcolm, M.B. Communicated by Professor Rutherford.

5. On Electrolytic Conduction. By Professor Tait.

FOURTEENTH ORDINARY MEETING.

Monday, 4th July 1898.

The Hon. Lord M'Laren, Vice-President, in the Chair.

The Gunning Victoria Jubilee Prize for 1893–96 was presented to John Aitken, Esq., for his varied and important Researches in the Physics of Meteorology.

The Chairman, on presenting the Prize, said :—

Mr John Aitken's outstanding contribution to science is his paper "On Dust, Fogs, and Clouds," which was published in the Society's *Transactions* (vol. xxx. pp. 337–368). Remarkable originality and ingenuity was shown in devising the apparatus which revealed and displayed the fact that the atmosphere everywhere contains small particles of dust, which serve as nuclei in the condensation of the aqueous vapour of the air into cloud and rain. It was shown at the same time that there are always present in the atmosphere great quantities of chloride of sodium and other kinds of dust, which, from their affinity for water, cause condensation to take place in unsaturated air, thus producing dry fogs, when they happen to be present in great quantities. But as regards those dust particles which have no affinities with water, condensation is delayed till supersaturation begins, and wet fogs are produced; and further, when this stage has been reached, there is a tendency to inequality in the size of the cloud particles which determines the fall of some of them through the others, and thus rain follows.

The great density of town fogs is occasioned by the fact that many of the products of combustion have strong affinities with water, thus favouring the generation of dry fogs.

It is shown by Mr Aitken's dust-counters that the numbers of dust particles in the atmosphere are subject to extreme variation—the numbers varying from almost *nil* at the Ben Nevis Observatory in certain types of weather to 100,000 in a cubic inch. Under Mr Aitken's direction the numbers of dust particles have been for some years observed regularly at this Observatory, and the importance of the results cannot be over-estimated in their bearings on the cyclones and anti-cyclones of North-Western Europe. This great discovery is leading the way in other important inquiries in meteorology, of which may be mentioned the formation of clear spaces in dusty air (*Transactions*, vol. xxxii. p. 239) in connection with the sudden and frequent hygrometric changes in the atmosphere, as shown by continuously self-recording hygrometers.

His series of papers in the *Proceedings* on "Thermometer Screens" is a valuable contribution to practical meteorology. Meteorologists are enabled by the methods proposed to make the closest approximation yet possible to an observation of the true temperature of the air.

Another very suggestive paper is the "Note on Hoar Frost" (*Proceedings*, vol. xiv. p. 121), in which such extraordinary accretions of ice as occur so frequently at the Ben Nevis Observatory on the windward side of posts and other objects exposed in the wind during certain types of weather are explained.

The splendid researches, thus briefly alluded to, form only a small fraction of Mr Aitken's work,—which includes valuable contributions to the physical explanation of dew; the effects of oil on stormy seas; of the moon on weather; sunsets; chromomictors; solar radiation; breath figures, etc. etc.

Professor JAMES EDWARD TALMAGE and Mr CHARLES TWEEDIE were admitted Fellows of the Society.

At the request of the Council, the ASTRONOMER ROYAL FOR SCOTLAND gave an Address on "The Total Solar Eclipse of 21st January 1898, with some account of Solar Observations generally." (*With illustrative Photographs.*)

The following Communications were then read:—

1. On a new Species of *Cephalaspis* found by the Geological Survey of Scotland in the Old Red Sandstone of Oban. By Dr R. H. TRAQUAIR, F.R.S. *Trans.*, xxxix. pp. 591–593.

2. On *Thelodus Pagei* (Powrie) from the Old Red Sandstone of Forfarshire. By the Same. *Trans.*, xxxix. pp. 595–602.

3. Report on Fossil Fishes collected by the Geological Survey of Scotland in the Upper Silurian Rocks of the Lesmahagow District. By the Same. *Trans.*, xxxix. pp. 827–864.

4. On the effect of Mixed Diet as regards Salivary Digestion. By Dr W. AITCHISON ROBERTSON. pp. 209–211.

Mr JOHN FINDLAY was balloted for, and declared duly elected a Fellow of the Society.

FIFTEENTH ORDINARY MEETING.

Monday, 18th July 1898.

The Hon. Lord M'Laren, Vice-President, in the Chair.

The following Communications were read:—

1. The Electrolysis of Ethyl Potassium Diethoxysuccinate. By Professor CRUM BROWN and Dr H. W. BOLAM. pp. 255–257.

2. The Freezing Point of Aqueous Solutions of Sodium Mellitate. (Preliminary Note.) By W. W. TAYLOR, M.A. Communicated by Professor CRUM BROWN. pp. 252–254.

3. On the Survey by the s.s. "Britannia" of the Cable Route between Bermuda, Turk's Islands, and Jamaica. By R. E. PEAKE, M.Inst.C.E. With Descriptions by Sir JOHN MURRAY, K.C.B., F.R.S., of the Marine Deposits brought home. Communicated by Sir JOHN MURRAY. pp. 409–429.

4. Notes upon some of the Deposits collected by the German ship "Gazelle" in 1874 and 1875. By Sir JOHN MURRAY, K.C.B., F.R.S.

5. Notes on Coral Reefs at Port Louis and Grand Port, Mauritius. By WILLIAM SHIELD, M.Inst.C.E.

6. Note on finding the log. sines and log. tangents of Small Arcs, etc. By Dr JAMES BURGESS. pp. 265–269.

7. Generalisation of Josephus' Problem. By Professor TAIT. pp. 165–168.

8. On the Compressibility of Sugar Solutions. By the Same. pp. 359–360.

The Chairman closed the Session with the following remarks:—

In the Review of the Session which I have been called upon to give, I shall confine my survey to statistics and a few general remarks.

I find that during the Session now brought to a close, 67 papers were read. Of these 16 belonged to the department of Physics, 6 to that of Mathematics, 1 to Astronomy, 6 to Chemistry, 2 to

Mineralogy, 5 to Geology, 4 to Physical Geography, 4 to Meteorology, 10 to Zoology, 1 to Botany, 9 to Physiology, 1 to Anatomy, 1 to Philology, and 1 to Archæology.

The President has favoured us with papers on Cosmic Physics and Electrodynamics, while the Physical and Chemical Laboratories of the Universities of Edinburgh and Glasgow have furnished important contributions to our publications, nor have the Physiological Laboratories of these Universities been less assiduous in giving us communications of great interest and value. A distinguished Fellow of the Society at the Cape of Good Hope still contributes papers on the Higher Algebra. The proximity of the Firth of Forth has led to communications on the structure and physiology of the organisms found within its waters; whilst the able officials of the Geological Survey of Scotland have kept a successful outlook for the traces of extinct organisms impressed on the rocks which come under their notice in the course of their operations, and have handed the fossils, thus found, for classification to an expert, who has described them to the Society. Sir JOHN MURRAY favoured us with an Address on the Structure and Origin of Coral Islands, while his example has led other naturalists to give papers on Antarctic Zoology and Atlantic Deposits. The Astronomer-Royal for Scotland illustrated, from personal observations, the grand phenomena of the Eclipse of the sun of 22nd January last; nor must I omit to mention that we have had a paper partly scientific and partly literary on the Crab in relation to Cancer in Mythology; and Professor CRUM BROWN, whose versatility we often have had occasion to admire, gave us a literary paper on the origin of some of the characters of the Phœnician Alphabet.

During the Session we have added to our number twenty-four Ordinary Fellows. Of these—two are Principals of Colleges, one a Professor, five are University Lecturers, and three are Doctors of Medicine.

Whilst welcoming the additions to our ranks, we have at the same time to lament the loss of ten Fellows who have been taken from us by death. These include—

Mr JAMES SYME, who for many years discharged the responsible duties of the office of General Manager of the British Linen Company's Bank.

Professor CALDERWOOD, who was somewhat unexpectedly elected to the Chair of Moral Philosophy in our University, but discharged its duties with great efficiency and success; and has written and edited several important works on mental science.

Professor HEDDLE, who contributed to our *Transactions* eight elaborate papers on the Mineralogy of Scotland; and whose extensive collection of Minerals, to the formation of which he had devoted the labour of a lifetime, now constitutes one of the treasures of the Edinburgh Museum of Science and Art.

ROBERT WILSON, who possessed great mathematical ability, and contributed a paper to the *Proceedings* of this Society on the Contact of Surfaces, in which he employed the resources of geometry in the elucidation of that difficult subject.

The Hon. BOUVERIE PRIMROSE, who for many years was Secretary to the Board of Manufactures, and evinced the interest he took in the prosperity of this Society, by bequeathing to it a sum of £200.

Sir JAMES BAIN, who held the office of Lord Provost of Glasgow, and took with him on his retirement from the civic chair of that great city a large measure of personal and official popularity.

Lord PLAYFAIR, who for eleven years held the Chair of Chemistry in the University of Edinburgh, relinquishing it for a parliamentary career, and who, as a legislator, advocated every useful measure for the promotion of science and education, as well as political and social progress.

I cannot conclude without mentioning that, during the Session, one of our Fellows has been made a Knight Commander of the Bath, and Member of the Prussian Order Pour le Mérite; another has received a Knighthood; and a third has been made a Companion of the Bath.

Meetings of the Royal Society—Session 1898-99.

The 116th Session.

GENERAL STATUTORY MEETING.

Monday, 28th November 1898.

The following Council were elected:—

President.

The Right Hon. Lord KELVIN, G.C.V., LL.D., D.C.L., F.R.S.

Vice-Presidents.

The Rev. Professor Flint, D.D.
Professor John G. M'Kendrick, M.D., LL.D., F.R.S.
Professor George Chrystal, LL.D.
Sir Arthur Mitchell, K.C.B., LL.D.
Sir William Turner, M.B., LL.D., F.R.S.
Professor Ralph Copeland, Ph.D.

General Secretary—Professor P. Guthrie Tait.

Secretaries to Ordinary Meetings.

Professor Crum Brown, F.R.S.
Sir John Murray, K.C.B., LL.D., F.R.S.

Treasurer—Philip R. D. Maclagan, F.F.A.

Curator of Library and Museum—Alexander Buchan, M.A., LL.D., F.R.S.

Councillors.

Professor D'Arcy W. Thompson, C.B.
The Rev. Professor Duns, D.D.
Lieut.-Col. Frederick Bailey (*late*), R.E.
Professor James Geikie, LL.D., F.R.S.
Sir John Batty Tuke, M.D., D.Sc.
A. Beatson Bell, Advocate.
Professor A. Shield Nicholson, M.A., D.Sc.
Professor John Gibson, Ph.D.
The Hon. Lord M'Laren, LL.D., F.R.A.S.
C. G. Knott, D.Sc.
Dr Alexander Bruce, M.A., F.R.C.P.E.
James A. Wenley.

By a Resolution of the Society (19th January 1880), the following Hon. Vice-Presidents, having filled the office of President, are also Members of the Council:—

His Grace The DUKE of ARGYLL, K.G., K.T., LL.D., D.C.L.
Sir DOUGLAS MACLAGAN, M.D., LL.D., F.R.C.P.E.

FIRST ORDINARY MEETING.

Monday, 5*th December* 1898.

The Rev. Professor Flint, Vice-President, in the Chair.

1. The Chairman, on opening the Session, made the following Statement:—

I have to congratulate the Society on the large number of papers—most of them involving patient and profound research—that were communicated to it during last Session. It will be remembered that some of them dealt with subjects appertaining to Egyptian and Phœnician archæology.

I have also to congratulate the Society on the increase of its members that has taken place during the last decade. In 1889 there were 487 members on the roll, in 1898 there were 517. The Fellows of the Society will have been gratified to observe that three of their number of last Session have received well-merited honours from the Queen.

A Fellow of this Society, Dr Thomas R. Fraser, Professor of Materia Medica and Clinical Medicine in the University of this City, has been appointed President of a Commission to report upon the plague in India, and several scientific experts have been associated with him. The members of the Commission were expected to reach Bombay towards the end of last month. Their inquiries will include—"The origin of the different outbreaks of plague; the manner in which the disease is communicated; and the effects of certain Prophylactic and Curative Serums that have been tried or recommended for the disease." They have, I am sure, the most ardent wishes of this Society for their success in the supremely important investigations which have been devolved upon them. May Medical Science, as represented by them, gain another of those victories over ignorance and disease which are the glory of its history.

I have much pleasure in announcing that the Queen has been pleased, on the recommendation of the Secretary for Scotland, to approve of the appointment of Professor D'Arcy Thompson, of University College, Dundee, to the office of scientific member of the Fishery Board for Scotland, vacant by the resignation of Sir

John Murray. Professor Thompson, as you are aware, was the British Delegate on the Behring Sea Fisheries, and was recently made a C.B. in recognition of his service.

Mr Mackay Bernard, one of our Fellows, merits the thanks of all interested in meteorology in Scotland, by preventing, through a liberal donation of £500, the closing of the Ben Nevis Observatories.

One of our Secretaries, Sir John Murray, in conjunction with other scientists, has zealously urged on Government the desirability of undertaking an antarctic expedition in the interests of science, and on other grounds; or, at least, of co-operating in such an undertaking by national subscription. Considering what additions to our knowledge in geology, natural history, meteorology, and other departments of science may confidently be looked for from an exploration of the vast and almost unknown antarctic region; and considering also what obviously special claims such an enterprise should have on Great Britain as the first naval and maritime power in the world; it may well be regretted that the efforts of Sir John Murray, and of those who have co-operated with him, should have been hitherto unable to overcome the *vis inertiæ* of the Government.

Possibly the Government may have had satisfactory reasons of a temporary kind for its decision, and if so, let us hope that it may reconsider it. Possibly it may have been influenced by the humanitarian consideration of the danger of loss of valuable life. But even that consideration, worthy of all respect as it is, may have an exaggerated weight attached to it; and further, it is, I believe, quite certain that the risk of loss of life in such an expedition as that proposed is very much less than it was not many years ago. Besides, ought not a government or national expedition to reduce unnecessary risk to a minimum, and be a guarantee that it would be conducted by the best attainable men—men known to be courageous without being foolhardy. Would the Government impartially re-consider its decision it could hardly fail, I think, to come to the conclusion that the balance of reason and of true and patriotic policy is distinctly in favour of the view urged on it by the scientists.

Although Sir John Murray, however, has not got things his own way with the Government, he is by no means to be thought of as, on the whole, a disappointed or unsuccessful man. An expedition, which he organised himself, at his own expense, to the islet known as Christmas Island, has produced admirable results. The island, an upraised coral atoll, like those on which Darwin based his view of the formation of coral islands, is about 200 miles South of Java; the depth of the ocean around it is of 3 and 4 miles; its area is about 45 square miles; it is covered by a dense forest, but is perfectly healthy; and its temperature is so equable and perfect that it might seem to have been in the mind's eye of the poets who have described the Islands of the Blessed.

It has a special interest to the Biologist owing to being the only tropical island which, until some eight years ago, had never been inhabited by either savage or civilised man. Hence it was highly desirable to have a complete scientific account of its fauna and flora before the arrival of introduced species. Mr Andrews of the British Museum has spent a year on the island, and has brought back large collections; and naturalists are looking forward with very great interest to his account of his explorations and description of his specimens. The first set of the natural history collections made by Mr Andrews has been presented to the British Museum by Sir John Murray.

The thread of my discourse takes me to another island—one with the exploration of which this Society has been intimately associated.

The British Museum authorities have organised a Scientific mission to the island of Socotra with these objects in view—(1) to make a careful survey of the island, (2) to solve the question as to the origin of the people and of their language, and (3) generally, to add as much as possible to what has been already ascertained as to its geology, geography, ethnography, botany, and zoology.

It will be remembered that Professor BAYLEY BALFOUR, under the auspices of this Society, visited the island in 1880 for the purpose of investigating its botany; and that although his time was very limited, extending only to 48 days, the results were extraordinarily rich. It so happened that in the spring of the following year a German scientific expedition—known as the "Riebeck

Expedition"—also explored the island; but that only turned out to the advantage both of Professor Bayley and our Society. Professor Bayley had given kindly counsel and aid to Dr Schweinfurth, the botanist of the Riebeck expedition, and Dr Schweinfurth, with rare self-abnegation and generosity, responded by subsequently sending his botanical collections to Professor Bayley, in order that the whole flora might be worked out in one. An entire volume of our *Transactions* (vol. xxi., published in 1888) is devoted to the Botany of Socotra. That volume contains Schweinfurth's botanical results worked up with those of the Balfour expedition, and nearly all that is known of the botany of Socotra, as the expedition of Mr and Mrs Bent, a couple of years ago, added little to it.

MM. OGILVIE GRANT and H. O. FORBES of the new expedition have just gone out, and will remain for five months. By the new expedition the survey question will, it is believed, be fairly tackled for the first time. The ethnographic question will be dealt with, and an attempt made to advance towards the settlement of it farther than Schweinfurth. As to the geology and biology of Socotra, previous expeditions have given definite knowledge, but further exploration will, of course, add much to it in the way of details. It is thought unlikely, however, that there will be anything so startling as the disclosures which the explorations of twenty years ago furnished. Dr Forbes, the botanist of the party, will endeavour to send home seeds and living plants of several species, valuable from a horticultural point of view. The *Begonia Socotrana* brought home by Balfour's expedition is now in general culture in Europe, and, as the parent of an entirely new race, has revolutionised Begonia culture.

In conclusion, Dr TRAQUAIR, from whom the Society has received so many valuable contributions, read in July last papers on "A new species of Cephalaspis discovered by the Geological Survey in the Old Red Sandstone of Oban"; and on "*Thelodus Pagei*, Powrie, from the Old Red Sandstone of Forfarshire"; as also a "Report on Fossil Fishes collected by the Gelogical Survey of Scotland in the Upper Silurian Rocks of the Lesmahagow District."

The last of these especially was of very great interest and importance. Fishes or Fish-Remains had not previously been recorded from the Silurian Rocks of Scotland, but the officers of the Geological Survey had, during the past season, brought together a large collection of well-preserved fossil fishes from rocks of the Silurian age in the Lesmahagow district. These were all new to science, and of the greatest possible interest from a biological as well as from a geological point of view. These fishes, having been intrusted by the Director-General of the Survey to Dr Traquair, formed the subject of the "Report," which, along with the other two Papers mentioned, will presently be published in the *Transactions* of the Society.

It being usual for the chairman of the Opening Meeting to advert briefly to each of the members of the Society who have died during the preceding Session, I will give a few short notices of the deceased, which are by no means intended to supersede more elaborate obituary notices, which I hope we will receive.

Brigade Surgeon **James Edward Tierney Aitchison** was the son of the late Major J. Aitchison, H.E.I.C.S., and was born in 1835. After studying at Edinburgh University, and graduating M.D. and L.R.C.P. in 1856, he entered the Bengal medical service in 1858, in which he remained for thirty years. He obtained the qualification of F.R.C.S. Edinburgh in 1863. In 1878 he served in the Afghan war. Edinburgh University conferred on him the degree of LL.D. in 1889.

He was distinguished as a botanical explorer, and during the last thirty years added much to our knowledge of oriental floras. In 1878 Lord Roberts, then in command of the Kuram field force, applied to the Government for his services as botanist with the Army. There afterwards appeared in the Journal of the Linnean Society in 1880 and 1881, two papers detailing his discoveries and observations in the valleys of the Kuram and Koraia rivers.

In 1884 Dr Aitchison was appointed scientific officer with the Afghan Delimitation Commission, and spent two years in Northern Baluchistan, the Helmund Valley, and the other districts in Central Asia traversed by the Commission. The results of these explorations were embodied in a paper printed by the Linnean Society in their Transactions for 1887. Dr Aitchison was an indefatigable collector. His Afghan and Central Asian collections amounted to more than 20,000 specimens of plants which were distributed to Kew and the other principal herbaria of the world. He was accustomed to add to them valuable original observations regarding their economic value. He was elected a Fellow of our Society in 1881, and a Fellow of the Royal Society of London in 1883. He died at Priory Terrace, Kew Green; on 30th September 1898.

Dr **John Caird** was a native of Greenock, born in December 1820. He studied at Glasgow University, and in 1845 graduated M.A. In the same year he was ordained minister of Newton-on-Ayr; in 1847 he was transferred to Lady Yester's Parish Church, Edinburgh; again, after an interval of two years, he became minister of the Parish of Errol; and in 1857 he was called to Park Parish Church, Glasgow. In 1860 the degree of D.D. was conferred on him by his Alma Mater; in 1862 he was appointed Professor of Divinity in Glasgow University; and in 1873, on the death of Dr Barclay, he became Principal and Vice-Chancellor of the College.

Dr Caird was one of the greatest preachers of this century. He had in a high degree, and in admirable combination, all the qualities of an attractive and impressive pulpit orator. He had also exquisite literary taste, and hence even his printed sermons have had great popularity.

He was likewise an independent and original thinker; and a stimulating and influential teacher of theology. His "Introduction to the Philosophy of Religion" and his volume on "Spinoza" attest his philosophical ability.

As Principal of the University of Glasgow he displayed much administrative capacity, and was greatly respected and admired.

He had a very distinct personality; one characterised by simplicity and strength, self-restraint, dignity, and susceptibility only to ambition of a noble kind.

His death occurred on the very day when his resignation of the Principalship, which he had held for a quarter of a century, took place, and on the 3rd of August last he was laid to rest in the town of his birth among many tokens of sorrow and regard.

Dr Caird was elected an Honorary Fellow of this Society in 1897.

Henry Calderwood was born on the 10th of May 1830 at Peebles, and was educated at the Royal High School and the

University of Edinburgh. Having studied for the ministry of the United Presbyterian Church, he was ordained pastor of the Greyfriars Church, Glasgow. This post he held till his appointment, in 1868, to the Chair of Moral Philosophy in the University of Edinburgh. Previously to that appointment, from 1861 to 1864, he was Examiner in Philosophy in the University of Glasgow, and in 1866, at the request of the Senatus, he conducted the class of Moral Philosophy in that University. At the early age of twenty-four, he published a work on the "Philosophy of the Infinite." From that time to this, he has given to the world a number of books, including "A Handbook of Moral Philosophy," a work "On the Relations of Mind and Brain," another entitled "Science and Religion." His last important work was on "The Evolution of Man's Place in Nature." In 1880 he was elected Moderator of the United Presbyterian Synod, and was the first chairman of the Edinburgh School Board. He took a deep interest in the movement for uniting the three Presbyterian Churches of Scotland. He received the degree of LL.D. from the University of Glasgow. He was elected a Fellow of this Society in 1869, and died on 19th November 1898.

Dr Calderwood was held in high esteem by all classes and conditions of men, and was active in the most varied spheres of usefulness. It was a necessity of his life to be zealously affected in whatever he deemed good. He was a most considerate and loyal colleague. He was a stout contender for what he held to be philosophical truth. He had greatly at heart educational progress and social reform. Few men took a broader or more sensible view of the relations between religion and science. He was an active, courageous, and patriotic citizen. He was a singularly fair-minded and peace-loving churchman. He was, emphatically, and in all relations, a wise and good man, whose memory well deserves to be cherished.

Dr HENRY MARSHALL was born at Clifton, Bristol, and received his early education at the old Bishop's College, Bristol, and subsequently went to the University of Edinburgh. In 1854 he was dresser, and afterwards assistant, under Professor Lister (now Lord Lister), whom Dr Marshall succeeded as house-surgeon. Dr Marshall became M.R.C.S. England in 1854, a year later obtained the degree of M.D. Edinburgh, and in 1859 became F.R. Edinburgh. He was surgeon to the Bristol General Hospital ten years, and on his resignation he became consulting surg He also lectured on Medical Jurisprudence at the Bristol Med School. He was elected a Fellow of our Society in 1869, and d on 24th April 1898.

Emeritus Professor MATTHEW FORSTER HEDDLE was born Malsettar, in Hoy, one of the Orkney Islands. He was educa at Edinburgh Academy, and subsequently at Merchiston Cas where he gained a school-prize for a herbarium, the plants of wh he had collected. On leaving school he went through the medi curriculum in the University of Edinburgh, and in 1851 he gra ated M.D., his thesis being "The Ores of the Metals." He t commenced practice in Edinburgh, but having found medical w uncongenial, he abandoned it. Soon after this, in 1856, he w to Faroe, and collected a large quantity of fine zeolites. In 1 Dr Heddle became assistant to the Professor of Chemistry at Andrews, and in 1862 was appointed to the Professorship. 1883 he relinquished his professorial duties and went to So Africa to report on certain mining possibilities there. He con buted to the Mineralogical Magazine a series of papers on " Geognosy of Scotland," and eight papers to the *Transactions* of Society, entitled "Chapters on the Mineralogy of Scotland." great collection of Scottish Minerals, the outcome of the labou a lifetime, became, partly by purchase and partly by gift, property of the nation, and is now displayed in the Edinbu Museum of Science and Art. He was elected a Fellow of Society in 1876, and died on 19th November 1897.

LYON PLAYFAIR was born at Meerut on 21st May 1819. was the second son of Dr George Playfair, Inspector-General Hospitals in Bengal. He studied at St Andrews, and afterwa at Glasgow. He afterwards studied chemistry at Univers College, London, under Graham, from whom he had previou taken lessons at the Andersonian Institution in Glasgow. Then went to Giessen to learn organic chemistry under Liebig. On return he managed extensive calico-printing-works at Clither In 1843 he went to Manchester, where he was appointed Profes of Chemistry at the Royal Institution. In recognition of services on a Commission to inquire into the sanitary condition of large towns, he was appointed in 1846 Chemist to the Museum of Practical Geology.

He was appointed Special Commissioner in the Department of Juries in the Great Exhibitions of 1851 and 1862, and for his services in the first of these he was rewarded with the Companionship of the Bath, and an appointment in the household of Prince Albert. In 1856 he became Professor of Chemistry in the University of Edinburgh, where the Prince of Wales and Prince Alfred were among his pupils. This post he held till 1869. Prior to 1862 he had been appointed Inspector-General of Government Museums and Schools of Science. A scheme for the reorganisation of the Civil Service, which bears his name, was the outcome of a Commission appointed in 1884, of which Playfair was the President.

At the general election of 1868 Playfair was returned to Parliament by the Universities of Edinburgh and St Andrews, and he retained the confidence of the constituencies for seventeen years. He was Postmaster-General in 1873, and in 1880 he was appointed Chairman of Ways and Means. That office he resigned in 1883, and was then made K.C.B. In 1885 and 1886 he was returned for the Southern Division of Leeds, and became Vice-President of the Council, and represented the constituency until he became a Peer in 1892. He was the author of a work which appeared under the title of Subjects of Social Welfare. He was elected a Fellow of this Society in 1859, and died on 29th May 1898.

The Honourable BOUVERIE FRANCIS PRIMROSE was the second son of the fourth Earl of Rosebery, and was uncle of the present Earl. He was born on the 19th September 1813, and educated at Trinity College, Cambridge. He rose through the several grades of the City of Edinburgh Rifle Volunteer Brigade to be Lieutenant-Colonel of the second battalion of that regiment. He was also a General in the Royal Company of Archers. In 1839 he received the appointment of Receiver-General of the Post Office in Scotland, which he resigned when he became Secretary to the Boards of Manufactures and Fisheries in Scotland. One of the leading events in his tenure of that office was the equipment of the National Gallery, which involved complicated negotiations and the expenditure of about £50,000. A large share of the labour connected with this work fell to him as Secretary. He also took a great interest in the Scottish Fisheries, and it was upon the passing of the Bill which established the Fishery Board for Scotland in 1882 that he retired from the position of Secretary of the joint Boards of Manufactures and Fisheries. He was elected a Fellow of this Society in 1849, and bequeathed to it a sum of £200. He died on 20th March 1898.

I cordially thank all those from whom I have received information contained in the statement now read, and above all, as having, of course, aided me most, our ever kind and zealous Librarian, Mr Gordon.

2. On the Miscibility of Liquids at Different Temperatures. By Professor Kuenen.
3. On Reversion in Birds and Mammals. (*With Lantern Illustrations.*) By Professor Ewart.

Mr James R. Appleyard, Mr James Chatham, Mr T. E. Gatehouse, Dr Ewen John Maclean, Dr E. W. W. Carlier, and Dr T. H. Milroy were balloted for, and declared duly elected Fellows of the Society.

SECOND ORDINARY MEETING.

Monday, 19th December 1898.

The Right Hon. Lord Kelvin, President, in the Chair.

Dr T. H. Milroy and Mr James Chatham were admitted Fellows of the Society.

The following Communications were read:—

1. Crane Scaffoldings:—their Mathematical Calculation and Probable Theoretical Deficiencies. By R. G. Alford, M.Inst.C.E. (*With Lantern Illustrations.*)
2. Note on Dew-Bows. By Drs C. G. Knott and R. A. Lundie. pp. 350–352.
3. On the Reflection and Refraction of Solitary Plane Waves at a Plane Interface between two Isotropic Elastic Mediums, Fluid, Solid, or Ether. By the President. pp. 366–379.
4. Equilibrium between Sulphuric Acid and Sulphates in Aqueous Solution. By Sydney A. Kay, B.Sc. Communicated by Professor Walker. pp. 484–522.
5. Dust Figures of Electrostatic Lines of Force. By David Robert-

SON, Esq. (*With Lantern Illustrations.*) Communicated by Professor JAMIESON, M.Inst.C.E. pp. 361–365.

6. On the Summation of the Series whose *n*th terms are u_n and $1/u_n$, where u_n denotes $(an+b)\left\{a(n+1)+b\right\}\ \left\{a(n+2)+b\right\}\ \ldots\ \left\{a(n+r-1)+b\right\}$. By Professor ANGLIN.

7. The action of Persulphates on Iodine. By HUGH MARSHALL, D.Sc. pp. 388–390.

THIRD ORDINARY MEETING.

Monday, 9th January 1899.

Sir William Turner, Vice-President, in the Chair.

The following Communications were read:—

1. Determination of the Sign of a Single Term of a Determinant. By THOMAS MUIR, LL.D. pp. 441–477.
2. The Energy of the Röntgen Rays. By the Rev. ALEXANDER MOFFAT, M.A., B.Sc. Communicated by Dr C. G. KNOTT. pp. 430–438.
3. On the Development and Morphology of the Marsupial Shoulder-Girdle. By Dr ROBERT BROOM. Communicated by Sir WILLIAM TURNER. *Trans.*, xxxix. pp. 749–770. (*Abstract*) pp. 482–3.
4. On an Interpretation of the Hydrokinetic Equations. By Professor TAIT.
5. On the Eliminant of a Set of Quadrics, Ternary or Quaternary. By E. J. NANSON. Communicated by Professor CHRYSTAL. pp. 353–358.

Professor CRUM BROWN moved, in accordance with the notice given by him at the General Statutory Meeting:—

"That the Ordinary Meetings of the Society be held on the first and third Mondays of November, January, February, March, May, June, and July, and on the first Monday in December, with the exception that when there are five Mondays in January, the Meetings shall be held on the second and fourth Mondays of that month."

Dr HEPBURN seconded the motion. Sir JOHN MURRAY moved, and Dr BUCHAN seconded, "that Professor CRUM BROWN'S motion be referred to the Council, and that the proposal as to changes should come before the Society as a recommendation from the Council."

Dr THOMAS ISHERWOOD was balloted for, and declared elected a Fellow of the Society.

FOURTH ORDINARY MEETING.

Monday, 23*rd January* 1899.

Sir Arthur Mitchell, K.C.B., Vice-President, in the Chair.

The following Communications were read :—

1. Symmetrical Solution of the Ellipse-Glissette Elimination Problem. By the Hon. LORD M'LAREN. pp. 379–387.
2. Experimental Contributions to the Theory of Heredity :—Reversion, Part II. By Professor COSSAR EWART, F.R.S.

FIFTH ORDINARY MEETING.

Monday, 6*th February* 1899.

The Right Hon. Lord Kelvin, G.C.V.O., President, in the Chair.

Mr GEORGE ALFRED NASH was admitted a Fellow of the Society.

The following Communications were read :—

1. On Nernst's "Osmotic Experiment," and a Definition of Osmotic Pressure. By Professor CRUM BROWN. pp. 439, 440.
2. On a Nomenclature of the Anhydrides of Acids proposed by the late Professor Andrews. By the Same.
3. Application of Sellmeier's Dynamical Theory to the Dark Lines D_1, D_2, produced by Vapour of Sodium. By the PRESIDENT. pp. 523–531.
4. On the Motion of Liquid in an Ellipsoidal Hollow—a continuation of a paper in *Proc. Roy. Soc. Edin.*, December 7, 1885. By the Same.
5. The Rainfall of Australasia. Part I.—The Mean Annual Rainfall of Australia. By A. J. HERBERTSON and P. C. WAITE.
6. The Multiplication of an Alternant by a Symmetrical Function of the Variables. By THOMAS MUIR, LL.D. pp. 539–542.

Dr ALLAN M'LANE HAMILTON, Dr DAVID W. FINLAY, Mr JOSEPH SLATER LEWIS, Mr GEORGE DUTHIE, and Mr JAMES M'CUBBIN were balloted for, and declared duly elected Fellows of the Society.

FIRST SPECIAL MEETING.

Thursday, 9*th February* 1899.

Professor Copeland, Astronomer-Royal for Scotland,
Vice-President, in the Chair.

At the request of the Council, Vice-Admiral MAKAROFF, of the Imperial Russian Navy, gave an Address "On some Important

Oceanographic Problems, and Novel Modes of Research." (*With Lantern Illustrations.*) pp. 391–408.

A large Model of the Russian "Ice-breaker" *Ermack* was exhibited.

SIXTH ORDINARY MEETING.

Monday, 20th February 1899.

Professor Chrystal, LL.D., Vice-President, in the Chair.

The following Communications were read:—

1. Note on the Tidal Currents of the North Sea. By ALEXANDER BUCHAN, Esq., LL.D., F.R.S. pp. 478–481.
2. The Experimental Bases of Professor Andrews' Paper on the Continuity of the Gaseous and Liquid States of Matter (*Phil. Trans.*, 1869). By Professor TAIT.
3. Magnetic Twist in Nickel. By Professor KNOTT. pp. 586–595. By permission of the Meeting, the Communication not having been announced in the Billet.

SEVENTH ORDINARY MEETING.

Monday, 6th March 1899.

Professor M'Kendrick, M.D., Vice-President, in the Chair.

The following Communications were read:—

1. On the Convection of Heat, Part I.:—On Newton's Law of Cooling. By Professor A. CRICHTON MITCHELL. *Trans.*, xl.
2. The Meteorology of Ben Nevis. By Dr ALEXANDER BUCHAN.
3. Note on Polarisation Phenomena observed in Quantitative Electrolytic Determinations. By HUGH MARSHALL, D.Sc. pp. 532–538.
4. Changes in the Metabolism due to Fever. By Drs NOËL PATON, DUNLOP, and IVISON MACADAM.
5. (*a*) The First Foundation of the Lung of Ceratodus;
   (*b*) The Embryonic Excretory Organs of Ceratodus. By GREGG WILSON, D.Sc. Communicated by Professor J. COSSAR EWART, F.R.S.

Mr THOMAS S. GOODWIN, Mr R. TATLOCK THOMSON, and Mr EDWARD GRAHAM GUEST were balloted for, and declared duly elected Fellows of the Society.

EIGHTH ORDINARY MEETING.

Monday, 20*th March* 1899.

Professor Copeland, Vice-President, in the Chair.

Mr David Brown was admitted a Fellow of the Society.

The following Communications were read :—

1. On the Temperature over the Floor of the Ocean, and the Maximum and Minimum Temperature of the Surface of the Ocean. By Sir John Murray, K.C.B.
2. Contributions to the Theory of Heredity, II. :—Intercrossing and Variation. By Professor J. Cossar Ewart, F.R.S.
3. On a Development of a Determinant of the mn^{th} order. By Thomas Muir, LL.D. *Trans.*, xxxix. pp. 223–228.

NINTH ORDINARY MEETING.

Monday, 3*rd April* 1899.

The Rev. Professor Duns, D.D., in the Chair.

The following Communications were read.

1. On the Restoration of Co-ordinated Movements after Nerve Section. By Robert Kennedy, M.A., B.Sc., M.D. Glasgow. Communicated by Professor M'Kendrick. (*With Lantern Illustrations.*) *Trans.*, xxxix. pp. 685–702. (*Abstract*) pp. 636–640.
2. On the Rimes in the Authentic Poems of William Dunbar. By Henry Bellyse Baildon, M.A. Cantab. *Trans.*, xxxix. pp. 629–664.

Mr James Taylor was balloted for, and declared duly elected a Fellow of the Society.

TENTH ORDINARY MEETING.

Monday, 1*st May* 1899.

Professor Copeland, Vice-President, in the Chair.

The following Communications were read :—

1. On the Application of Force within a limited space required to produce Spherical Solitary Waves, or trains of Waves, of both or either Species, equivoluminal and irrotational, in an Elastic Solid. By the Right Hon. Lord Kelvin, G.C.V.O., President.

2. Preliminary Note of Experiments showing Heat of Combination in the formation of Alloys of Zinc and Copper to be negative when the proportion of Copper is less than about 30 per cent. By ALEXANDER GALT, Esq., D.Sc. pp. 619–621.

3. Changes that occur in some Cells of the Newt's Stomach during Digestion. By E. WACE CARLIER, M.D., B.Sc. pp. 673–691.

4. On the Leakage of Electricity from Charged Bodies at Moderate Temperatures. By Professor J. C. BEATTIE, D.Sc.

5. On the Anatomy of *Prosthecocotyle torulosa* (Linstow) and *Prosthecocotyle heteroclita* (Dies). By Dr O. FUHRMANN, University of Geneva. Communicated by Sir JOHN MURRAY, K.C.B. pp. 641–651.

6. On the Linear and Vector Function. By Professor TAIT. pp. 547–549.

The Right Hon. MITCHELL THOMSON, Lord Provost of Edinburgh, Dr ANDREW FREELAND FERGUS, and Mr W. LAMOND HOWIE were balloted for, and declared duly elected Fellows of the Society.

SECOND SPECIAL MEETING.

Monday, 8th May 1899.

The Rev. Professor Flint, D.D., Vice-President, in the Chair.

At the request of the Council, Mr CHARLES W. ANDREWS, of the British Museum, gave an Address on "The Exploration of Christmas Island, and in particular its Geological Structure." (*With Lantern Illustrations and Exhibition of Specimens.*)

ELEVENTH ORDINARY MEETING.

Monday, 15th May 1899.

Professor M'Kendrick, Vice-President, in the Chair.

The following Communications were read :—

1. Note on Magnetic Strains in Bismuth. By Dr C. G. KNOTT.

2. Fog-Bows, etc., seen at Ben Nevis since 1887. By R. T. OMOND, Esq.

3. Note on Fog-Bows. By Professor TAIT.

4. On a Practical Method of Enlarging and Deepening the Field of a Compound Microscope. By Mr W. FORGAN. Communicated by Dr ALEXANDER BRUCE.

5. On some Remains of Scottish Early Post-Pliocene Mammals. By the Rev. Professor DUNS. No. II. pp. 692–699.

TWELFTH ORDINARY MEETING.

Monday, 5th June 1899.

Mr A. Beatson Bell in the Chair.

The following Communications were read:—

1. Note on a Persymmetric Eliminant. By **Thomas Muir**, LL.D. pp. 543–546.
2. Contributions to the Life-Histories of the Cod and the Whiting. By **A. T. Masterman**, D.Sc. *Trans.*, xl.
3. The Hydrolysis of Thallic Sulphate. By **Hugh Marshall**, D.Sc. pp. 596, 597.

Mr **Alexander G. Ramage** was balloted for, and declared duly elected a Fellow of the Society.

THIRTEENTH ORDINARY MEETING.

Monday, 19th June 1899.

Sir William Turner, M.B., Vice-President, in the Chair.

Mr **Alexander G. Ramage** and Dr **T. H. Bryce** were admitted Fellows of the Society.

The following Communications were read:—

1. On the Eliminant of a Set of General Ternary Quadrics. By **Thomas Muir**, LL.D. *Trans.*, xxxix. pp. 667–684.
2. On the Structure and Affinities of a Lepidodendron Stem from the Calciferous Sandstone of Dalmeny. (*With Lantern Illustrations.*) By **A. C. Seward**, M.A., F.R.S., and **A. W. Hill**, B.A. Communicated by **R. Kidston**, F.G.S. *Trans.*, xxxix.
3. On Duplicitas Anterior in an early Chick Embryo. (*With Lantern Illustrations.*) By **T. H. Bryce**, M.A., M.B., Queen Margaret College, Glasgow University. pp. 622–630.
4. The Trap-Dykes of the Orkneys. (*With Lantern Illustrations.*) By **J. S. Flett**, M.A., B.Sc. *Trans.*, xxxix. pp. 865–905.
5. On the Vascular System of the Hypocotyl and Embryo of *Ricinus communis*. By **Edith Chick**, B.Sc., Quain Student, University College, London. Communicated by Sir **John Murray**, K.C.B. pp. 652–672.
6. Note on Mr J. O. Thompson's Paper on the Torsional Vibrations of Wires. By **W. Peddie**, D.Sc. pp. 598–600.
7. Obituary Notice of Major-General Gosset. By Mr **John Winzer**. pp. v., vi.

FOURTEENTH ORDINARY MEETING.

Monday, 3rd July 1899.

Professor Copeland, Astronomer-Royal for Scotland, Vice-President, in the Chair.

The following Communications were read :—

1. The Non-alternate ± Knots of the Tenth Order. By Professor C. N. LITTLE. Communicated by Professor TAIT. *Trans.*, xxxix. pp. 771–778.
2. Contributions to the Craniology of the People of the Empire of India. Part I.—The Hill Tribes of the North-East Frontier, and the People of Burma. By Professor Sir W. TURNER, F.R.S. *Trans.*, xxxix. pp. 703–747. (*Abstract*) pp. 550–552.
3. Decorated and Sculptured Skulls from New Guinea. By the Same. pp. 553–572.
4. An Improved Form of Craniometer for the Segmentation of the Transverse, Vertical, and Antero-posterior Diameters of the Cranium. By DAVID HEPBURN, M.D. pp. 601–618.
5. On the Meteorology of Ben Nevis in Clear and in Foggy Weather. By J. Y. BUCHANAN, M.A., F.R.S. *Trans.*, xxxix. pp. 773–826.

Dr E. H. SNELL was balloted for, and declared duly elected a Fellow of the Society.

THIRD SPECIAL MEETING.

Monday, 10th July 1899.

Professor Copeland, Vice-President, in the Chair.

Mr JAMES M'CUBBIN was admitted a Fellow of the Society.

At the request of the Council, Professor CARGILL G. KNOTT, formerly of the Imperial University of Japan, gave an address "On Earthquake Vibrations, their Propagation through the Earth, and their Bearing on the Question of the Earth's Internal State." (*With Lantern Illustrations.*) pp. 573–585.

FIFTEENTH AND LAST ORDINARY MEETING.

Monday, 17th July 1899.

The Hon. John Abercromby in the Chair.

Mr JAMES TAYLOR was admitted a Fellow of the Society.

PRIZES.

The Keith Prize for the period 1895–97 was presented to Dr THOMAS MUIR, for his continued communications on Determinants and Allied Questions.

The General Secretary read the following special account of the reasons for the award:—

In awarding the Keith Medal and Prize for a second time to Dr THOMAS MUIR, the Royal Society desires to express its sense of the importance of the continuation of Dr Muir's Researches in the Theory of Determinants and its Applications. Since 1883 he has enriched the *Proceedings* and *Transactions* of the Society with a continuous stream of papers, many of them containing new results of permanent importance, others co-ordinating, extending, and tracing to their sources theorems already known, and all of these marked by the peculiar analytical sagacity which is familiar to the reader of Dr Muir's mathematical writings.

Prominent among these memoirs is a series of papers on the *Theory of Determinants in the Historical Order of its Development.* The series contains the most complete bibliography of the subject in existence, accompanied by a critical analysis and comparison of the various memoirs from the dawn of the theory to its latest modern development. This monograph will form a monument of the learning and mathematical acumen of its author more enduring than the Keith Medal; for it cannot fail to remain a permanent chapter in the history of mathematics, which will no doubt be added to by future investigators, but which will never be wholly superseded.

The Makdougall-Brisbane Prize for the period 1896–98 having been awarded by the Council to Dr WILLIAM PEDDIE, for his papers on the Torsional Rigidity of Wires, the prize was presented, the General Secretary reading the following statement as to the reasons for its award:—

The Makdougall-Brisbane prize for 1896–98 is awarded to Dr WILLIAM PEDDIE in recognition chiefly of his experimental researches in physical science. In his investigations on the torsional rigidity of wires, he has studied the effect of large oscillations on the elasticity of the wire, thus supplementing in a very important direction Lord Kelvin's well-known results for small oscillations. In the second paper, communicated to the Society in 1896 (*Transactions*, vol. xxxviii.) the formula which had been found to represent with great accuracy the relation between range of oscillations and number of oscillations, was much more fully verified, and a relation was found to exist between two of the parameters under varied conditions of fatigue of the wire and initial ranges of oscillation. In the third paper (*Transactions*, vol. xxxix., 1898), the law of change of the parameters was further investigated, and an important common feature holding throughout the whole series of experiments was brought to light, indicating the existence of an absolute constant, probably characteristic of the material composing the oscillating wire. By a neat piece of statistical mathematics, Dr Peddie showed how not only the phenomena of torsional oscillations, but also other known phenomena, such as deviation from Hooke's Law and the relation between torsion and set, could be deduced theoretically as illustrative of Maxwell's views of the constitution of a molecular solid. In its application to vibrations this theoretical investigation required the time of outward swing through a given range to be less than the time of inward swing—the experimental verification of which was one of the new results contained in the paper.

In another contribution to the *Transactions* of the Society, Dr Peddie gave a very thorough investigation into an apparently unique case of colour-blindness. It was a case of what is termed yellow-blue or violet-blindness, but it differed from known examples of such cases in the total absence of appreciation of green, and in the fact that the visible spectrum suffered no shortening at the violet end.

Among other papers communicated to the Society and published in the *Proceedings*, the one on the Law of Transformation of Energy and its Applications calls for more than passing notice. In it various known expressions connecting physical quantities are deduced by simple reasoning based upon generalised Carnot's cycles—the usual methods of derivation of these results employing processes of higher mathematics.

The Neill Prize for the period 1895-98 was presented to Professor COSSAR EWART, for his recent investigations connected with Telegony.

Sir John Murray read the following statement as to the reasons for the award :—

The Council have awarded the Neill prize to Professor COSSAR

Ewart for a series of papers published partly in the *Proceedings* of the Society and partly elsewhere. In these papers Professor Ewart has recorded an elaborate series of experiments designed to investigate several questions of great importance bearing on the theory of heredity. More especially, he has inquired into the subject of Telegony, and as the result of his experiments in crossing several mares with a male zebra, and subsequently with a sire of their own stock, he considers that strong evidence is given against the theory of infection by a previous sire, which has been so strongly believed in by breeders of stock. He considers that the character observed in the offspring can be more satisfactorily explained by the simpler doctrine of Reversion or Atavism. Although reversion does not invariably occur, even when extreme forms are crossed, it is the usual result of intercrossing and hybrid breeding. His experiments also show that intercrossing leads to variation, and thus gives species a chance of adapting themselves to changes in the environment. He attaches great importance to the influence of prepotency, so that a distinct variety may not only hand on its own characteristics, but also absorb the chief points of the variety with which it is mated. In the course of these experiments he has obtained a number of horse embryos at various stages of development, and has in consequence been able to work out many interesting facts in the life-history of this animal.

The following Communications were read :—

1. On Magnetism and Molecular Rotation. By the President. pp. 631–635.
2. On a Bathymetrical Survey of the Scottish Fresh-water Lochs. By Sir John Murray and Mr F. P. Pullar, F.R.G.S.
3. A New Osteometric Board (with Exhibition). By Dr Hepburn.

Donations to the Library of the Royal Society from 1898 to 1899.

I. TRANSACTIONS AND PROCEEDINGS OF LEARNED SOCIETIES, ACADEMIES, etc.

Adelaide.—Royal Society of South Australia. Transactions and Proceedings. Vols. XXI. 2, XXII. 1898–99. 8vo.

Observatory. Meteorological Observations, 1894–96. 3 Vols. 4to.

American Association for the Advancement of Science.—46th Meeting (Detroit, Mich.). 1898.

Amsterdam.—Kon. Akademie van Wetenschappen. Verhandelingen. Afd. Natuurkunde. 1ste Sectie. Deel VI. 1898–99. 2te Sectie. Deel VI. 1897–98.—Afd. Letterkunde. Deel II. 1-2. 1898.—Verslagen en Mededeelingen.—Letterkunde. 4de Reeks. Deel I., II. 1897–98. 8vo. Verslagen der Zittingen van de Wis-en Naturkundige Afdeeling. Deel V. 1897. VI. 1898.—Jaarboek, 1897–98.—Proceedings of the Section of Sciences. Vol. I. 1899. 8vo. Poemata Latina.

Wiskundig Genootschap. Nieuw Archief voor Wiskunde. 2e Reeks, Deel IV. 1–3. 1899. Opgaven VII. 4–7, VIII. 1.—Revue Semestrielle des Publications Mathématiques. Tom. VI., VII. 1898-99.

Flora Batava. 319-326 Afleveringen. (*From the Dutch Government.*)

Kon. Zoologisch Genootschap "Natura Artis Magistra." 1838–98. 4to. 1898.

Australia.—Australasian Association for the Advancement of Science. Report, 7th Meeting (Sydney), 1898. 8vo.

Baltimore.—Johns Hopkins University. American Journal of Mathematics. Vols. XIX., 3, 4, XX., XXI., 1-3. 1898-99. 4to.—American Chemical Journal. Vols. XIX., 3–10, XX., XXI., 1–5. 1898-99.—American Journal of Philology. Vols. XVII., 4, XVIII., XIX. 1898-99.—University Studies in Historical and Political Science. Series XV.-XVII.—University Circulars. Nos. 121-130, 132-36, 139-141. 1898-90.—Memoirs from the Biological Laboratory. IV. 1, 2. 1899.

Johns Hopkins Hospital. Bulletin, Nos. 80-97. Reports, Vols. I., III., VI., VII. 1896-99.

Maryland Geological Survey. Publications. Vols. I., II. 1897-98.

Peabody Institute. Annual Reports, 1898–99.

Bangalore, India. Meteorological Results of the Observations taken at Bangalore, Mysore, Hassan, and Chitaldroog Observatories. 1896-98. By John Cook. 3 vols. 4to.

Basel.—Naturforschende Gesellschaft. Verhandlungen. Bde. XI., 3, XII., 1. 1897-98. 8vo.

Batavia.—Magnetical and Meteorological Observatory. Observations. Vols. XIX, XX.. 1896-97.—Regenwaarnemingen in Nederlandsch-Indie. 18e-19e Jaarg. 1896-97. 8vo.

Bataviaasch Genootschap van Kunsten en Wetenschappen. Verhandelingen. Deel 50, 3, 51, 1. 8vo.—Tijdschrift voor Indische Taal-Land-en Volkenkunde. Deel XXXIX., 3, 5, 6, XL., XLI., 1-4. 1898-99. 8vo.—Notulen, Deel XXXV., XXXVI. 1, 2. 1898-99, 8vo.

Kon. Natuurkundig Vereeniging. Natuurkundig Tijdschrift voor Nederlandsch-Indie. Deel 57-58. 1898-99. 8vo.

Belfast.—Natural History and Philosophical Society. Proceedings, 1896-98. 2 Vols. 8vo.

Bergen. — Museum. Aarsberetning. 1897-99. 3 Vols. 8vo. — An Account of the Crustacea of Norway. By G. O. Sars. Parts 9-14. 1898-99. 8vo.

Berlin.—K. Akademie der Wissenschaften. Abhandlungen. 1897-98. 2 Vols. 4to.—Sitzungsberichte. 1897, 40-53. 1898-99, 1-38. 4to.

Physikalische Gesellschaft. Fortschritte der Physik im Jahre 1892, 1896, 1897. 1te Abtheil. — Allgemeine Physik, Akustik. 2te Abtheil.—Optik, Wärmelehre, Elektricitätslehre. 3e Abtheil.—Physik der Erde. Berlin. 8vo.—Namenregister. Bde. XXI.-XLIII. 1865-87. 8vo. 1897-98. 2 vols. Verhandlungen, 1898-99. 8vo.

Deutsche Meteorologische Gesellschaft. Zeitschrift. 1898-99. 2 Vols. 4to.

Deutsche Geologische Gesellschaft. Zeitschrift. Bde. XLIX., 3, 4, L., LI., 1, 2. 1898-99. 8vo.

Physikalisch-Technische Reichsanstalt. Die Thätigkeit der Phys.-Technischen Reichsanstalt im 1897-98, 1898-99. 4to.

Kgl. Technische Hochschule. Programm, 1897-98, 1899-1900.—Chronik der Kgl. Technischen Hochschule. 1799-1899. 4to. Die reine Mathematik in den Jahren 1884-99. 8vo. 1899.

Bern.—Beiträge zur geologischen Karte der Schweiz. Lief. XXVIII. (Texte). Neue Folge. Lief. VIII. 1898. 4to. (*From the Commission Fédérale Géologique.*)

Naturforschende Gesellschaft. Mittheilungen. Nos. 1436-1450. 1897. 8vo.

Berwickshire.—Naturalists' Club. Proceedings. Vols. XV. 2, XVI. 1898-99. The Session Booke of Bonckle. 8vo.

Blue Hill (U.S.).—*Meteorological Observatory.* See *Cambridge* (U.S.).

Bologna.—Accademia d. Scienze dell' Istituto di Bologna.

Memorie. Ser. V., Tom. V., VI. 1895-97. 4to. Rendiconti, Nuova Serie. Vol. I., 1-4. 1898.

Bombay.—Government Observatory. Magnetical and Meteorological Observations for 1896–97. Bombay. 4to.—Brief Sketch of th Meteorology of the Bombay Presidency. 1897–98. 4to.

Bombay Branch of the Royal Asiatic Society. Journal. Vol. XIX. No. 53, XX. No. 54. 1898–99. 8vo.

Archæological Survey of Western India. Progress Report. 1897–98. 4to.

Bonn.—Naturhistorischer Verein der Preussischen Rheinlande und Westfalens. Verhandlungen. Jahrg. 54–56 (1). 1898–99. 8vo.

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Bordeaux.—Société des Sciences Physiques et Naturelles. Mémoires. 5e Série, Tomes. I.–IV. 1898–99. 4to.

Société de Géographie Commerciale. Bulletin. 1898–99. 8vo.

Bosnia-Herzegovina.—Das Veterinärwesen in Bosnien und der Hercegovina seit 1879. Sarajevo, 1899. 8vo. (*From the Government.*)

Boston.—Boston Society of Natural History. Memoirs. Vol. V. 3–5. 1898–99. 4to.—Proceedings. Vols. XXVIII., Nos. 1–16. 1898–99. 8vo.

American Academy of Arts and Sciences. Proceedings. Vols. XXXII. Nos. 15–17., XXXIII., XXXIV., XXXV., 1–3. 1898–99. 8vo.

Bremen.—Naturwissenschaftlicher Verein.—Abhandlungen. Bde. XV. 2, XVI. 1898–99. 8vo.

Brera.—See *Milan.*

British Association for the Advancement of Science.—Report of the Meetings at Toronto, 1897; Bristol, 1898. 8vo.

Brunswick.—Verein für Naturwissenchaft. Jahresberichte. 1895–98. 8vo.

Brussels.—Académie Royale des Sciences, des Lettres, et *des Beaux-Arts de Belgique.* Mémoires. Tome 53, 1898. 4to.—Mémoires Couronnés. Tomes 48, 53–55, 57. 1896–98. 8vo.—Mémoires Couronnés et Mémoires des Savants Étrangers. T. 54–56. 1896–98. 4to.—Tables Générales des Mémoires, 1772–1897. 8vo, 1898.—Bulletin. 1898. 3e Série. Classe des Sciences. Tome XXXVII. Nos. 1–8.—Classe des Lettres et des Sciences Morales et Politiques. 1899. 1–8.—Table Générale du Bulletin. 3e Sér., Tomes I.–XXX. 1881–1895. 8vo.—Annuaire, Années, 1898–99. 8vo.—Biographie Nationale, XIV., XV., 1. 1896–98. 8vo.

Observatoire Royal. Annales Astronomiques. Tome VII. 1896. —Annales Météorologiques. Tom. III., IV. 1895.—Annuaires. 1889–97. Bibliographie Générale de l'Astronomie, par J. C. Houzeau et A. Lancaster. Tom. I. 2e Ptie. 1889. 8vo.

Bucharest.—Academia Romana. Analele. Tom. XX.–XXI. 1899. 4to. —Also Publications relating to the History, etc., of Roumania. 1898–99.

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Buda-Pesth.—Magyar Tudományos Akadémia (Hungarian Academy). Mathemat. és természettud. közlemények (Communications Math. and Nat. Sciences), XXVII., 1, 3. 1899. Nyelvtud. közlemények (Philology), XXVII. 3, 4, XXVIII., XXIX. 1, 2; Mathemat. és természettud. Értesitö (Bulletin, Math. and Nat. Sciences), XV. 3, 5, XVI., XVII. 1, 2; Nyelvtudom. Értekezések (Philol. Memoirs), XVII. 1–2; Történettud. Ertekezések (Historical Memoirs), XVII., 2–10, XVIII., 1–6; Társadalmi Értekezések (Memoirs, Political Sciences), XII. 3; Archaeologiai Közlemények, XX. 1897.—Almanach, 1898–99. —Mathematische und Naturwissenschaftliche Berichte aus Ungarn. Bde. XIV.-XV. 1896–97. And other Publications of the Hungarian Academy, or published under its auspices.

Kir-Magy. Termeszettudomanyi Tarsulat. (Royal Hungarian Society of Nat. Sciences). Francé: Craspedomonadinák. Kohaut: Magyarorszag szitakötoféléi (Libellulidae Hungariae). Kurländer: Erdmagnetische Messungen in Ungarn. Szadeczky: Zempléni szigethegység geológiája.

Hungarian Ministry of Public Instruction.—Landwirtschaftliche Statistik der Länder der Ungarischen Krone. Bde. 2, 3. 1897. 4to.

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Museo Nacional. Anales. T. VI. 1899. 4to.—Communicaciones. T. I. 1–4. 1898–99. 8vo.

Calcutta.—Asiatic Society of Bengal. Proceedings. 1897–99. 8vo.—Journal (Philology, Natural History). Vols. 66, 2–4, 67, 68. (Anthropology), Vol. 67, 1–2. 68, 1. 1898–99. 8vo. Içvaru-Kaula. The Kaçmiraçabdāmrta. A Kāçmīrī Grammar written in the Sanskrit Language. Ed. by G. A. Grierson. Parts 1 and 2. 1897–98. 8vo.

Indian Museum. An Account of the Deep-Sea Madreporaria collected by the R.I.M. Survey ship 'Investigator.' By A. Alcock. 1898. 4to.

Ophiuroidea. By R. Koehler.

Brachyura. By A. Alcock.

Descriptive Catalogue of the Indian Deep-Sea Fishes, collected by R.I.M. 'Investigator.' By A. Alcock.

Royal Botanic Gardens. Annals. Vol. VIII., 1–4. Fol. 1898. See also *Indian Government.*

California.—Academy of Sciences. Proceedings. 3rd Ser. (Geology), Vol. I. Nos. 2–6; (Botany), Vol. I. Nos. 2–9; (Zoology), Vol. I. Nos. 2–12. 1898.—Occasional Papers. No. 5.—Reptiles of the Pacific Coast. 1899. No. 6. New Mallophaga. III. 1899. 8vo.

California.—University of California. Registers and Annual Reports. 1897–98.—Reports of Agricultural College. 1895–97.—Bulletin of the Geological Department. Vol. II. No. 4.—And Miscellaneous Pamphlets.

Lick Observatory. Publications. No. II. No. III. 1894–95. 4to.

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(U.S.).—Harvard College. Annual Reports. 1896–98. 4to.

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OBITUARY NOTICES.

Rev. John Wilson, M.A. By **C. G. Knott,** D.Sc.

(Read December 20, 1897.)

Rev. John Wilson, M.A., was born in Montrose on 21st November 1847. After graduating as Master of Arts of Edinburgh University, he entered the New College, and was in due course licensed as a preacher by the Free Presbytery of Edinburgh. He never, however, sought an ecclesiastical charge, but devoted himself to lightening the labours of his father, who was rector of the James Wilson Academy in Bannockburn. Latterly Mr John Wilson had entire charge of this school until, in 1887, it ceased to have a separate existence and was merged in the Public School. In addition to his work in Bannockburn he established and conducted evening science classes in Stirling long before work of this kind was taken up by Educational Boards.

He had a strong bent towards mathematical studies, and in 1876 he communicated to this Society an interesting paper "On Parallel Motions," a subject at that time fresh and fascinating. In illustration of this paper he exhibited several models neatly constructed by himself, by means of which accurately straight lines could be drawn. He was elected a Fellow in 1878.

In 1887 Mr Wilson settled in Edinburgh as a tutor in mathematics and natural philosophy, and soon acquired a sound reputation as a conscientious and efficient teacher of these subjects. His *Notes on Physics and Natural Philosophy*, an epitome of physical principles arranged alphabetically, admirably fulfilled the purpose intended, as many grateful graduates can testify.

In 1888 he became Treasurer of the Edinburgh Mathematical Society, an office which he filled with great acceptance till 1895, when he was elected Vice-President. In the succeeding year he was chosen President. He was also an active member of the Educational Institute of Scotland, his position as Secretary of the local branch entailing upon him a considerable amount of organising work. He was an ardent Free Churchman and took a keen interest in the Home Mission work of the Free High Church, in which he also served as interim session-clerk for two years.

His death on 8th December 1896, after a few days' illness, gave a great shock to his many friends, to whom he had endeared himself by his quiet, unobtrusive, but truly manly worth.

Edmund Chisholm Batten.

(Read January 31, 1898.)

During the past year (1897) the Royal Society of Edinburgh has lost one who took a lively interest in its welfare, and was, almost to the last, a constant attendant at its annual meetings.

Mr Chisholm Batten, who was a J.P. for Inverness-shire, was born in 1817, at Kingston, near Yeovil, Somerset.

He was the head boy of Sherborne School, and in 1834 proceeded to the University of Edinburgh. The *Life and Letters of Principal J. D. Forbes* (better known as Professor Forbes, the celebrated Alpine traveller, and discoverer of the theory of glaciers) tells how the young English student was the favourite pupil and life-long friend of the young Scotch Professor of Natural Philosophy.

Subsequently he was called to the English Bar, and *The Gentleman's Magazine* for 1843 records: "On August 1, at Windlesham, Edmund Batten, barrister-at-law, [was married] to Jemima, only sister of 'The Chisholm.'" On the Chisholm's death, in 1858, this lady became the representative, the heiress-at-law, of the last three chiefs, her father and her two brothers.

Edmund Batten then assumed the prefix of Chisholm, by Royal Licence, and from that time his annual visit to Scotland, kept up till 1896, was usually extended to the Highlands.

Literary tastes seem to have been inherited with his manor of Thornfalcon in Somerset, for his ancestor, Robert Batten (whose estate at Pitminster was sold to buy that manor), is credited with having written, over the initials R. B., in the *Spectator*, to his friend Sir Richard Steele.

He was an original and prominent member of the Somersetshire Archæological and Natural History Society. He contributed to it almost annually a paper on biography or history, suggested by local architecture; he thus gave interesting accounts of the foundation of various churches and schools in that part of England, which had been built and endowed by pious benefactors of former times; and traced, through the houses in which they took refuge, the similarity of the wanderings toward the south coast of Charles II. after Worcester, and of his son the Duke of Monmouth after defeat at Sedgemoor.

His connection by marriage with Inverness-shire led to his writing a work entitled *The Charters of the Priory of Beauly, with Notices of the Priories of Pluscardine and Ardchattan, and of the Family of the Founder, John Bysset* (the common ancestor of Lovat and of The Chisholm), published for the Grampian Club in 1877. He also published, in 1889, *The Register of Bishop Fox, while Bishop of Bath and Wells, with Life*, temp. *Henry VII. and Henry VIII.*

His antiquarian pursuits did not interfere with regular practice at the Bar. He wrote *A Practical Treatise on the Law relating to the Specific Performance of Contracts*; and in conjunction with Mr Henry Ludlow, *A Treatise on the Jurisdiction, Pleadings, and Practice of the County Courts in Equity.*

Mr Chisholm Batten was interested in many societies. Besides being a Fellow of the Royal Society of Edinburgh, he was a member of the Northern Meeting, the Highland Society of London, the Somersetshire Society, the Tithe Redemption Trust, the Somersetshire Discharged Prisoners Aid Society, and the Somerset Archæological and Natural History Society.

He was elected a Fellow of this Society in 1857, was one of the early members of the British Association, and was for more than fifty years a member of the Athenæum Club, 1846–1897.

Mr Chisholm Batten died at Thornfalcon, on Saturday, 13th February 1897, and was there, beside his wife, who died in 1883, in the forty-first year of their marriage, laid to rest under the shadow of the church which he had repaired from floor to roof.

J. F. C. B.

Major-General Gosset. By John Winzer, F.R.S.E.

(Read June 19, 1899.)

The Royal Society of Edinburgh has lost a distinguished Fellow in the death of Major-General Gosset, R.E., F.R.S.E.; he died at 70 Edith Road, West Kensington, on the 19th May 1899, aged 77.

Major-General Gosset had a varied and distinguished career. At the Military Academy he was an able and industrious student, so got nominated to the Corps of Royal Engineers in 1840. In 1842 he was appointed to the Ordnance Survey, and on this great national work he performed many very responsible and important duties. He was employed as Trigonometrical and Astronomical Observer, and assisted, in 1844, at the great longitude operations at Valentia, in Ireland; and the Astronomer-Royal, Professor Airey, wrote of him: "Lieutenant Gosset is a most zealous and able officer, and how fortunate for the satisfactory termination of the work was the choice of the Valentia Observer." From observing duties Lieutenant Gosset was placed in command of a Detail Survey Division for the surveying of counties in Scotland, and made surveys of Wigtown and Kirkcudbright, and commenced the surveying of the City and County of Edinburgh, but before finishing the latter work he was appointed Executive Officer at the Ordnance Map Office, Southampton, a post he could not have filled without possessing varied qualifications and marked ability. It was whilst superintending the surveying of the City of Edinburgh, in 1850, that Captain Gosset was elected a Fellow of the Royal Society of Edinburgh. In 1855 Captain Gosset was selected to fill the important post of Survey-General in the Island of Ceylon, a position in which he displayed great administrative ability, and rendered valuable services to the island. He found the Survey Department weak in numerical strength and inefficient, and in three years, under Sir Henry Ward's approval, he reorganised and added over twenty able assistants, most of whom were from the Ordnance Survey, to his staff, and left the department sufficiently strong and efficient to meet the great and growing demands for Crown lands and surveys, and for increasing the revenue from sales of Crown lands from £6000 to £37,000, which went on increasing to £70,000 per annum, at the same time, relatively to the amount of work done, reducing the operative expenses by many thousands of pounds per annum. The Governor of Ceylon, Sir Henry Ward, himself a very able and indefatigable man, wrote in his message to the Legislative Council: "Captain Gosset was a man of great ability and untiring industry, and he was eminently practical."

Severe labour in a tropical climate compelled Captain Gosset, in 1858, to seek change of climate and rest in England, and his inability to resume his post in Ceylon led him to accept an appointment in British Columbia, where he filled the offices of Treasurer, Commissary, Postmaster-General, Master of the Mint, and Director of the Gold Assay Department, giving great satisfaction in all departments to the Home Authorities.

On promotion, Colonel Gosset returned to the duties of his corps, and was Commanding Royal Engineers in Ireland, Plymouth, and Woolwich, until he attained the rank of Major-General, and retired from the army at the end of 1873. At this time Major-General Gosset was too young for a totally inactive life being congenial to him, so he obtained an appointment as Inspector of Art Schools, under Science and Art Department, South Kensington Museum, and continued to fill this post until the age clause compelled him to retire at the end of 1894. As inspector, Major-General Gosset did more than satisfy the Science and Art Department.

Recapitulation of Major-General Gosset's official services during a period of fifty-four years, 1840 to 1894: rising in military rank from Lieutenant to Major-General; measuring the heavens and the earth on the great national work of the Ordnance Survey; filling the position of Surveyor-General in the Island of Ceylon; Treasurer, Commissary, Postmaster-General, Master of the Mint, and Director of the Gold Assay Department in British Columbia; Commanding Royal Engineers at the chief military stations in Great Britain; and Inspector of Art Schools under Science and Art Department, South Kensington Museum.

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